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Maximum Likelihood Detection of Low Rate Repeat Codes in Frequency Hopped Systems

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Maximum Likelihood Detection of Low Rate Repeat Codes in Frequency Hopped Systems

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***Abstract*—In time diversity wireless and satellite communication, multiple copies of the same signal segment are transmitted during different time intervals to improve signal detection. If the communication system is frequency hopped, coherent detection is infeasible. In the traditional approach to deal with this problem, the receiver uses only the reference signals for phase shift correction and combines soft symbol decisions obtained independently from each copy. In this paper, we develop the corresponding theoretical maximum likelihood (ML) detection problem, its solution, and a computationally efficient algorithm that is an approximate ML solution. We present several simulation experiments and results. The experiments include phase drifts allowable in practical systems and randomized variations in the locations of reference symbols. Results indicate power savings of up to 2.0 decibel (dB) over the traditional method for different system configurations. They also show that for short data segments used in systems over severely degraded channels, the results from the theoretical solution and our algorithm are virtually indistinguishable. The main impact of this research is that survivable and protected communication systems can take advantage of our new signal combining algorithm that offers considerable power savings.**

Keywords—diversity communication; joint phase estimation and detection; maximum likelihood; signal combining; frequency hopping; repeat codes

I. INTRODUCTION

Satellite, airborne, and terrestrial wireless communications through severely degraded and/or contested channels require the use of low rate coding. A practical approach to achieving such low rate codes is to first construct forward error correction (FEC) codes of half or one-third rate and then transmit multiple copies of these code words. Protected communication systems employ frequency hopping and the dwell time of each carrier frequency is too short for carrier phase acquisition. This prevents coherent demodulation [1] at the receiver. The symbol sequence transmitted over a dwell time is called a hop. A hop includes reference symbols of known values in addition to the data symbols. In severely degraded channel conditions, the system uses larger and larger symbol transmission times to increase the signal to noise ratio (SNR). Under such conditions, a hop would contain only a modest number (for example, a few tens) of data symbols and a few reference symbols. Transmitted hops are subjected to unknown phase shifts during radio propagation. Received hops should be demodulated and their soft decisions input to the decoder. Typically, the demodulator uses a local oscillator with an arbitrary phase and outputs a complex valued number for each symbol. We refer to each of these complex numbers (corresponding to a symbol) as a signal. The rest of the receiver should be designed to deal with the unknown arbitrary phase shift in the sequence of such signals produced by the demodulator. Noise on reference symbols introduces errors in the estimation of such phase shift. It is important to minimize this effect through the proper use of all of the received reference and data signals in a hop.

Multiple copies of a hop are subjected to independent unknown phase shifts and additive noise. At a first glance, it might appear that the best approach for the receiver to deal with two copies is to apply the ML phase shift and detection estimation algorithm due to Mackenthun [2] separately for each of the two hops, obtain the sequences of symbol or bit log likelihood ratios (LLRs), and then add the LLRs. The following argument shows that such an approach is suboptimal. When we apply the ML algorithm separately on the two copies, the algorithm allows for the detected data symbols at a particular position (say the seventh position in the two hops) to be different. Clearly, this is suboptimal and a good algorithm should constrain that the symbol detections at the corresponding positions of the two hops be the same. In this paper, we study joint ML phase estimation and detection of multiple copies of a hop.

Some background and related literature are reviewed in Section II. Section III is devoted to the development of the ML signal combining problem and its solution. The global objective function for c copies of an M-ary Phase Shift Keyed (MPSK) hop with n data symbols and r reference symbols in each copy is developed and simplified. Complexities of the theoretical ML solution are discussed. Our signal combining algorithm, which is an approximation to the ML solution, is developed in Section IV. Section V presents several simulation experiments and results. The experiments include different burst modes (different numbers of data and reference symbols in a hop), different types of modulation, phase drift caused by local oscillator frequency error, and randomized locations of reference symbols within a hop. Section VI concludes the paper with highlights of the results and the impact.

II. BACKGROUND AND RELATED LITERATURE

Consider a coherent communication system [1] that receives two noisy signal segments corresponding to identically transmitted copies. Let E_s be the energy per data symbol at the receiver in each copy and let N_1 and N_2 be the spectral densities of the additive white Gaussian noise (AWGN) corrupting the two copies, respectively. If the corresponding noise standard deviations are σ_1 and σ_2, combining the two signal segments is through weighted averaging, with weights proportional to $\frac{1}{\sigma_1}$ and $\frac{1}{\sigma_2}$ (Chase [3]). If $\sigma_1 = \sigma_2$, the process of combining is simply averaging the two signal sequences before obtaining soft decisions. In this case, the effect of transmitting two copies of a data segment is the equivalent to having transmitted only the original segment (and no copy) at double (or 3 dB above) the power. As mentioned earlier, wireless and satellite communication systems that use frequency hopping cannot implement coherent demodulation, and the transmitted hops include known reference symbols to help with demodulation and to obtain soft decision. Consider MPSK systems with n data symbols and r reference symbols in each hop. Assume that all the reference symbols are transmitted with a zero phase.[1] The simplest approach to estimate the phase shift is to average the r received reference signals. A simple way to imagine the joint maximum likelihood phase estimation and data symbol detection is to construct the likelihood function for each of all the M^n possible symbol sequence detections and select the one with the highest likelihood function. If necessary, we can then obtain the best estimate of the phase shift by assuming that the best sequence detection is correct. An alternative approach for joint ML detection and estimation is to consider all possible values of phase shifts, evaluate the sequence-detection and the corresponding likelihood function for each case of the phase shift, and then select the best sequence detection. In this procedure, as we increase the candidate phase shift variable, corresponding symbol decision boundary radii rotate. When (and only when) a boundary crosses a data signal, the detection of only that symbol changes. This induces a change in the behavior of the likelihood function at every value of the phase shift variable at which a data signal falls on one of the decision boundary radii. The continuous variation of phase shift variable between two successive decision boundary crossings is irrelevant. This leads to an efficient algorithm of computational complexity $n \log n$ developed by Mackenthun [2] and independently by Sweldens [4]. Motedayen-Aval and Anastasopoulos [5] generalize this result and identify a class of problems for which the hard or soft symbol sequence decisions in the presence of unknown parameters can be exactly evaluated with only polynomial complexity with respect to the sequence length.

III. ML SIGNAL COMBINING WITH UNKNOWN PHASE SHIFTS

Consider the problem of sequence detection from two copies of a hop received with unknown and independent phase shifts and AWGN. The joint probability density function we need to maximize for signal combining of two copies is

$$\frac{1}{\pi^{2(n+r)}\sigma_1^{2(n+r)}\sigma_2^{2(n+r)}} \exp\left[-\frac{1}{\sigma_1^2}\sum_{i=1}^{n+r}\left\|a_i e^{jb_i} - e^{j(\theta_i+\varphi_1)}\right\| - \frac{1}{\sigma_2^2}\sum_{i=1}^{r+r}\left\|c_i e^{jd_i} - e^{j(\theta_i+\varphi_2)}\right\|\right]. \tag{1}$$

In the above probability density function, the sequences $\{a_1, \cdots, a_{n+r}\}$ and $\{b_1, \cdots, b_{n+r}\}$ are the magnitude and phase parts of the first copy of the received hop. The sequences $\{c_1, \cdots, c_{n+r}\}$ and $\{d_1, \cdots, d_{n+r}\}$ are those of the second copy of the hop. σ_1^2 and σ_2^2 are the variances of the complex AWGN in the two hops, respectively. θ_i is the phase angle of the data symbol i. If these two variances are different, we can normalize the magnitudes of the signals in the received hops. Therefore, we can assume without loss of generality that $\sigma_1^2 = \sigma_2^2 = \sigma^2$. The phase angles φ_1 and φ_2 are the unknown phase shifts in the two hops. As earlier, there are r reference symbols in each hop. The optimization problem becomes

$$\min_{\varphi_1,\varphi_2,\{\theta\}}\left\{\sum_{i=1}^{n+r}\left\|a_i e^{jb_i} - e^{j(\theta_i+\varphi_1)}\right\| + \sum_{i=1}^{n+r}\left\|c_i e^{jd_i} - e^{j(\theta_i+\varphi_2)}\right\|\right\}, \tag{2}$$

where $\boldsymbol{\theta}$ is the sequence $\theta_1, \ldots \theta_n$ of phase angles for the n data symbol decisions and $\{\boldsymbol{\theta}\}$ is the set of phase angle sequences of all possible data sequence decisions. With this background on two copies of a hop, we can generalize to multiple copies. Now, let there be c copies of hops. Let each hop contain n data symbols and r reference symbols for a total of $n + r$ symbols per hop. The order of interleaving the data symbols and reference symbols within the hop is irrelevant. We just need to be able to rearrange them at the receiver to restore the same order for both the hops. Therefore, we assume without loss of generality that the sequences of transmitted symbols in all the copies of a hop are identical. Let the magnitude of the i-th data signal in the j-th received copy of the hop be a_{ij} and the phase angle of the same be b_{ij}. The unknown phase shift of the j-th copy is φ_j. The magnitude of the i-th reference signal in the j-th copy of the hop is s_{ij} and the phase angle of the same is t_{ij}. We can express the joint density function as earlier and mathematically manipulate the expression that is required to be

[1]In practice, each reference symbol can be any valid symbol. The receiver is required to know the exact value of the reference symbol used at each location. The choice of the reference symbol does not affect the BER.

maximized. After considerable manipulation, we obtain the following expression for optimization.

$$\max_{\boldsymbol{\varphi},\{\boldsymbol{\theta}\}} f(\boldsymbol{a},\boldsymbol{b},\boldsymbol{s},\boldsymbol{t},\boldsymbol{\varphi},\boldsymbol{\theta}) = \max_{\boldsymbol{\varphi},\{\boldsymbol{\theta}\}} \sum_{j=1}^{c} \left[\left\{ \sum_{i=1}^{n} a_{ij} \cos(\varphi_j - b_{ij} + \theta_i) \right\} + \left\{ \sum_{i=1}^{r} s_{ij} \cos(\varphi_j - t_{ij}) \right\} \right]. \quad (3)$$

On the left hand side (LHS) of the above equation, the bold letters represent the sequences (or vectors) corresponding to the subscripted variables found on the right hand side (RHS). This optimization problem has c continuous phase angle variables, φ_j, each varying over $[0,\ 2\pi)$ radians, and n discrete variables θ_i, each of which takes one of the M phase angles in the MPSK system.

As in the case of optimization for only one copy of a hop, for any candidate phase vector parameter $\boldsymbol{\varphi}$, there exists a correspondingly best sequence of symbols decisions, $\boldsymbol{\theta}^*(\boldsymbol{\varphi})$ and these are straightforward to evaluate, separately, for each data signal at index $i = 1, \cdots, n$. That is, θ_i^* is obtained by optimizing as follows:

$$\max_{\theta_i} f_i(\boldsymbol{\varphi}, \theta_i) = \max_{\theta_i} \sum_{j=1}^{c} a_{ij} \cos(\varphi_j - b_{ij} + \theta_i). \quad (4)$$

Note that if the phase shift parameter $\boldsymbol{\varphi}$ is given, the reference signals do not influence the symbol detections. Alternatively, if we are given a sequence of symbol decisions $\boldsymbol{\theta}$, we can evaluate the correspondingly best reference phase vector $\boldsymbol{\varphi}^*$, with components φ_j^*, $j = 1, \cdots, c$. The objective function with known θ_i values and unknown φ_j is mathematically manipulated and this results in

$$f(\boldsymbol{\varphi}) = \sum_{\boldsymbol{j=1}}^{\boldsymbol{c}} \left\{ \cos(\varphi_j) \left[\sum_{i=1}^{n} a_{ij} \cos(b_{ij} - \theta_i) + \sum_{i=1}^{r} s_{ij} \cos(t_{ij}) \right] + \sin(\varphi_j) \left[\sum_{i=1}^{n} a_{ij} \sin(b_{ij} - \theta_i) + \sum_{i=1}^{r} s_{ij} \sin(t_{ij}) \right] \right\}. \quad (5)$$

Let

$$A_j = \left[\sum_{i=1}^{n} a_{ij} \cos(b_{ij} - \theta_i) + \sum_{i=1}^{r} s_{ij} \cos(t_{ij}) \right], \quad (6)$$

$$B_j = \left[\sum_{i=1}^{n} a_{ij} \sin(b_{ij} - \theta_i) + \sum_{i=1}^{r} s_{ij} \sin(t_{ij}) \right], \quad (7)$$

$$C_j = \sqrt{A_j^2 + B_j^2}, \quad (8)$$

and ψ_j uniquely defined by

$$\sin(\psi_j) = \frac{B_j}{C_j}, \text{ and} \quad (9)$$

$$\cos(\psi_j) = \frac{A_j}{C_j}. \quad (10)$$

We now have

$$f(\boldsymbol{\varphi}) = \sum_{j=1}^{c} C_j \cos(\varphi_j - \psi_j). \quad (11)$$

Maximizing $f(\boldsymbol{\varphi})$ is now very simple. The solution is given by

$$\varphi_j^* = \psi_j, \qquad j = 1, \cdots, c. \quad (12)$$

Now, consider the behavior of the objective function over the range of all possible c phase shift variables, the c-dimensional hypercube with each edge of the cube extending over $[0, 2\pi)$. This hypercube is implicitly partitioned into many sub-regions such that the best sequence detection is the same for all phase angle vectors in a given sub-region. The boundaries of these sub-regions are nonlinear (as opposed to being hyper-planes). The objective function is continuous everywhere in the hypercube, but it is not differentiable at points on the hyper-surface boundaries separating these sub-regions. In general, some of these (at least one) sub-regions have a point each at which all partial derivatives of the objective function vanish. If we start with any point in such a sub-region, evaluate the correspondingly best data symbol detections, and re-estimate the phase shift corresponding to these detections, the result is the point with zero partial derivatives and a local optimum. If we start from any point in a sub-region that does not have a point with zero partial derivatives, evaluate the correspondingly best data symbol detections, and re-estimate the phase shift vector based on the detections, we would have moved out of the sub-region into a different sub-region. This can be confirmed by re-evaluating the best detections for the re-estimated phase shift vector and comparing it with the detections in the first iteration. Proofs of these properties of the objective function follow as extensions to corresponding properties in the case of one copy and are omitted to satisfy page limitations. Now, the global optimization procedure is easy to conceive, as follows:

1. Begin with an empty list of local optima.
2. Identify and list one phase shift vector inside each sub-region of the partition of the c-dimensional hypercube, the feasible region of phase shift vectors.
3. Pick a phase shift vector from the list in Step 1. Evaluate the correspondingly best data sequence detection. Re-estimate the phase shift vector for the identified data sequence detection and re-evaluate the sequence detection for the re-estimated phase shift.
   a. If the two sequence detections are not identical, the sub-region does not contain a local optimum. Discard the results over this sub-region.
   b. If the two sequence detections are identical, we have a local optimum. Insert it into the list of local optima.
4. Repeat Step 3 for each phase shift vector in the list created in Step 2.

5. Search the list of all local optima and select the global optimum. Output the corresponding sequence joint ML detection and phase shift vector estimate.

The difficulty with this ML solution is that the boundaries separating the sub-regions are curved hyper-surfaces as mentioned above and are computationally cumbersome to evaluate. Therefore, identifying one phase shift vector inside every sub-region in Step 2 is an impediment to implementing the ML solution. To keep the computations at an acceptable level, we propose the following algorithm as an approximation. Later on, we point out through simulation results that this algorithm is a very nearly globally optimal for the application domain under consideration.

IV. THE SIGNAL COMBINING ALGORITHM

A local initial point for the phase shift parameter vector is obtained by maximizing the sum of components in the objective function corresponding to the reference signals and one particular data signal. That is, let φ_{ij} represent the initial phase estimate for the j-the copy based on data signal i only. This is determined by maximizing as follows:

$$h_i = \max_{\varphi_{ij},\theta_k} \sum_{j=1}^{c} \left\{ a_{ij}\cos(\varphi_{ij} - b_{ij} + \theta_k) + \sum_{l=1}^{r} s_{ij}\cos(\varphi_{ij} - t_{lj}) \right\}. \quad (13)$$

The above maximization is easily carried out by manipulating the argument as a linear combination of $\cos(\varphi_{ij})$ and $\sin(\varphi_{ij})$ as follows. Let

$$h_{ik} = \sum_{j=1}^{c} \left\{ a_{ij}\cos(\varphi_{ijk} - b_{ij} + \theta_k) + \sum_{l=1}^{r} s_{ij}\cos(\varphi_{ijk} - t_{lj}) \right\} \quad (14)$$

$$= \sum_{j=1}^{c} \cos(\varphi_{ijk}) \left(a_{ij}\cos(b_{ij} - \theta_k) + \sum_{l=1}^{r} s_{ij}\cos(t_{lj}) \right) + \sin(\varphi_{ijk}) \left(a_{ij}\sin(b_{ij} - \theta_k) + \sum_{l=1}^{r} s_{ij}\sin(t_{lj}) \right). \quad (15)$$

Let

$$A_{ijk} = \left(a_{ij}\cos(b_{ij} - \theta_k) + \sum_{l=1}^{r} s_{ij}\cos(t_{lj}) \right), \quad (16)$$

$$B_{ijk} = \left(a_{ij}\sin(b_{ij} - \theta_k) + \sum_{l=1}^{r} s_{ij}\sin(t_{lj}) \right), \quad (17)$$

$$C_{ijk} = \sqrt{A_{ijk}^2 + B_{ijk}^2}, \quad (18)$$

and ψ_k, uniquely determined by

$$\cos(\psi_{ijk}) = \frac{A_{ijk}}{C_{ijk}}, \quad \text{and} \quad (19)$$

$$\sin(\psi_{ijk}) = \frac{B_{ijk}}{C_{ijk}}. \quad (20)$$

Then,

$$h_{ik} = \sum_{j=1}^{c} C_{ijk}\cos(\varphi_{ijk} - \psi_{ijk}), \quad (21)$$

$$h_i = \max_{\varphi_{ij},k} \sum_{j=1}^{c} C_{ijk}\cos(\varphi_{ijk} - \psi_{ijk}). \quad (22)$$

Let $k = k^*$ maximize h_{ik}. Then the j-th component of the local reference phase vector estimate corresponding to data signal i is given by

$$\varphi_{ij} = \psi_{ijk^*}. \quad (23)$$

The initial vector of phase shifts corresponding to data signal i is given by

$$\boldsymbol{\Phi}_i = (\varphi_{i1}, \cdots, \varphi_{ic}), \;\; i = 1, \cdots, n. \quad (24)$$

These $\boldsymbol{\Phi}_i$, $i = 1, \cdots, n$ now constitute the list of all the initial phase vectors replacing the list in Step 2 of the globally optimum ML signal combining solution developed in Section 3. This completes the development of our algorithm for signal combining.

V. SIMULATION EXPERIMENTS AND RESULTS

We conducted extensive simulation experiments and compared the performances of a few methods for phase estimation, detection, and signal combining. The first of these is the traditional method that uses only the reference symbols for phase correction and evaluates soft bit decisions separately in each copy and then adds them for signal combining. The second method uses the ML algorithm for phase estimation and soft decision evaluation independently for each copy and then adds them for signal combining. Finally, our new joint ML-based signal combining algorithm is simulated. The above algorithms are applied to three configurations of a hop. These are the 5+9 mode which has 5 reference symbols and 9 data symbols per hop, the 2+12 mode, and the 3+54 mode. Recently, we obtained virtually identical numerical results (as from the above joint ML-based algorithm) with a more efficient algorithm that matches phase shifts before joint phase estimation and detection.

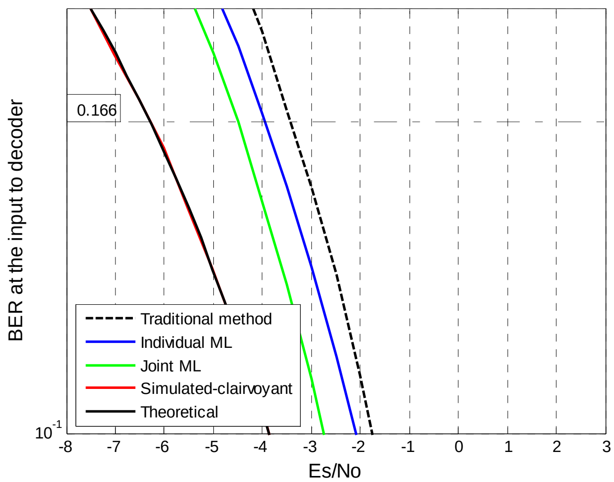

Figure 1. Comparison of BERs for 4 copies, QPSK, 5+9

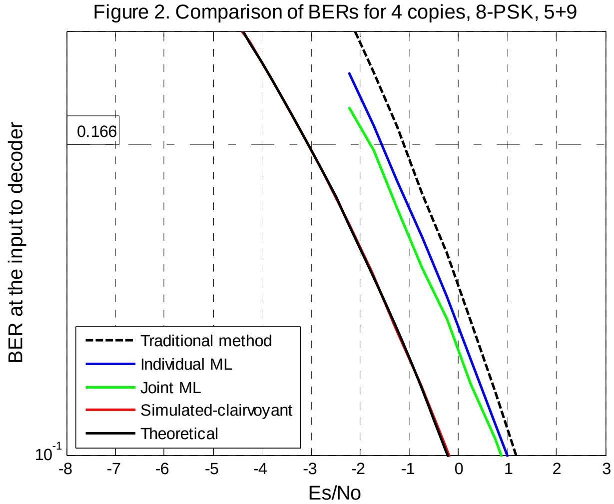

Figure 2. Comparison of BERs for 4 copies, 8-PSK, 5+9

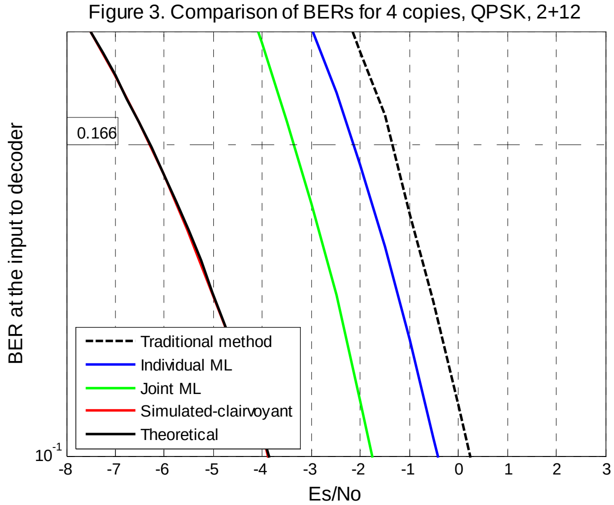

Figure 3. Comparison of BERs for 4 copies, QPSK, 2+12

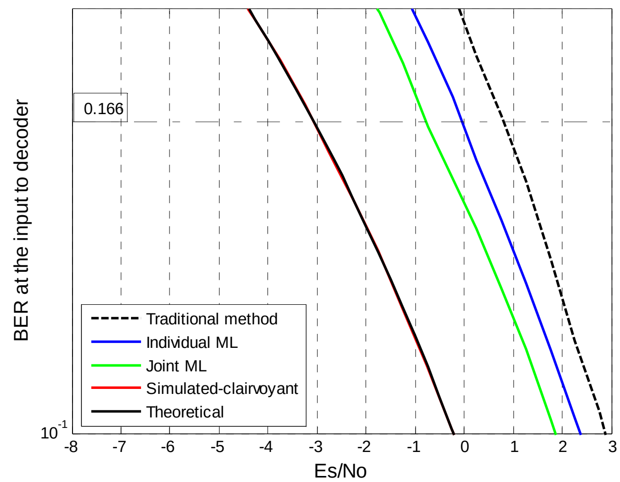

Figure 4. Comparison of BERs for 4 copies, 8-PSK, 2+12

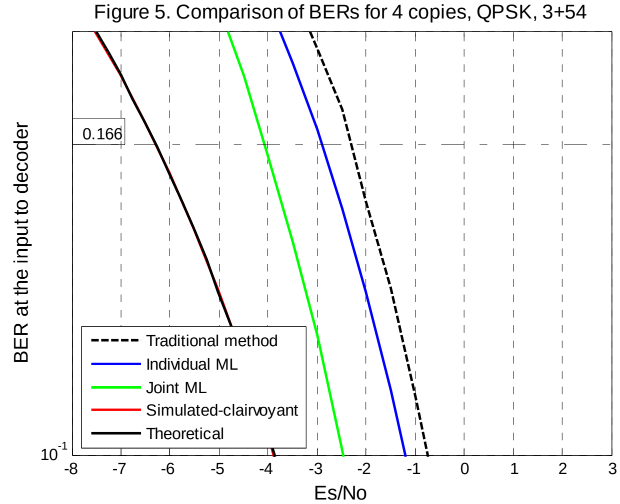

Figure 5. Comparison of BERs for 4 copies, QPSK, 3+54

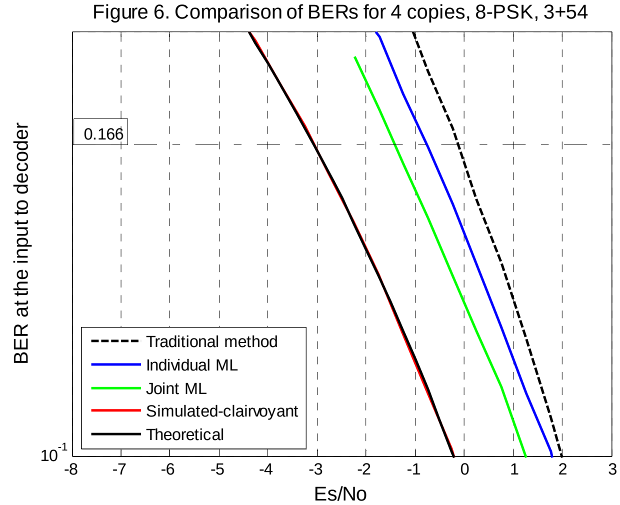

Figure 6. Comparison of BERs for 4 copies, 8-PSK, 3+54

We experimented with quadrature phase shift keying (QPSK) and 8-PSK (phase shift keying with eight symbols in the constellation) modulation schemes corresponding to 2 and 3 bits per symbol. These experiments are first conducted with an accurate carrier frequency and a uniformly distributed random phase shift. That is, in this case, there is no phase drift over the successive symbols in a given hop and the location of reference symbols is irrelevant. In another set of experiments, we added a phase drift for each hop to simulate a frequency error. In different hops, the total phase drift over the entire hop is uniformly distributed between $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$, a nominally acceptable range in military satellite communication (MILSATCOM). The reference symbols also experience phase drift and so their locations affect the detection performance. To account for this, the locations of reference symbols are randomized over the entire hop. Indeed, their locations over multiple copies of the same hop are independently chosen at the transmitter (but they are known at the receiver).

In our simulation experiments, all the data symbols in every original hop are generated statistically independently and with equal probabilities. These are considered to be segments of one-third rate FEC encoded and interleaved code words. The soft decision outputs from the signal-combined detection procedure would be the input to the FEC decoder. The phase shift experienced by every hop is simulated as independent random numbers in the range $[0, 2\pi)$. All the copies of a hop are subjected to AWGN with the same noise power spectral density. In order to illustrate all the capabilities of our joint ML-based signal combining algorithm, we plot results for the experiments with phase drifts and randomized locations of reference symbols. Results of the other signal combing methods in the plots are based on no phase drift. The locations of reference symbols are irrelevant when there is no phase drift. Results for combining four copies are plotted in Figures 1 through 6. We conducted experiments with two copies also and some summary results of these experiments are given in Table 1.

The horizontal axis represents the ratio of the energy in each data symbol of each copy to the noise power spectral density. This is denoted by $\frac{E_s}{N_o}$. The vertical axis represents the bit error rate resulting from the different signal combining algorithms. Soft bit decisions from the signal-combined detection will be input to the FEC decoder. The bit error rate (BER) required at the input to the decoder should be 0.166 or less in order to result in a BER of 10^{-5} or better at the output of the on-third rate decoder. The range of $\frac{E_s}{N_o}$ over which the results plotted is the same for all the experiments and is chosen to highlight the plots around the BER value of 0.166 for each plot. Table 1 below lists the savings in dB that our signal combining algorithm offers over the power that would be required for the traditional signal combining method that uses only reference symbols for phase shift correction, for the case of BER = 0.166. The columns listed as "Indep. ML" shows the power savings over the traditional method for the case of using the ML method separately for each copy before adding soft decisions. The best entry in the table is 2 dB for four copies in the QPSK 2+12 mode.

Table 1. Power savings in dB over the traditional method at BER = 0.166

Mode	Two copies		Four copies	
	Indep. ML	Joint ML	Indep. ML	Joint ML
QPSK 5+9	0.0	0.4	0.7	1.1
8-PSK 5+9	0.1	0.1	0.7	0.7
QPSK 2+12	0.1	0.9	1.2	2.0
8-PSK 2+12	0.1	0.4	1.3	1.6
QPSK 3+54	0.0	0.7	1.0	1.8
8-PSK 3+54	-0.1	0.3	0.9	1.3

VI. CONCLUSION

Low rate repeat codes are useful for satellite communication in severely contested environments. Frequency hopping is often employed in these systems for protection. The hypothetically best gain in $\frac{E_s}{N_0}$ that can be realized by retransmitting once after an original hop is transmitted is 3 dB and this requires coherent detection which is not feasible in FH systems. In our simulation experiments, we observed a gain of only 2.3 dB instead of the 3 dB for the QPSK 5+9 mode. For four copies in the same mode, the gain is only 4 dB instead of 6 dB. In severely contested environments, recovering even a fraction of a dB with a better signal combining algorithm is very valuable. In response to this requirement, we have developed a new signal combining algorithm that is based on the maximum likelihood approach. Our results show an improvement of up to 2 dB over the traditional method of signal combining for different burst modes. The signal combining algorithm is very robust against allowable local oscillator frequency error and randomized locations for reference symbols. The latter feature is very useful in preventing a prospective specialized jammer from concentrating its power on reference symbols only.

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