MTR 98W0000153

MITRE TECHNICAL REPORT

The Architecture of CAPE Models

March, 2000

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Dept. No.: W062 **Project No.:** 51MSR8CA-AA

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Section 1

Introduction

1.1 Purpose of the Report

1.1.1 Background of the Report

MITRE is developing a methodology for analyzing the performance and cost of future national command-control and information systems, emphasizing so-called sensor to shooter systems. The methodology is named CAPE, which stands for *C4ISR analytic performance evaluation.* The CAPE methodology supports MITRE in its role as a systems engineer for national C4ISR systems.[1]

CAPE involves the extensive use of mathematical models and computers to estimate system performance and cost. Though many MITRE project briefings and reports describe the results of CAPE modeling, heretofore MITRE has not provided general documentation of how its staff constructs and uses mathematical models in applying the CAPE methodology.

1.1.2 Objective

This report provides general documentation of the purposes, technical basis, and fundamental methods of CAPE modeling.

1.1.3 Anticipated Benefits

The report will help MITRE s sponsors to understand the technical basis of CAPE modeling. It will clarify the CAPE methodology for them, create a common terminology and framework for discussing C4ISR modeling with MITRE staff, and provide them the information necessary to judge the general suitability of CAPE models for specific analyses. The report will also serve MITRE s own staff for internal training in CAPE modeling and for the development of databases and libraries to support the CAPE methodology. It will facilitate communication between the builders of CAPE models and their MITRE colleagues.

1.2 Organization of the Report

The report covers the following topics:

1. *Introduction* — Purpose of the report, organization, background, related documents

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[1] C4ISR stands for Command, Control, Communications, Computers, Intelligence, Surveillance, and Reconnaissance. See the Glossary for further information.

2. *Overview of the CAPE Methodology* — Purpose, scope, and uses of the CAPE methodology; distinctive principles of CAPE modeling; existing CAPE models; hardware and software for CAPE models
3. *The Inputs and Outputs of CAPE Models*
4. *Modeling Operational Processes with CAPE* — Characteristic C4ISR processes and how they are represented in CAPE models
5. *APPENDICES* — Justification of the CAPE Principles, Deriving a Model of Target Movement, Glossary

1.3 The Origin of CAPE Modeling

In the late 1980s, Henry A. Neimeier began developing a collection of useful analytic techniques for modeling system performance, applying them in many MITRE projects.[2] The techniques include methods for modeling analytic queues and using analytic risk evaluation, as well as other methods described below. They also include techniques for implementing such modeling with two software packages named Analytica (originally named Demos) and Extend.

In the mid-1990s, Roy C. Evans, Jr., originated a continuing and expanding series of top-level C4ISR investment analyses for MITRE s sponsors. He built the first mission-oriented analytical model, CAPSTONE, using Demos. As input to Demos, he used Extend to calculate C4ISR system-level estimates of technical performance.

Evans then enlisted Neimeier to bring his analytic system modeling techniques to bear on particular signals-intelligence system-design issues in the context of national and theater imagery and other ISR capabilities. The analysis that resulted was named Sensor to Shooter. Neimeier first used Extend simulations for this work. He later transferred the simulation model to Demos.

In 1997, under Evans s direction, several members of MITRE s staff, including Neimeier, were using analytic system modeling techniques and Analytica to model different aspects of the performance of national surveillance and reconnaissance systems. It was then that the name CAPE originated, at Dr. Russell Richards's suggestion, as an umbrella phrase to signify the general modeling methodology the analysts were using and applying to C4ISR studies. The analysts work was part of the Department of Defense s C4ISR Mission Assessment study (CMA). In addition, new models of battlespace awareness (BAM) and air/land/maritime force engagement (ALMEM) were created and first used at about this time. The CMA and its follow-on studies had participants from many Defense Services and

2 See the bibliography in the section immediately below. Also see Analytic in the Glossary.

Agencies, as well as from several Federally Funded Research and Development Centers and other Defense contractors.

During the CMA study, analysts at both MITRE and the Secretary of Defense s C4ISR Decision Support Center (DSC) embraced the CAPE methodology when they found that current military operations research models and approaches would not satisfy their analytic needs. They needed tools to explore a large set of C4ISR study questions quickly, identify key factors, compare alternative hypothetical combinations of new C4ISR capabilities in terms of mission effectiveness, and to provide an analytic umbrella connecting the results of detailed system level analyses in a sensible, mission-oriented context. These analysts knew that the existing models either did not address the C4ISR domain explicitly, addressed it poorly, or they required too much time and effort for data collection and analysis.[3]

1.4 Related Documents

1.4.1 Technical Papers on Techniques of Modeling

Analytic queue

- Neimeier, H. A., A New Paradigm for Modeling the Precision Strike Process (Unclassified), in the *Proceedings of the 1996 IEEE Military Communications Conference* (SECRET)
- Neimeier, H. A., Analytic Network Queuing, a paper presented in Stirling, Scotland, at the International System Dynamics Conference, 1994.

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3 Here is the basis for these findings: Admiral Owens, Vice Chairman of the Joint Chiefs of Staff, asked the Military Operations Research Society to conduct a workshop on Joint Warfare Capability Assessments (JWCAs) in October 1995. Dr. Stuart Starr of MITRE chaired the C2IW working group. That working group concluded that the tools to support JWCAs in the area of C2IW were deficient. Admiral Owens then created the C4ISR Decision Support Task Force to look at current US capabilities to do C4ISR studies. Dr. Russell Richards of MITRE chaired the working group on assessing the capabilities of current tools. His working group found that the tools (models and simulations) built by the Services had a stovepipe orientation and most had major deficiencies with respect to supporting Joint assessments of C4ISR. The working group produced a paper summarizing the input from 80 or so organizations (government, FFRDC, and contractors). In all, a total of over 700 tools were identified as being used to support C4ISR analyses. However, the working group s assessment was that there were many serious deficiencies for supporting JWCAs, JROC decisions, QDRs, and so forth. The DSTF recommended the establishment of the C4ISR Decision Support Center and the C4ISR Joint Battle Center to provide core organizations with a focus on joint studies of C4ISR. Both organizations were created.

Analytic risk evaluation

- Neimeier, H. A., Analytic Uncertainty Modeling Versus Discrete Event Simulation, *Phalanx,* March 1996.
- Neimeier, H. A., Use of Response Surfaces and Utility Functions in Nonlinear Multiattribute Risk Simulation, Sixth AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Bellevue, Washington, 4—6 September 1996.
- Neimeier, H. A., Analytic Uncertainty Modeling, a paper presented in Stirling, Scotland, at the International System Dynamics Conference, 1994.
- Neimeier, H. A., Nonlinear Multiattribute Risk Simulation, unpublished paper, no date
- Neimeier, H. A., Nonlinear Multiattribute Risk Simulation Using Response Surfaces, unpublished paper, no date

1.4.2 Technical Papers on CAPE Analyses

- Parker, S. K., *Modeling the Collection and Exploitation Trade-Off in the C4ISR Mission Assessment Study,* MITRE Technical Report 98W0000042, May 1998.
- Belldina, J. S., Neimeier, H. A., Pullen, K. W., Tepel, R. C., *An Application of the Dynamic C4ISR Analytic Performance Evaluation (CAPE) Mode,.* MITRE Technical Report 98W0000004, December 1997.
- Richards, R. F., Neimeier, H. A., Hamm, W. L., Alexander, D. L., Analytical Modeling in Support of C4ISR Mission Assessment (CMA), a paper published in the proceedings of the International Command and Control Conference, 1997.
- Neimeier, H. A., A New Paradigm for Modeling the Precision Strike Process, a paper presented at MILCOM 96.
- Neimeier, H. A., Precision Strike Process for Mobile Targets, 64th MORS Symposium, Fort Leavenworth, Kansas, 18—20 June 1996.

1.4.3 Briefings on CAPE Topics

- Kuskey, K. P. and Tepel, R., A High-Level Model of Target Location, Movement, and Engagement, 67th MORS Symposium, West Point, New York, 24 June 1999.
- Belldina, Jeremy, Weapon/Target Pairing Algorithm Used in Dynamic CAPE, INFORMS Military Applications Society Conference, Huntsville, AL, May 1998.

- Neimeier, H. A., C4ISR Analytic Performance Evaluation (CAPE) Models, INFORMS Military Applications Society Conference, Huntsville, AL, May 1998.
- Parker, S. K., Modeling the Performance of Imagery Exploitation Systems, INFORMS Fall Conference, Dallas, Texas, October 1997.
- Neimeier, H. A., Precision Strike Attack Process for Mobile Targets, 64th MORS Symposium, Fort Leavenworth, Kansas, 18—20 June 1996.

1.4.4 Other Documentation of CAPE Models

- Rueda, Catalina, Documentation of the Dynamic CAPE Model, August 1998, unpublished.
- Rueda, Catalina, Documentation of the E-CAPE Model, August 1998, unpublished.

1.4.5 Other Papers

- Davis, P., Starr, S., Thomas, C., Report of Synthesis Panel, MORS Mini-Symposium on Quick Reaction Analyses and Methodologies (QRAM), October 1—3, 1996.
- Neimeier, H. A., and Gulledge, T., Functional Economic Analysis of Purchasing at MITRE, a paper presented in Stirling, Scotland, at the International System Dynamics Conference, 1994.
- Neimeier, H. A., *MITRE Abbreviated Functional Economic Analysis Tool (MAFEA),* MITRE Technical Report 92W0000037, February 1992.
- Neimeier, H. A., *Performance and Sizing Models to Support IRS Tax Modernization System,* MITRE Technical Report 88W00232, June 1989.
- Neimeier, H. A., and Scholl, M. M., *A Modeling Approach for Tax System Modernization (TSM),* MITRE Technical Report 88W00231, January 1989.

Section 2

Overview of the CAPE Methodology

The CAPE methodology, while broad, is applied to the domain of C4ISR systems within the context of federal decision making. This limited domain shapes the purpose, scope, and uses of CAPE analysis.

2.1 The Purposes of CAPE Analysis

Viewed from a variety of perspectives, one may describe the purposes of CAPE analysis as follows:

- Support top-level, cross-functional planning and analysis in C4ISR organizations, where questions must be answered quickly, typically within two months, or be overcome by events.
- Provide a framework for discussion of complex future systems — an analytical framework within which many people can share and test their intuition, judgment, and knowledge about the behavior of hypothetical new systems.
- Visualize, understand, and estimate the benefits and costs of implementing alternative choices for future investments, doctrine, and organization of C4ISR functions and systems. Do this at the level of operational requirements and broad capabilities more than at the level of detailed system design.

2.2 Scope of CAPE Analysis

The elements of CAPE analyses include operational concepts, scenarios, environments, sensors, targets, collection processes, processing-exploitation-dissemination (PED) processes, command-control processes, communications, computers, and warfare processes. CAPE modelers spend much of their time gathering and analyzing information to build up their knowledge about the elements.

The elements in CAPE analyses are usually conceptualized at the abstract level of aggregates, averages, and probability distributions instead of at the level of individual actors and events.

The results of CAPE analyses, stated in terms of measures of effectiveness and performance, are usually developed at the analytical level of averages (expectations) and sensitivity analyses, not at the more detailed analytical level of probability distributions or risk profiles. On the other hand, probabilistic information is usually used in CAPE analyses, and probability distributions are sometimes calculated as intermediate results.

2.3 Uses of CAPE Analysis

Analysts investigate two primary questions with the CAPE methodology: (1) Which system capabilities and characteristics have the most impact on benefit and cost? and (2) What are the estimated benefits and costs? The range of actual and potential inquiry with CAPE models is summarized as follows.

2.3.1 Estimation of C4ISR Performance and Benefits

The emphasis in CAPE analysis has been on performance and benefits. The CAPE methodology has been used to

- Identify first-order relationships between C4ISR system characteristics and warfighting measures of effectiveness (MOEs) and measures of performance (MOPs);
- Analyze C4ISR investment tradeoffs in terms of the investment-to-MOE connection, i.e., the effects of investments on warfighting effectiveness across the whole sensor-to-shooter chain;
- Quantify the contribution of alternative future C4ISR systems and concepts to warfighting effectiveness; and
- Quantify the benefits of improvements in various systems and concepts using measures of performance and measures of effectiveness

2.3.2 Estimation of C4ISR Costs

While most CAPE analyses have emphasized system effectiveness, the CAPE methodology may also be used to analyze the cost implications of C4ISR systems. It is suitable for modeling the costs of acquisition, operations and maintenance, logistics, and R&D. It is suitable for calculating rough order of magnitude (ROM) cost estimates for alternative mixes of C4ISR systems. It has been used to estimate operational costs.

2.3.3 Resource Allocation

The CAPE methodology is compatible with the use of mathematical programming techniques to optimize the allocation of resources over competing designs or requirements for system elements. To date this method has not been utilized.

2.4 Distinctive Principles of CAPE Modeling

The purpose and scope of the CAPE methodology have led MITRE s staff to three basic principles to guide all CAPE modeling. An understanding of the principles goes a long way toward explaining why any particular CAPE model was constructed the way it was.

2.4.1 Principle 1. Model Complex Uncertain Phenomena with Aggregate Probabilities

Represent uncertain actors, situations, and events with probability distributions, either continuous or discrete, which aggregate more complex underlying phenomena. This approach averages over many potentially complex interactions instead of modeling the interactions explicitly. Practically, this approach enables rapid experimentation with the models to discover key factors and relationships. Here are two examples:

Discrete random variable. Let us say the weather at any place and moment may be clear, cloudy, or rainy. In CAPE models, we average over all the complexity of moment-to-moment and place-to-place fluctuations in weather and all the ways that military systems respond to the fluctuations. We assign unconditional probabilities, such as .5, .25, and .25, to represent the concept that whenever a sensor looks at any location the weather will be clear, cloudy, or rainy, respectively, regardless of what the weather was at any earlier time. These probabilities will represent the weather throughout the model, averaging over all the hour-to-hour and place-to-place variations in weather that might be represented in an event-driven simulation model.

Continuous random variable. An actor on the battlefield who stops will usually move again some time later. In CAPE models, we aggregate over all possible reasons for stops and starts and assign a probability density function to the pause time per stop. We do this not for individual actors, but for a whole class of actors. For instance, the pause (dwell time) per stop for a mobile command post could be an exponentially distributed random variable with a mean of 30 minutes. This probability density function for the random variable would then represent all kinds of mobile command posts throughout the analysis, averaging over all the complex interactions of one actor with another and with the environment that might cause the pause to be small or large.

2.4.2 Principle 2. Strive for Simple Analytic Calculations of Probabilistic Results

There are three aspects to this principle, all of which reduce the amount of calculation needed to answer a question about the main effects being modeled. First, always calculate results simply as averages (probabilistic expectations) without variances or risk profiles (probability distributions). In other words, focus calculations on the main effect itself, not on the variability of the main effect. Plan to use sensitivity analysis to explore the variability, but only as needed. Second, in probabilistic calculations, where several random independent variables may take on a range of values and combine in various ways as functions of the random variables, try hard to keep the whole analysis in terms of the parameters and moments of the distributions. In other words, avoid calculating the actual probability distributions of intermediate functions of random variables unless the distributions are very important to your results. Third, only as a last resort use numerical analysis or random-number generation and statistical sampling to find the moments or the distributions of the combined functions.

2.4.3 Principle 3. Simulate Simply

When a CAPE model is used to calculate a state history for a C4ISR dynamic probabilistic system, use a non-sampling, fixed-time-step method that approximates the inputs of successive time-steps as the expected values of the outputs of previous steps. This principle using expected outputs of one time step as deterministic inputs for the next time step focuses CAPE analysis on the main (first order) effects in the state history and avoids extensive calculations. It is inexact, but it is the most useful calculation that can be done without also calculating the probability distributions or moments of the outputs and using random-number generation and statistical sampling.

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The three principles of the CAPE methodology produce C4ISR models that differ greatly from many existing models used for military operations research. Such existing models are used to statistically estimate the performance of systems by repeated next-event simulations of the systems operation. MITRE staff believe that the simplifications of the CAPE methodology, which avoids statistical sampling, are more appropriate to the class of problems they address than the other models. There are several reasons (see Appendix A).

Implicit within the principles of CAPE modeling is the requirement that the analysts who build the models are sophisticated probabilistic thinkers. They should have a strong grasp of probability mathematics, dynamic systems, and of dynamic probabilistic systems, so that they can make the simplifications inherent in the principles while preserving the correspondence between the CAPE models and the real world.

2.5 Existing CAPE Models

The following descriptions of current CAPE models illustrate the variety of subject matter the CAPE methodology can cover.

Air-Land-Maritime Engagement Model (ALMEM) —ALMEM is a dynamic model derived from *Dynamic CAPE.* It calculates joint performance in theater level conflicts in terms of attrition of enemy and own forces. Unlike *Dynamic CAPE,* for which the analyst determines the lengths of battle phases, *ALMEM* calculates the lengths of the battle phases, using force ratios and other factors. *ALMEM* finds out how long the battle lasts. It also finds out how much the battle costs, including such costs as materiel, operations and support, and veterans benefits. Some 200 variations of *ALMEM* have been used for various analyses. *ALMEM* was employed in Global Engagement 97 to model the impact of C4ISR actions on scenario outcome. Other versions were used to evaluate new theater architectures (DSC Task 2, IMINT/MTI CDA, and IOSA-2).

Close CAPE - Models ground combat as limited to a single division. It includes ground-oriented sensors, special operations, placement of sensors on roads, area versus linear sensors, intelligence fusion (signals, imagery, and human intelligence). It uses scenarios from the Close Support End-to-End Assessment (CSEEA) study.

Cruise Missile Defense — This model calculates the percentage of cruise missiles that can be engaged and destroyed for a given set of missile flight geometries. Defensive systems that are modeled include, airborne ISR sensors, ground based air defense systems, fighter aircraft defensive systems, on-board and off-board aircraft sensors and communication capabilities.

Deep CAPE - Models the air strike mission against targets that are as deep as 350 km to determine the sensitivity of various measures of effectiveness to variations in such factors as imagery quantity and timeliness, communications delay, and shooter characteristics.

Dominant Battlefield Awareness (DBA) - DBA is defined as sensing and understanding what is happening and then communicating that information in timely fashion to the forces that need to know what is happening. Based upon red forces and their movement characteristics, the DBA model calculates the DBA that can be achieved for a given mix of ISR capabilities. The model includes the effects of environment and enemy behavior on the amount and quality of data that can be collected and disseminated on a timely basis by intelligence, surveillance, and reconnaissance (ISR) assets. The measure of performance is the amount of information (area and points) collected and disseminated on a timely basis. The model also includes an aggregate cost estimate of the ISR assets needed to provide a given level of DBA.

Dynamic CAPE - Models the effectiveness of the strike mission and supporting intelligence systems as affected by various strike and intelligence factors, including theater environment, target, sensor mix, sensor platform. Models 50 time periods, remaking platform-weapon-target assignments for each time period. With this model forces can be deployed gradually over time; and weapons, platforms, sensors, and targets are attrited daily.

E-CAPE - Models the imagery-exploitation subsystem of the intelligence collection process to determine the sensitivity of exploitation throughput and delay to various factors such as the percentage of imagery sent out of theater for exploitation, the number of imagery analysts in theater, the exploitation rate, and the use of automated exploitation tools.

Fuse CAPE - Evaluates multi-sensor fusion over a large environmental envelope.

Geo CAPE - Extends *Deep CAPE* to model the effects of terrain masking and foliage and to produce a measure of battlefield awareness, calculated as the percentage of targets for which the location is accurately known. The computer display shows a map of the conflict region with target icons that are color-coded to show how well blue forces can identify their location.

Global CAPE - Models naval forces and operations in addition to Air Force missions and sensors. Weapons include strategic ballistic missiles.

Integrated Overhead SIGINT Architecture (IOSA) Model - This model calculates the total volume of signals emitted that are of interest for a specified mission, the number of signals that are closed and the amount of data that is collected, received and down-linked by the overhead systems. The model groups detailed signals of interest into signal families with like electrical characteristics to reduce the computational requirements. The model allows the user to specify the mission of interest (e.g. JSEAD, DCA, counter proliferation, treaty oversight, etc.), the architecture of the overhead SIGINT system (altitude, aperture size, access parameters, etc.), the percent contribution of the overhead system versus the airborne collectors, and the country of interest.

Probability of Closing the Kill Cycle - This model calculates the likelihood that a mobile target can be engaged by a weapon system if it has been detected by an ISR system. The model uses the speed of the mobile target after it begins to move, the average time between mobile target moves, the accuracy and latency of the ISR sensor and the response time and footprint of the weapon system to calculate the likelihood that the weapon system successfully kills the mobile target.

SEAD CAPE - Models and evaluates the suppression of enemy air defenses over a large environmental envelope. New SA10 and SA12 surface to air missiles are evaluated.

SEASCAPE - Models and evaluates present and future naval task force survivability against present and future anti-ship missile defense. Autonomous, task force cooperative engagement capability (CEC), and integrated CEC operations modes are evaluated.

Tactical Ballistic Missile Defense (TBMD) Model — This model allows the user to specify the characteristics of the attacking TBM (e.g. geographic launch and aim points, burn time, velocity at burnout, etc.) and the number and geographic distribution of all of the family of system (FOS) TBM components that are to be modeled. The model calculates the number of TBMs that survive each segment of the TBM's flight regime (boost, mid-flight, exo-atmospheric, and endo-atmospheric). The actual parameters of the TBM's ballistic flight path are calculated and the times to detect and engage along with the probability of kill for defensive systems are combined to assess the defensive system capabilities.

2.6 Software and Hardware for CAPE Models

2.6.1 Overview of Software and Hardware

MITRE staff have built CAPE models with Analytica and Extend, which are commercial off-the-shelf software applications. Analytica and Extend run on both Macintosh and Windows personal computers.

Analytica is the preferred tool to rapidly prototype new CAPE models, often in a matter of hours. Analytica s graphic user interface makes it easy for analysts to demonstrate models to subject matter experts and then to adapt the models on-the-fly to reflect reality more accurately. However, Analytica is not currently computationally efficient for dynamic models. Extend is efficient. Another current drawback of Analytica is that it does not provide a procedural programming language. When a CAPE model involves a complex algorithm, such as a numerical integration, a programming language is virtually indispensable. Extend has such a language.

Therefore, once a CAPE model stabilizes (few changes are being made), MITRE staff often convert the model from Analytica to Extend to reduce running time or to enable better algorithms. When dynamic models are involved, Extend is 30 to 500 times faster than Analytica and requires one-tenth the memory. (MITRE staff have partially automated the conversion of models from Analytica to Extend.)

Though MITRE staff have selected Extend for CAPE modeling, any other software development package producing compiled code (e.g., C or FORTRAN) could be used.

2.6.2 Analytica

Lumina Decision Systems, Inc. of Los Gatos, California is the source of Analytica.[4] The software is available for Windows 95/98/NT 4.0 and the Macintosh OS. Lumina describes Analytica as follows: Analytica is a visual software tool for creating, analyzing, and communicating quantitative models. It provides an alternative to the spreadsheet, providing graphical influence diagrams to show qualitative structure of models, hierarchical models to organize complicated models into manageable models, and intelligent arrays with the power to scale simple models up to handle large problems.

CAPE modelers use Analytica for its intelligent arrays, which might be described as hierarchical, multi-dimensional spreadsheets linked via equations. The intelligent arrays support quick visual modeling of multi-dimensional concepts, automatically determining the order of calculation for all intermediate results as the user reconfigures and reconnects various parts of the model. Another capability of Analytica is dynamic modeling, where the outputs of one time period are used as inputs for the next time period. CAPE modelers use this Analytica capability, but not extensively because it slows down the computations. Instead, when dynamic modeling is important, MITRE s CAPE modelers use Extend.

Analytica has a strong capability for probabilistic modeling through Monte Carlo methods (and variations thereof). However, the CAPE methodology avoids such probabilistic

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4 Lumina Decision Systems, Inc.: 1-408-354-1841 or toll-free 1-877-6-LUMINA; or http://www.lumina.com

methods in the interests of quick calculation. Thus the CAPE methodology uses Analytica minus its probabilistic support.

While Analytica may be used to represent influence diagrams, it does not have mathematical programming algorithms to solve influence diagrams (or decision trees) for optimal decisions. If CAPE modelers should wish to find optimal mixes of systems, they will need to extend Analytica s capability, either by embedding optimization into an Analytica model or by drawing upon another software tool that has solution algorithms, such as DPL by Applied Decision Analysis, Inc., of Menlo Park, California.[5]

2.6.3 Extend

Imagine That, Inc. of San Jose, California is the source of Extend.[6] The software is available for DOS, Windows 3.1/95/NT and the Macintosh OS. Summarizing from the web page: Extend is a dynamic, iconic simulation environment with a built-in development system for extensibility. It enables a discrete-event, continuous, or a combined discrete-event/continuous process or system to be modeled. Extend has libraries of pre-built blocks that can be accessed from a drop and drag menu. For custom effects, functions of several blocks can be combined into one through hierarchy or by using an equation editor. Or custom blocks can be built using Extend s built-in C-like language (Mod-L) and dialog editor. Models built with Extend are compiled before operation.

Extend is used by MITRE staff to implement CAPE models because Extend is a convenient general programming environment providing fast compiled code, and because it provides good visualization functions for graphically portraying results. For CAPE models, any compiled language with good graphical functions could be used instead of Extend. MITRE staff do not use Extend s probabilistic simulation capabilities for CAPE models, since the principles of the CAPE methodology rule out probabilistic simulation in the interests of short computations and simple sensitivity analyses. (On the other hand, MITRE staff do use Extend s simulation capabilities outside of CAPE models to develop parameters for use in CAPE models.) Though speed and memory are gained by converting a CAPE model from Analytica to Extend, one loses Analytica s automated intelligent array capability to choose the order for calculating all intermediate results. With Extend, the model builder must determine and specify the order in advance or provide an algorithm for choosing the order. If the model is changed, the order may need to be revised.

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5 See Applied Decision Analysis s DPL web page: www.adainc.com/sw/whatis.html.

6 Imagine That, Inc.: 1-408-365-0305; or http://www.imaginethatinc.com

Section 3

The Inputs and Outputs of CAPE Models

3.1 Introduction

Each CAPE model is built in more or less modular fashion, but there is currently little commonality or uniformity of the modules across the models. That is, a modular CAPE architecture has not been formalized. Though all CAPE models reflect the same design principles and have similar inputs, outputs, and processes, in a formal sense each is *ad hoc.* MITRE staff are considering how to modularize the elements of the models.

As a basis for describing CAPE models in this report, we consider the three broad ways the models represent the real world:

1. Inputs — decisions, initial conditions, constraints, assumptions, uncertain events;
2. Processes — interactions of the inputs; and
3. Outputs — measured results of the processes.

This section of the report describes and illustrates the inputs and outputs that have been modeled in various CAPE analyses. By reviewing the section, the reader will appreciate the level of aggregation and fidelity used in the CAPE methodology. Because the emphasis in CAPE is on performance, the order or presentation begins with the outputs and then describes the inputs that cause the outputs. Section 4 of the report describes the processes by which inputs are transformed to outputs.

3.2 Outputs — Measures of Performance and Effectiveness

The most important outputs for several CAPE models are described here. As is implicit in Principle 2, the outputs are always expected values, not probability distributions.

3.2.1 ALMEM

- Lengths of campaign phases
- Percentage of targets killed
- Attrition of own forces
- Attrition of enemy forces
- Territory controlled
- Cost — 20 year marginal cost of force structure and C4ISR

3.2.2 Deep CAPE

Deep CAPE s outputs are for a chosen day in the conflict (e.g., day 1, day 17). They show the successive reduction, by process phase, in the proportion of targets at risk.

- Target elements sensed per day
- Percentage of targets collected that are downlinked for processing
- Percentage of targets that get through processing with sufficient time left for attack
- Percentage of targets with adequate TLE (target location error) and time to attack
- Percentage of targets that are at risk
- Costs

3.2.3 Dynamic CAPE

The following cumulative and/or daily-expected results are calculated, by day, for 50 days:

- Targets killed and remaining targets (fixed and mobile/relocatable)
- Targets at risk (percentage of remaining targets that can be successfully found and attacked before they are expected to move)
- Autonomous kills (supplement to kills from planned sorties)
- Attrition of strike aircraft
- Attrition of sensor platforms
- Daily cost to conduct the war (weapons expended, logistics, sensor attrition, and aircraft attrition)
- Remaining strike sorties
- Remaining sensor platforms
- Remaining weapons
- Phase results (results for target elements by processing phase)
  - Target elements sensed per day
  - Target elements downlinked per day
  - PED of target elements (target-element images exploited per day)

- – Targets at risk (target elements per day that can be successfully found and attacked)

3.2.4 E-CAPE

- Spare time (the time available to attack targets after exploitation is complete)
- Exposure (percentage of targets with positive spare time after PED)
- Average exploitation delay in theater and out of theater
- Throughput (percentage of collected imagery that is exploited)
- Communications bandwidth required to pass imagery

3.2.5 SIGINT (signals intelligence) Models

MITRE staff have generally used CAPE models of SIGINT to develop measures of performance to use in other CAPE models that develop measures of effectiveness. The following measures of performance are representative of many others used in SIGINT models:

- Percentage of all vehicles bringing drugs into a country that can be tracked with SIGINT
- Percentage of a hypothetical new surface-to-air missile that can be located

3.2.6 Tracking Models (single-sensor and multi-sensor intelligence fusion models)

The following measures are usually found as functions of the number of sensors of various types, the number of elements per target tracked as a group (e.g., company or brigade), and sensor revisit times.

- Proportion of targets located
- Probability of maintaining birth-to-death track for a target through several move cycles
- Proportion of targets currently seen and tracked correctly
- Target location error (TLE), given that a target is tracked

3.3 Inputs — the Actors and the Stage

While the outputs of CAPE models are invariably expected values, the inputs take several different forms. The following sections describe illustrative inputs in terms of what they are (content) and how they are represented (form). The form of the input can be a simple index, a constant, a table of constants, a probability, a table of probabilities, a probability density function, or an equation.

3.3.1 Theater Environment Charactistics

The theater environment in the CAPE models is typically described using the following factors:

3.3.1.1 Country

The scenario or location of interest is represented as an index (e.g., North Korea, Iraq, Iran) that will condition other input tables and constants.

3.3.1.2 Range Bands

Typically, the theater is divided into four range bands. The range bands represent strips of the country that are as wide as the theater and have various depths. Their depths are generally chosen to coincide with the range bands of the Deep Attack Weapons Mix Study (DAWMS), as follows:

(1) 0-40 km

(2) 40-150 km

(3) 150-340 km

(4) 340+ km

The range bands are represented as an index, with the names above.

3.3.1.3 Range-band Area

The size of each range band, in square kilometers or square nautical miles, is represented as a table that is indexed by country and by range band.

3.3.1.4 Weather

Weather can affect the effectiveness of the sensors as well as the ISR sortie rates in some models. Typical weather conditions in CAPE models are

(1) Clear

(2) 50% cloudy

(3) Full clouds

(4) Light rain

(5) Medium rain

(6) Heavy rain

The weather is represented by an index and a table, as follows:

- Weather is defined as an index, with the names given above; and
- Weather Proportion is defined as a probability table, indexed by Weather and by country, giving the percentage of time each indexed country experiences each type of weather.

3.3.1.5 Foliage

Foliage can affect the effectiveness of sensors. Typical foliage categories in CAPE models are

(1) Clear

(2) Urban

(3) Scrub

(4) Forest

The foliage in a particular country is described by the percentage of the area that is covered by the type of foliage. Foliage coverage is represented by an index (Foliage) and a table (Foliage Proportion) that is indexed by foliage and by country.

3.3.1.6 Terrain Masking

The terrain of a country may inhibit a sensor from accessing targets of interest. Terrain masking (delimitation) is modeled as the percentage of a country s area that is not visible because of its terrain. Terrain masking is represented as a table of such percentages indexed by country. In addition to the country, the altitude of the collection platform can affect the amount of delimitation. Sensors on high altitude platforms are less likely to be affected by terrain considerations. In some models the amount of terrain masking is represented as a function of the platform altitude.

3.3.2 Operational Environment Characteristics

The following factors are used to describe the operational environment in a CAPE model.

3.3.2.1 Phase of War

The phase of the war may be described in terms of (1) pre-SEAD[7] and post-SEAD, or (2) phases 1, 2, 3, 4. Phase is represented as an index. The phases in the CAPE model are used in describing which assets are available at what point in the conflict. They also may be used to determine whether airborne platforms can over fly the theater or must standoff.

3.3.2.2 Operations Tempo

If troops are moving more frequently, then it is more difficult to monitor them. The frequency of troop movement affects both collection requirements (how often collection must be done) and the ability to collect, process, exploit and disseminate the image of a target before the target moves. Operations tempo is represented as a table of frequencies of troop movement indexed by rangeband and phase of war.

3.3.2.3 IPB (Intelligence Preparation of the Battlefield)

If good IPB can be accomplished, the locations of many targets will be available at the start of the conflict. IPB is modeled as the percentage of target locations that are known at the start of the conflict. IPB is represented as a table of percentages indexed by Good IPB and Poor IPB.

3.3.3 Target Charactericstics

3.3.3.1 Target Class

Targets are modeled in terms of broad target classes. The target classes used in most of the CAPE models are those used in the DAWMS, which categorize approximately 200 targets into seven classes:

(1) Short intercept

(2) Short mobile column (<10 vehicles)

(3) Long mobile column (>10 vehicles)

(4) Small long dwell target

(5) Large long dwell target

(6) Small fixed target

(7) Large fixed target

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[7] SEAD stands for suppression of enemy air defenses. It denotes a supposed time in a scenario at which one has defeated an enemy s air defenses.

3.3.3.2 Target Quantity

The number of targets in a range band on any given day is represented deterministically as a table of quantities indexed by target class and range band. It is assumed that the targets are uniformly distributed throughout each range band.

3.3.3.3 Target Area

The target area (size) for a specific target in a target class is considered to be a random variable, since some targets in a class are larger than others. Further, the target area is represented as a continuous random variable for which our uncertainty may be described using a triangular probability density function. The three parameters of the triangular distribution are the minimum, mode, and maximum target size, which all vary by target class. Target area is represented by a table of such parameters indexed by target class. In calculations, the algebraic equations for a triangular distribution, involving the three parameters, also represent the target area.

3.3.3.4 Target Pause

The length of time a mobile or relocatable target stays in one location after it stops. Target pause is modeled as a continuous random variable with our uncertainty characterized by a triangular probability density function. It is represented as a table of min-max-mode parameters of the triangular distribution, indexed by target class.

3.3.3.5 Move Time

Move time is the length of time a mobile or relocatable target moves after leaving a location before it stops again. Move time is often modeled as a constant for each target class. It has been represented as a table of constants indexed by target class. It can also be modeled as a random variable and represented as a table of parameters of a probability density function, indexed by target class.

3.3.3.6 CCD (Concealment, Cover, and Deception)

CCD models the percentage of targets not seen due to the fact the red force sometimes employs a CCD measure. The percentage depends on the target class and whether the target is moving or stationary. CCD is represented as a table of percentages indexed by target class and target mode (fixed or mobile/relocatable).

3.3.3.7 Elements Per Target

Some targets contain more than one element (e.g., mobile column greater than 10 vehicle). This is used when considering weapons allocation and when determining the resolution required for detection of the target. Elements per target is represented as a table of constants indexed by target class.

3.3.3.8 Target Value

Some targets are more important than others are. When calculating the platform plan or the weapon-target-pairing algorithm, the value of a target can be used to select important targets for attack. The target value is modeled as a constant for each target class. It is represented as a table of constants indexed by target class.

3.3.3.9 Signals Target Classes

Signals target classes are represented as an index. Some signals target classes used as indices in CAPE models are as follows:

(1) Leaders

(2) Non-government organizations

(3) Groups

3.3.3.10 Emitter Class

Each signals target uses signal-producing devices, which are called emitters, such as telephone, mobile phone, radar, and radio. Emitters are modeled in broad signal families, not as specific emitters. Emitters are represented by an emitter class index, whose members include such titles as

(1) All UHF radios

(2) Special UHF radios

(3) Mobile telephones

(4) Iridium telephone

3.3.3.11 Emitter Quantity

The quantity of emitters is represented as a table of quantities of emitters per signals target, indexed by emitter class and signals target class.

3.3.3.12 Emitter Characteristics

Emitters are modeled in terms of various electrical characteristics, such as power, shape of the power, waveform, and bandwidth. The emitter characteristics are represented in a table of characteristics indexed by emitter class.

3.3.4 Collection Mix Characteristics

The term collection mix is used to describe some combination of actual and/or proposed collection platforms whose performance is evaluated using a CAPE model. The following variables are used to describe the platforms in a collection mix. Not all of the variables described below are used in all CAPE models.

3.3.4.1 Platform Type

The different types of collection platforms (e.g., U-2, Global Hawk, P-3) are represented as an index. The platform may be an aircraft or a satellite. With current CAPE models, the sensors on a platform are implicitly part of the platform type. For instance, a Global Hawk with a SIGINT package would be a different platform type than a Global Hawk with an IMINT package.

3.3.4.2 Mix Quantities

This is the quantity of each type of collection platform in the scenario of interest. The quantities may be varied by the analyst, but they are treated as constants for any particular run of the model. Mix quantity is represented as a table of platform quantities indexed by platform type and country (scenario).

3.3.4.3 Deployment Rate

The deployment rate indicates when and in what quantity a particular type of collection platform is available for use. The deployment rate may be specified in terms of pre-SEAD/post-SEAD, phase of the war, or the day of conflict, depending upon the CAPE model. The deployment rate is a function of the time phased force and deployment list (TPFDL). Deployment rate is represented deterministically as a table indexed by platform type and time (e.g., day, phase). The table specifies either (1) the number of collection platforms arriving in each time period, or (2) the percentage of the total deployable number arriving in each time period.

3.3.4.4 Platform Plan

The platform plan is used to assign a platform(s) to collect in a particular range band. The platform plan may also be a function of pre-SEAD/post-SEAD, the phase of the war or the day of conflict. For each platform, it is the proportion of each range band that is to be covered by the platform. While set up to handle any proportion, in practice it is usually zero or 100 percent. Occasionally it is 50 percent. The platform plan is an input to the model. It does not necessarily represent the optimal allocation of the collection assets. The platform plan is represented as a table of proportions indexed by platform type and range band.

3.3.4.5 Impact of Weather on Sensor Type

Weather conditions may degrade the ability of sensors to collect usable imagery. Degradation is modeled as the probability that an image is not useful due to a weather condition. The effect of weather varies by sensor type, with electro-optical (visual) imagery being most sensitive to the weather and SAR being the least sensitive. The impact of weather is represented as a table of degradation probabilities indexed by weather condition and by sensor type.

3.3.4.6 Impact of Foliage on Sensor Type

The type of foliage in a country may degrade the usefulness of a given sensor. This loss is modeled as the probability that a collected image is not useful because of foliage. The impact of foliage is represented as a table of degradation probabilities indexed by foliage type and by sensor type.

3.3.4.7 Sortie Rate

The number of potential sorties per sensor platform per day is modeled as a constant. Sortie rate is represented as a table of constants indexed by platform type.

3.3.4.8 Impact of Weather on Sortie Rate

Weather conditions may prevent an airborne sensor platform from being able to sortie. The probability that the mission cannot be performed is represented in a table indexed by weather condition and platform type.

3.3.4.9 Correlation

Several sensors in the mix may see and collect information on the same target. This phenomenon is accounted for by correlation, a percentage of sensed targets that are not duplicates. A correlation of 90 percent, for instance, would mean that ten percent of sensed targets are duplicates, and 90 percent of sensed targets are separate targets. Correlation is represented as a constant.

3.3.4.10 Collection Platform Attrition Rate

Due to mechanical failures or enemy air defenses, collection platforms may be lost. The attrition rate is used to account for the loss of platforms. It decreases the number of platforms as a function of time and sortie rate. It is a constant for each collection platform. It is represented as a table indexed by collection platform type.

3.3.5 Collection Platform Characteristics

The discussion of platform characteristics is divided into general characteristics and then characteristics specific to various types of sensors. For ease of presentation, the descriptions below say that the characteristics are represented by separate tables. However, the usual practice is to build one large table indexed by platform type and platform characteristics.

The one table contains all the virtual tables described below. In calculations, Analytica s slice function is used to retrieve the data for each virtual table.

3.3.5.1 General Platform Characteristics

3.3.5.1.1 Duty Cycle

The duty cycle is the reciprocal of the number of platforms required to maintain a twenty-four hour orbit. Duty cycle models the percentage of a platform s service time in which the platform provides continuous coverage. The duty cycle considers flight endurance as well as maintenance and downtime. The equivalent number of twenty-four hour orbits is calculated by multiplying the number of platforms by the duty cycle. Duty cycle is represented by a table of duty cycles indexed by platform type.

3.3.5.1.2 Altitude

The altitude of a sensor platform influences (1) the distance its sensors can see into a country, (2) the effects of terrain masking, and (3) the survivability of the platform in regions where there are enemy air defenses. The altitude may or may not be directly used in a calculation, but if not, it may influence the analyst s decision when defining the platform plan. The altitude is represented as a table of altitudes indexed by platform type.

3.3.5.1.3 Range

The distance an airborne platform can fly from its base/ground station influences the range bands in which it can be employed. The range is not necessarily represented in a CAPE model or used directly in a CAPE calculation, but it is used by the analyst when defining the platform plan.

3.3.5.1.4 Required Stand-off Distance

This is the distance that an airborne collection platform must standoff to avoid air defenses pre-SEAD. This factor is used to calculate which range bands the sensor can access and therefore what percentage of targets in the range band the sensor can image. It also affects terrain masking. The required standoff distance is represented as a table of distances indexed by platform type.

3.3.5.2 For Imagery Collection

3.3.5.2.1 Area Collection Rate

An area collection rate is specified for each of a platform s imagery sensors. It is the amount of area imagery the platform can collect per hour with the sensor. The area collection rate is represented as a table of rates indexed by sensor type and by platform type.

3.3.5.2.2 Point Collection Rate

A point collection rate is specified for each of a platform s imagery sensors. It is the number of point images the platform can collect per hour with the sensor. The point collection rate is represented as a table of rates indexed by sensor type and by platform type.

3.3.5.2.3 Sensor Resolution

The resolving power of a sensor is modeled as a single parameter, the sensor resolution, which is the smallest area that the sensor can discriminate. (Sensor altitude affects sensor resolution, so sensor resolution and platform altitude must not be varied jointly when sensitivity analysis is done.) CAPE analysts have used the same resolution for both point and area collection, though different resolutions could be used if the specific information were available. The sensor resolution is compared to the target area of a target class (a random variable) to determine the probability the sensor will detect/identify targets in the target class. Sensor resolution is represented as a table indexed by sensor type. It could also be indexed by target type if specific information were available.

3.3.5.2.4 Area Imagery Proportion

This is the percentage of time that a sensor collects imagery at the area rate instead of the point rate. The area imagery proportion may be represented as a constant across all platform types or as a table indexed by platform type.

3.3.5.2.5 Collection Efficiency

This is the percentage of time a collection platform is collecting imagery. It is always less than 100% because of turns, sensor mode transitions, and the geographical laydown of the collection requirements. It accounts for the lost collection time when the platform is moving between targets or moving the sensor. It is applied as a percentage that reduces the effective collection rate. The same collection efficiency is assumed for point collection as for area collection. It is the same for all platforms. It is represented as a constant.

3.3.5.3 For Signals Collection

The sensor and platform characteristics for signal collection are too sensitive to discuss in this report. Generally, they are related to the sensitivity of the sensor, the flexibility to direct the sensor to collect signals from emitters repeatedly, the capability of the sensor to listen to the full range of the emitter s frequency range, and the capacity to transmit information to processing stations.

3.3.5.4 Other Collection

3.3.5.4.1 Sensor Circular Error Probable (CEP)

Sensor CEP is used as target location error (TLE) for fixed targets. TLE is used in the weapon-allocation process (see p. 42). For mobile targets, it is combined into a weighted average along with the accuracy of TLE cues from other intelligence sources (see below, section on Cueing). It is represented as a table indexed by sensor type and platform type.

3.3.5.4.2 Downlink Capacity

This is the communications capacity (bandwidth) between a sensor platform and a processing facility, represented as a table of constants indexed by platform type.

3.3.5.4.3 Cueing

- TLE cue — The target location error (TLE) of other intelligence sources that cue the sensor. It is modeled as a constant for each target class, and it is represented by a table of constants indexed by target class.
- Other INT cue — The proportion of targets that have been cued from other intelligence sources. It is assumed to be independent of target class. It is represented as a constant.

3.3.6 Imagery Exploitation Characteristics

The process of imagery exploitation is represented with varying levels of fidelity within the CAPE models. It is usually modeled simply in terms of the time it takes to exploit an image after the image is collected. The time includes a waiting time, since there will usually be a backlog of images to exploit, plus the time to do the exploitation itself.

The E-CAPE model, which focuses on the exploitation process, includes different exploitation sites and the use of automated exploitation aids.[8] It also models the exploitation time as a function of workload. The formulas for the exploitation time (see p. 36) make use of the following factors:

3.3.6.1 Imagery Arrival Rate

The average hourly number of satisfactory images arriving for exploitation, which is represented in CAPE models as a dependent variable that is calculated from various inputs and other intermediate results.

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8 The use of E-CAPE is documented in Parker, S. K., *Modeling the Collection & Exploitation Trade-Off in the C4ISR Mission Assessment Study,* MITRE Technical Report 98W0000042, May 1998.

3.3.6.2 Imagery Arrival Distribution

The variation in the inter-arrival times of collected imagery. It is modeled by the coefficient of variation, which is the quotient of the standard deviation of the inter-arrival times and the average inter-arrival time. It is represented as a constant, though it may be varied by the analyst in sensitivity analysis.

3.3.6.3 Number of Imagery Workstations

The workstations are used to exploit the arriving satisfactory images. The number of workstations is represented as a constant. (By specifying the number of workstations instead of the number of analysts, the implicit modeling assumption is that the workstations are always staffed.)

3.3.6.4 Exploitation Rate

The average number of satisfactory images exploited per hour per workstation represented as a constant.

3.3.6.5 Exploitation Distribution

The variation in the time required to exploit an image is modeled by the coefficient of variation, which is the quotient of the standard deviation of the service time and the average service time. It is represented as a constant.

3.3.7 Weapon Characteristics

3.3.7.1 Engagement Time Requirements

The minimum time from the tasking of an aircraft or missile until the weapon engages the target, represented as a table of time requirements indexed by platform type and range band.

3.3.7.2 Weapon TLE (Target Location Error) Requirements

The accuracy with which the target s location must be known in order to use a weapon effectively. TLE for a target is compared to the weapon TLE requirement when allocating weapons. Weapon TLE requirements are represented by a table of TLEs indexed by weapon.

3.3.7.3 Inventory

The number of weapons in the inventory at the start of the scenario, represented as a table indexed by type of weapon.

3.3.8 Strike Platform Characteristics

3.3.8.1 Potential Sortie Rate

This is the number of aircraft that can be launched per day. The initial value of the potential sortie rate is represented in a table indexed by platform type. In a dynamic model, the initial value changes with aircraft attrition and with the deployment of new platforms. Another variable, the assigned sortie rate, counts the actual strike sorties, differentiated by platform type.

3.3.8.2 Strike Platform Attrition Rate

The probability that a strike platform is lost when it is sent on a mission (sortie), represented as a table indexed by platform type, range band, and phase of the conflict.

3.3.8.3 Expected Kills Per Sortie (EKS)

An aircraft may carry several weapons (i.e., munitions) and use them to attack several targets. The expected kills per sortie is the expected number of targets destroyed per sortie. It is represented as a table of EKS constants indexed by strike platform type, munition, and target type.

Section 4

Modeling Operational Processes with CAPE

Several techniques for modeling operational processes are used so repeatedly in CAPE analyses that they may be said to be characteristic of CAPE models. Most of the techniques help implement the second principle of the CAPE methodology, doing probabilistic calculations analytically and simply. This section describes the techniques in the context of the operational processes they have supported. However, the techniques are not limited to the specific processes identified here. They are general modeling techniques that support the three CAPE principles of modeling (see p. 8).

4.1 Determining the Efficiency of Collection Processes

4.1.1 Overview of Imagery Collection

CAPE modelers have approached the imagery-collection process in four basic steps. The four steps and the basic input parameters used in each step are as follows:

1. For each specific platform/sensor combination, determine the proportion of targets that will be masked from collection. Use these input parameters:

- Weather, terrain, and foliage theater-environment characteristics
- CCD (concealment, cover, and deception) target characteristic

2. For each platform/sensor combination, determine the effective collection rate (images per hour of operation). Use these input parameters:

- Area and point collection rates
- Area imagery-proportion
- Collection efficiency

3. For each range band, determine the number of targets (by target type) that could theoretically be observed for each sensor/platform combination, given a plan for the allocation of sensors to the range band. Use these input parameters:

- Platform plan
- Correlation
- Collection rate
- Range band area

- Sortie rate
- Stand-off distance, range band depth, sensor range
- Note: If the sensor is standing off, and the sensor range can only reach half-way into the range band, then it will have access to at most half of the targets, just due to the effects of standoff. You then must also consider the area to be covered and the collection rate.

4. For each platform/sensor combination in each range band, determine the expected number of targets (by target type) accurately identified and located. Use these input parameters:

- Target resolution by target class
- Impact of weather on sortie rate
- Fraction of targets pausing (calculated from move and pause time)

4.1.2 Overview of Signals Intelligence Collection

Because of information sensitivity, all that can be said about modeling the SIGINT collection process is that it follows these general steps:

1. For a given emitter, determine the probability that it is detectable, in terms of signal-to-noise ratio.

2. If detectable, determine the probability that an antenna can be aimed at the emitter frequently enough to collect signals.

3. If detectable and collectable, determine the probability that the sensor s radio receivers are tunable through the full range of the emitter s spectrum.

4. If detectable, collectable, and tunable, determine the probability that the information from the sensor s receiver can be transmitted to the processing station.

4.1.3 Modeling the Masking Due to Weather, Foliage, and Terrain

CAPE models use many tables of conditional probabilities to calculate intermediate or final results (outputs) as unconditional probabilities or expectations. Such tables and calculations are the staples of CAPE models, being the major part of every CAPE analysis. An important case is modeling how the detection of a target by a sensor depends on the weather, the foliage, and the terrain.

The detection issue can be posed as a question to answer, What is the probability the target can be seen by the sensor, considering that the target may be masked by weather, foliage, or terrain? To get at this question, CAPE modelers typically assume that the masking effects of weather, foliage, and terrain are independent of one another, in this sense:

- The effects of weather do not depend on the foliage or terrain;
- The effects of foliage do not depend on the weather or the terrain; and
- The effects of terrain masking do not depend on the weather or the foliage.

Such assumptions allow one to treat weather, foliage, and terrain as if they were three separate and independent masking factors that might obscure the target from the sensor.

Given the assumptions, one sets up a method to calculate an unconditional probability of non-masking for each of the three factors, multiplies the resulting three unconditional probabilities together, and calls the result the probability that the target will not be masked, considering all the possibilities of weather, foliage and terrain. The probability the target can be seen despite weather, foliage, and terrain is just this calculated probability of non-masking.

To develop one of the three unconditional probabilities, that for weather, one would proceed as follows:

- Create a table of conditional probabilities of non-masking, specifying the probability of non-masking for each type of weather.
- Set up another table of unconditional weather probabilities, specifying one probability for each type of weather.
- Calculate the unconditional probability of non-masking by multiplying the weather-conditioned probabilities of non-masking with the unconditional probabilities of weather and adding up the products.

Here are examples of the two tables of probabilities and of the corresponding calculation of the probability of non-masking for the weather factor:

Weather Condition	**Probability of Masking**	**Probability of Non-Masking**
Clear Weather	0	1
Cloudy Weather	0.5	0.5
Rainy Weather	0.9	0.1

Weather Event	Probability of Event
Clear Weather	0.5
Cloudy Weather	0.25
Rainy Weather	0.25

$$\Pr(\textit{Non} - \textit{Masking by Weather}) = 0.5 \times 1.0 + 0.25 \times 0.5 + 0.25 \times 0.1 = 0.65$$

Suppose the probabilities of non-masking for foliage and terrain are calculated (by similar combinations of conditional and unconditional probabilities) to be 0.70 and 0.75, respectively. Then the unconditional probability that the target will not be masked by any of the three factors (weather, foliage, or terrain) is the product of the non-masking probabilities for the factors:

$$\Pr(\textit{Non} - \textit{Masking}) = 0.65 \times 0.70 \times 0.75 = 0.34$$

Looking at it the other way, in terms of masking, the probability that the target *is* masked by at least one of the three factors is the complement of this probability:

$$\Pr(\textit{Masking}) = 1.0 - 0.34 = 0.66$$

The foregoing illustration shows the typical CAPE use of conditional probabilities to calculate unconditional probabilities. A related CAPE technique uses tables of expectations (for instance, the expected number of targets in a range band) to calculate other expectations. For instance, one might calculate a probability of 0.34 (as above) that a target is not masked by weather, foliage, or terrain and then find in a table that the expected number of such targets in a specific range band is 100. These two numbers would be multiplied to find the expected number of non-masked targets in the range band, which is 34.

4.1.4 Accounting for Target Size — Using Thresholds for Target Resolution

Targets may be too small to reliably collect. So even though weather, foliage, and terrain masking factors are absent, one may not be able to use a sensor to detect or identify a target. This phenomenon is typically modeled in the CAPE methodology by identifying a threshold resolution for the sensor, in the same dimension as target size (e.g., the dimensions of a vehicle, or the power of a radio transmitter), and then comparing the threshold to the size of any particular target. If the target is larger than the threshold, then one assumes that the sensor data can be used to detect or identify the target. (In general, there would be different threshold resolutions for detecting, classifying, and identifying targets.)

Because the CAPE methodology models targets in the aggregate, with target classes instead of with individual targets, the threshold must be applied with all the sizes of targets in the class to determine the probability that a randomly selected target from the class will be detected. In order to make such a determination, one typically specifies a probability density function (PDF) for the size of targets in a target class. Then one uses the PDF to compute the probability that a target from the class will be detected. This is the probability that a target from the class is larger than the threshold resolution of the sensor.[9]

For imagery, the threshold resolution and target size are specified in terms of target length. Target length is considered to be a continuous random variable, and uncertainty in target length is modeled by a PDF, typically a triangular distribution.

The triangular PDF for target length has three parameters:

- *Min* (no targets have a smaller size than *Min*),
- *Mode* (the most likely size), and
- *Max* (no targets are larger than *Max*).

All targets are detectable if the sensor s resolution threshold is smaller than *Min*. No targets are detectable if the threshold is larger than *Max*. Some targets are detectable if the threshold is between *Min* and *Max*.

Let s stand for the threshold resolution for a sensor. Then when a triangular PDF is used to model uncertainty in target length, the formulas for the probability that a target is detectable are as follows:

1. When the threshold s is less than *Min* or greater than *Max*,

$$\Pr(\textit{Target is detectable}) = 0$$

2. When s is less than or equal to *Mode*, but greater than *Min*,

$$\Pr(\textit{Target is detectable}) = 1 - (s - \textit{Min})^2 / ((\textit{Mode} - \textit{Min})(\textit{Max} - \textit{Min}))$$

3. When s is greater than or equal to *Mode*, but less than *Ma*x,

$$\Pr(\textit{Target is detectable}) = (\textit{Max} - s)^2 / ((\textit{Max} - \textit{Mode})(\textit{Max} - \textit{Min}))$$

A simpler representation of target length would be a uniform PDF, where all sizes fall between a *Min* and a *Max*, and there is no most likely size — all sizes are equally likely. In this case, the probability that the target is detectable is even easier to calculate:

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9 Mathematically, if $G(x)$ is the cumulative distribution function for X, the size of the target, and if s is the sensor s threshold for detecting targets, then the probability of detecting the target is taken to be $\Pr(\textit{Detect}) = 1 - G(s)$.

$$\Pr(\textit{Target is detectable}) = \begin{cases} 1 & \textit{for} \quad s \le Min \\ (Max - s)/(Max - Min) & \textit{for} \quad Min \le s \le Max \\ 0 & \textit{for} \quad s \ge Max \end{cases}$$

Finally, the size may be expressed in units of time. For instance, if a target must move for at least five minutes to be detected and located for targeting, then the move-time becomes the size. The move-time distribution might be either triangular or uniform, and treated as above. It might also be modeled with an exponential distribution over times ranging from zero to infinity. If so, and if μ is the average move-time, then the probability that the move-time T_m is greater than a threshold s is calculated as

$$\Pr(T_m > s) = e^{-s/\mu}$$

4.2 Modeling the Delay in Sensor-to-Shooter Response

4.2.1 Background

After taking a reconnaissance photograph, it takes some time before the film can be processed. It then takes additional time to get the film to someone to analyze. It takes more time to analyze (exploit) the photograph to determine what information it contains. Finally, it takes time to interpret that information, decide what it means (in the context of other information), and act upon it. Thus it may take hours or days between the taking of a photograph and the launch of a strike sortie against targets shown in the photograph. The target may move in the meantime.

Many improvements in C4ISR systems are aimed at shortening the time between sensing the target and striking the target. Therefore, to analyze such improvements, CAPE modelers need a method for modeling the delay between sensing and shooting. To satisfy this need, they have generally modeled the delay with (1) a probabilistic queue for the sense-to-decision time, and (2) a deterministic fly-out time for the strike once it is launched.

A queue is a waiting line consisting of people or items awaiting service. Arrivals occur when more people or items come for service. Those who arrive usually have to wait in the queue for others to be served before they are served. The time they have to wait depends on how many others are in line ahead of them and on how fast the others will be served.

4.2.2 Calculating the Delay

CAPE modelers generally work at such a high level of aggregation that they can estimate only the arrival rate of work into a service center and the service rate of the center. They may also estimate the variances of the rates. They will generally not know how work is divided or processed within the service center. In particular, they will not know how many servers are working simultaneously. With this information, they model the service center and queue as a single-server system, not because they know it is a single server system, but because they do not know enough to say how many servers there might be. The result is that their calculations of delay through the system will tend to systematically underestimate the delay that would be calculated for more servers. However, so long as the system is well utilized (near capacity), the error may not be great.[10] In CAPE models, the systems used for analyzing imagery and for planning strikes do tend to be working at or near capacity. That is because skilled intelligence analysts during wartime are scarce in comparison to the volume of information they are asked to analyze.

The CAPE model for delay is a first-in, first-out (FIFO) queue with general inter-arrival times, general services times, and a single-server (G/G/1). Henry Neimeier identifies the model as an approximate solution to the general inter-arrival and general service time queue.[11] With this model, the formula for the average delay at a node (service center with a single server) is written as follows:

$$\mathit{Average\ Delay} = \left[\left(\frac{U \times \left(C_a^{\,2} + C_s^{\,2}\right)}{2 \times (1 - U)}\right) + 1\right] \times \frac{1}{S_r}$$

Where

- U is the process utilization (see formula below)
- C_a is the coefficient of variation of the inter-arrival times[12]
- C_s is the coefficient of variation in service times
- S_r is the average service rate, i.e., the average number of services performed per unit of time

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10 For two systems with equal average service times and arrival rates, but different numbers of servers, the system with the most servers will have the longest service time per server. Hence it will have the longest total delay (waiting time plus service time). The difference in total delay is magnified when the utilization is low, causing service time to be the main component of total delay.

11 See Neimeier, H. A., Analytic Network Queuing, *International Systems Dynamics Conference Proceedings*, Stirling, Scotland, 1994.

12 A random variable s coefficient of variation is its standard deviation divided by its mean. Its standard deviation is the square root of its variance.

The process utilization (U) of the system is

$$U = \frac{\text{Average arrival rate}}{\text{Average service rate}} = \frac{\text{Average arrival rate}}{S_r}$$

For the case where the inter-arrival times and service times can be modeled with an exponential probability distribution, which is a typical assumption for modeling queues, the coefficients of variation both equal 1.0, so the formula for average delay is simplified, as follows:

$$AverageDelay = \frac{1}{\text{Average service rate} - \text{Average arrival rate}}$$

4.2.3 Correcting for Actual Delays

In general, the formulas for delay will underestimate the delay if the system has more than one server (see footnote 10). A way of correcting for this underestimate, if more information is available, would be to take information about the actual delay associated with a given arrival rate and service rate, compare it to the delay calculated with the formula, and develop a correction factor (actual/formula) to be applied to formula calculations. Such correction would adjust the rates to fit the delay. So long as the correction is relatively small compared to the formula s calculation, the estimate may be acceptable. In general, it is difficult to predict the corrections needed for different rates without a detailed underlying model.

4.2.4 Calculating the Throughput

If the utilization of the system is less than one, then the throughput (percentage of arriving work that receives service) of the system is 100 percent. However, if the utilization is greater than one, the throughput is the reciprocal of the utilization, and the average delay formula presented above is not applicable. This situation has been (or can be) handled three different ways in CAPE models:

1. *Set 95 percent as the highest allowable utilization.* In cases where the utilization would be greater than 95 percent, assume that the excess over 95 percent is discarded or diverted to another service system. In this case, the highest possible delay for work performed at the service center is the delay associated with the 95 percent utilization. This case is equivalent to setting a maximum length to the queue awaiting work at the service center and rejecting work that would extend the line awaiting work beyond the maximum.
2. *Set a constraint on delay.* If there is more work, potentially causing a longer delay than the constraint, either discard the work or divert it to another service system. This approach is also equivalent to setting a maximum length to the queue.

3. *Keep a backlog.* Using either of the approaches above, do not discard the excess work, but store it for later service if the arrival rate should decrease.

4.3 Movement of the Target Beyond Strike Range

For many sensor-to-shooter (STS) systems, significant delay occurs between sensing a mobile ground target and engaging it with a weapon system. One major purpose of new investment in STS systems is to shorten the delay, so that the mobile target is more likely to be found and engaged successfully.

A model of target movement is at the heart of many CAPE analyses of STS systems, representing mobile ground targets in their race against the imagery-based STS system. Target movement involves the interaction of complex probabilistic phenomena.

To enable CAPE analysis of investments in STS systems, a probabilistic model of target movement has been derived from reasonable probabilistic assumptions using basic probability calculus. The derived model includes related probability distributions (cumulative distribution functions) for

- The length of the pause that remains after a mobile target is observed pausing;
- The distance the target may have moved at any specific time after it is observed; and
- The distance the target may have moved when a strike sortie arrives to attack it, where the time of arrival is itself a random variable.

These probability distributions are calculated from more basic probabilistic models of target movement time, pause time, and velocity, as well as from the revisit time used for managing surveillance and reconnaissance of targets. The distributions are used to calculate the probability that a target observed pausing will be within engagement range when a sortie, launched on the basis of the observation, arrives to attack the target. This involves a model of the sensor-to-shooter response time.

Appendix B contains details of the derived probability distributions. The derivations have been implemented in software to make them available for incorporation in analyses. They exist as Visual Basic code and as Analytica models.

4.4 Strike Engagement — Catching the Target Before it Moves Too Far

In accounting for target size and using thresholds for target resolution (above, p. 34), a threshold is compared to a random variable. For instance, a sensor s threshold resolution of 10 meters is compared to target size, a random variable that might vary between one and 50 meters. The basic analytical question associated with thresholds is What is the probability that the random variable is larger than the threshold?

In more complex comparisons, the basic question remains the same, but two random variables are compared to each other instead of one random variable being compared to a threshold. Specifically, to model the success of a strike mission with the CAPE methodology, two random variables are compared:

- T_m, the time from the moment a sensor collects data about a target until the target moves a safe distance away and can no longer be found by a strike mission that is sent to attack it; and
- T_s, the time from the moment a sensor collects data about a target until a strike mission arrives to engage the target.

If T_m is greater than T_s, then the target can be successfully attacked. If T_m is smaller than T_s, the target cannot be successfully attacked. As with the threshold method, the basic analytical question associated with such comparisons is What is the probability that T_m is greater than T_s? One may also ask, If T_m is greater than T_s, how much greater is it?

To answer the analytical questions, three approaches may be taken. The first is based on the target-movement model reported in Appendix B. The second is based on the Fractile Method (described below, p.49). The third is a heuristic method based on the threshold approach for target resolution (described above, p.34).

4.4.1 Applying the Target-Movement Model

Appendix B presents an overview of equations that represent target movement in connection with the strike mission. The last section of the Appendix presents two approaches for calculating the answer to the analytical questions stated in this section. The first approach depends upon the specification of a cumulative distribution function (CDF) for the time T_s. A double numerical integration is performed using this CDF. The second approach depends upon the specification of a mean and variance for T_s. The mean and variance are then used in a single numerical integration to approximate an answer to the analytical question.

4.4.2 Applying the Fractile Method

If one has probability distributions or data sets for T_m (perhaps from Appendix B) and for T_s, the answers to the analytical questions may be approximated to any degree of precision by using the Fractile Method. The Fractile Method is a special technique of CAPE analysis (see p. 49). It is applied as follows to answer the analytical questions:

1. To find the probability that T_m is greater than T_s, first approximate the continuous random variables T_m and T_s by generating discrete random variables T_m and T_s (each with N values). Then find the number M of combinations of T_m and T_s in which T_m is greater than or equal to T_s. Finally, divide M by the total number of combinations, N^2. The quotient $M \div N^2$ is the probability that T_m is greater than T_s.

2. To find the cumulative distribution function for $Z = T_m - T_s$ for the case when T_m is greater than T_s, first generate the variables T_m and T_s as defined above. Then find the differences $Z = T_m - T_s$ for all combinations of T_m and T_s. Finally, discard all negative values of Z. The remaining values of Z are equally probable. Collectively they approximate the random variable Z when Z is known to be non-negative.

4.4.3 Applying Threshold Heuristics

CAPE modelers have used a heuristic approach based on threshold comparisons to model the success of strike missions. The logic of the heuristic approach is as follows:

- Because the target s pause may be just ending or just beginning when the sensor finds the target, one expects the distribution function for T_m to extend from zero to several hours. Thus the standard deviation of T_m could be fairly large.
- For T_s, the complex process of imagery exploitation, target analysis, and initiation of attack sorties is a sequence of tasks performed in serial order. Hence one expects there to be a basic minimum time to accomplish all the tasks — the sum of the minimum times it takes to perform each separate task. In an efficient system, one might expect the standard deviation of the total time T_s to be small to moderate as compared to the basic minimum time.
- If the standard deviation of T_s is small relative to its minimum, and if the standard deviation of T_m is large compared to the minimum of T_s, then it is (heuristically) reasonable to approximate the random variable T_s by a single value, T_s, and then compare T_s to T_m through use of a distribution function for T_m. CAPE modelers have used the expected value of T_s as the representative value T_s. They have typically used a triangular distribution function for T_m.

Following this logic, approximate answers to the analytical questions are developed as follows:

1. Find the expected value of T_s, and call it μ_S.
2. Define a cumulative distribution function (e.g., triangular) for T_m, and call it $G(t_m)$.
3. Calculate the probability that T_m is greater than T_s:
   (a) Use the threshold method (see p. 34) to calculate the probability that T_m is larger than the expected value of T_s, μ_S. That is, calculate the probability
   $\Pr(T_m > \mu_S) = 1 - G(\mu_S)$.

   (b) Take the probability just calculated, and call it the probability that T_m is larger than T_s. That is, assign the probability
   $\text{Prob}(T_m > T_s) = \text{Prob}(T_m > \mu_S) = 1 - G(\mu_S)$.

Such an assignment does not follow from the axioms of probability calculus, but it may, in conditions known to the analyst, be a good assignment as an approximation.

4. Calculate the cumulative distribution function for $Z = T_m - T_s$ for the case where T_m is greater than T_s:

   (a) Use $G(t_m)$ and the expected value μ_S to compute a conditional distribution function for the size of T_m when it is bigger than μ_S. That is, calculate the conditional distribution
   $G(t_m \mid T_m > \mu_S) = G(t_m)/(1 - G(\mu_S))$.

   (b) Define $Z = T_m - \mu_S$. Thus Z represents the amount of time remaining after launching a sortie to attack the target until the target moves. The distribution function for Z, a positive number by definition, is
   $G(z \mid T_m > \mu_S) = G(z + \mu_S)/(1 - G(\mu_S))$.

   (c) Take the distribution just calculated and call it $H(z \mid T_m > T_s)$, the conditional distribution for Z when T_m is greater than T_s. That is,
   $H(z \mid T_m > T_s) = G(z \mid T_m > \mu_S) = G(z + \mu_S)/(1 - G(\mu_S))$
   Such an assignment does not follow from the axioms of probability calculus, but it may, in conditions known to the analyst, be a good assignment as an approximation.

It is up to the analyst to assess the quality of the threshold heuristics in specific applications. They are not necessarily close approximations. The smaller the standard deviation of T_s, and the larger the standard deviation of T_m, the better the approximation.

4.5 Weapon-Target Pairing

4.5.1 Introduction

Targets are located by surveillance and reconnaissance and then attacked with weapons. Improvements in surveillance and reconnaissance can pay off with increases in the number of targets attacked and eliminated. To model this relationship between operational intelligence and operational results, one must account for the assignment of strike sorties to targets. In military operations, such assignment results from a complex process of target analysis. A CAPE model must represent the function of target analysis in order to link surveillance and reconnaissance to the operational results.

In Dynamic CAPE the strike sortie is the basis of target analysis. Target analysis is modeled in terms of the sorties that could be launched in a given time period. The model consists of the following procedure, called the weapon-target-pairing algorithm:

- Rank all the kinds of individual strike sorties that could be flown in terms of their desirability;
- Starting at the top of the list, fly as many of the first type of sortie as possible, considering the numbers of targets left to strike, weapons left to allocate, and platforms available to fly; and
- Continue down the list, flying as many sorties of each type as possible, considering all the same factors.

While this procedure allocates all sorties for the time period at once, as if for a daily air tasking order, it is meant to approximate the allocations that would be made when sorties are allocated continuously throughout a time period.

The weapon-target-pairing algorithm is highly aggregate, not modeling any specific sub-functions of target analysis. It is heuristic, not based on any mathematical programming technique or on any target analysis technique. It is intuitively appealing because it is logical and seems analogous to the way target analysis should take account of many factors.

The weapon-target pairing algorithm has three broad steps, explained below: (1) Calculate pairing values; (2) Rank sorties by pairing value; and (3) Select sorties.

4.5.2 Calculate Pairing Values

Use a pairing value formula to calculate a value for each possible strike sortie that may be flown, considering all possible combinations of platform types, munitions, target types, and range bands. Various pairing formulas have been used. The basic formula is as follows:

$$Pairing\ Value = T_V \times EKS \times P_C \times P_{COM}$$

Where the parameters are

- Target value, T_V (optional)[13]
- Expected kills per sortie (*EKS*), from a table of *EKS* indexed by platform type, munition, and target type.
- P_C, the probability that the sortie can reach the target s observed location before the target moves out of the strike system s engagement range. Call P_C the probability of *capture,* with the connotation that the strike sortie s target acquisition system can find

[13] Seemingly the most important factor, it has been difficult to develop consensus on the relative values of the various target types. In analyses with Dynamic CAPE, all target types in all range bands were treated as equal in value because consensus could not be achieved.

the target. Capture does not guarantee a successful attack, but it is a necessary precondition for success. The probability is calculated for three different cases:

(a) For mobile targets that are located when they are stationary, and are not known to have moved, P_C is calculated from various parameters of the target movement model (see Appendix B) and from a model of the sense-to-respond time for the total sensor-to-shooter system. The probability varies by sensor platform type, target type, range band, strike platform, and strike weapon.

(b) For mobile targets that are located when they are moving, P_C is taken to be 1.0, under the assumption that the target is being continuously tracked.

(c) For non-mobile targets, P_C is taken to be 1.0, under the assumption that the geolocation error of the sensor system is nil as compared to the requirements of the weapon system. That is, if the sensor produces an adequate image to identify the target, it will have a small geolocation error.

- P_{COM}, communications throughput, the probability that the exploited information for a collected target is provided to target analysts. In practice, the same probability has been assigned for all target classes and range bands in the same theater. It has differed only with theater. It would affect the allocation of strike sorties across theaters.

The pairing-value formula may be interpreted as follows: The pairing value is a benefit received for launching one sortie. The benefit comes from killing the number *EKS* of targets that have value T_V. The benefit is discounted by the factors P_C and P_{COM}.

4.5.3 Rank Sorties by Pairing Value

Sort, in decreasing order of the pairing value, all the possible sorties made up of combinations of platform type, weapon type, target type, and range band.

4.5.4 Select Sorties

Beginning with the sortie at the top of the list, and continuing down the list until as many platforms, weapons, or targets are allocated as possible (without duplication), do these steps:

1. Determine the number of sorties to fly:

   (a) Determine the theoretical number of sorties it would take to eliminate all the *N* targets requiring service during the time period, given one could eliminate *EKS* targets certainly with each sortie. (If, for example, the *EKS* is 0.67, then it theoretically takes ($1 \div EKS$) or 1.5 sorties to eliminate one target.) The number of sorties it would take to eliminate *N* targets is $N \div EKS$. (In the CAPE methodology, *N* is calculated as an expected value elsewhere in the model, and then it is used to calculate $N \div EKS$.)

(b) Determine the number of sorties that could be flown with the number of weapons available to be allocated. This number is just the total number of weapons available divided by the loadout (number of weapons per sortie).

(c) Determine the number of sorties that could be flown with the number of strike platforms available, using the potential sorties that could be flown per day (sortie rate, p. 24).

(d) The number of sorties to fly is the minimum of the numbers calculated in (a), (b), and (c) above, rounded down to the nearest whole number.

2. Subtract the numbers of weapons and platforms allocated to the number of sorties just chosen above from the numbers available for allocation to targets.

3. Subtract the number of targets attacked by the sorties from the total of targets requiring service.

4. Pick the next sortie on the sorted list, and return to Step 1.

4.6 Calculating Dynamic Results — Setting Up Time-Step Simulations

4.6.1 The Basic Approach for CAPE Simulations

Following CAPE Principle 3 (p. 10), all simulations with CAPE are deterministic and time-step. That is, no random-number generation is done, no statistics of such random numbers are calculated, and the output state descriptions of one time period are used as inputs to compute the output state descriptions of the next time period.

In practice, many CAPE inputs are probabilistic, and therefore the output state descriptions for any given time period are necessarily probabilistic, not deterministic. Nevertheless, in the CAPE approach to simulation (e.g., as exemplified in Dynamic CAPE), one uses the expected values of the probabilistic outputs from one time period as if they were the deterministic inputs for the next time period. This is a heuristic approximation, and its validity needs to be assessed by the analyst for each specific model. Its results are indicative of patterns that might be seen over time, but the results are not definitive for those patterns. It is the best that can be done without increasing the computations many fold by using random number generation and statistical methods.

4.6.2 A Method for Verifying CAPE Simulations

Though it has not been done yet, one could use a probabilistic approach to test the accuracy of Principle 3 s heuristic approximations. The objective of such testing would be to confirm, for a limited set of scenarios, that the expected values calculated with Principle 3 s heuristic approximation are comparable to those that would be found through probabilistic methods. The calculations internal to each time period would be the same, but the outputs of the time periods would be randomly generated samples instead of expected values. These random samples would feed the next time period as inputs. The only difference in calculation would be in generating the random samples instead of the expected values.

Consider the calculations in the Dynamic CAPE model. The time-varying input variables for simulation are numbers of targets (by target class and range band), numbers of strike platforms available to be flown (by platform type), and numbers of weapons available for sorties (by weapon type). In each time period, these numbers change to model the effects of attacking the targets with the platforms and weapons. They also change to model the deployment of new resources and to model the doctrine followed in different phases of the war. Figure 1-1 illustrates the changes (Δ s) for the first three time periods. The changes for a time period are a function of the targets, weapons, and platforms available at the start of the period. The same CAPE model is used to calculate the changes in the different time periods. It produces different outputs for the periods because it has different inputs for each period.

Figure 1-1. Time-Step Simulation

The original numbers of targets, weapons, and platforms are successively reduced or increased in the time periods. The model for each time period is the same, but the inputs differ and so the outputs differ, too.

	Period 1 Model		Period 2 Model		Period 3 Model	
Targets →	ΔTgts →	Targets' →	ΔTgts →	Targets'' →	ΔTgts →	Targets'''
Weapons →	ΔWpns →	Weapons' →	ΔWpns →	Weapons'' →	ΔWpns →	Weapons'''
Platforms →	ΔPltfms →	Platforms' →	ΔPltfms →	Platforms'' →	ΔPltfms →	Platforms'''

4.6.3 Generating Random Samples to Verify CAPE Simulations

Currently, the Dynamic CAPE analysis generates probabilities at each time period that various targets will be destroyed and that strike aircraft will be lost in combat. It uses these probabilities to calculate the expected numbers of targets destroyed and aircraft lost. If one attacks N targets of a specific type in a range band, and if the probability of destroying one such target is P, then the expected number of targets destroyed is $P \times N$.[14]

A statistical approach for verifying the Dynamic CAPE calculations would use the same probabilities (P)at each time period to generate random samples instead of to calculate expected values for the numbers of targets destroyed and aircraft lost. The random variables would be the inputs for the next time period. They would substitute for the expected numbers used as inputs in the CAPE heuristic approximation.

For instance, one may generate a random sample M of the number of targets destroyed when N targets are attacked and when the probability of destroying a single target is P. One generates this random sample as follows: First generate N independent uniform random variates,[15] and then count how many of them are less than or equal to P. The count is the sample that was to be generated, M.

In each place where CAPE s heuristic simulation calculates $P \times N$ to get the expected number of targets destroyed, one generates the random sample M instead. This requires more calculation than merely multiplying $P \times N$ to get the expected number. However, it is not an impossible amount of calculation.[16]

4.7 Special Techniques — Calculation of Probability Distributions

4.7.1 Introduction

The purpose of CAPE modeling is to support exploratory studies of future C4ISR systems. For such use, the typical probabilistic information available to modelers is notional, intuitive, or based on professional judgment. In some cases, however, CAPE modelers have access to high quality statistical information that they wish to use in their models. The

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14 P must be derived from the number of targets attacked, the number of weapons used in the attack, and the probability of destroying the target with a single weapon. The factors described in the section on weapon target pairing would be involved: *EKS*, P_C, and P_{COM}.

15 A standard uniform random variate is a real number selected from the closed interval (0 to 1.0). It is a *uniform* random variable because the probability that a random sample is within any arbitrary sub-interval of (0 to 1.0) is equal to the size of the sub-interval. For example, the probability that the variable turns out to fall in the sub-interval (0.2 to 0.5) is 0.3.

16 Assuming that it takes one multiplication and one addition to generate a random sample, and assuming that multiplication takes much more time than the addition, one could argue that it takes N multiplications to calculate M and only one multiplication to calculate $P \times N$. So the increase in calculation to generate a random sample instead of an expected value is proportional to N.

information may be in the form of historical data or of statistics (the mean and variance) for historical data. This section of the paper describes how CAPE modelers incorporate such probabilistic information.

4.7.1.1 Functions of Random Variables

When high quality statistical information is available, it can be used to improve the validity of CAPE models. This is done primarily by more careful calculation of the results when two or more random variables interact to create another (dependent) random variable. In such cases, one is using higher quality information to calculate better information about a function of random variables.

4.7.1.2 Pairwise Decomposition of Functions

When more than two random variables combine in a complex function to form another random variable, a basic strategy for developing probabilistic information about the function is to decompose the function into sub-functions that combine two random variables at a time. Then one develops the probabilistic information for the full function by successively combining information about only two random variables at a time. For instance, the function $Z = ((X - Y)/S) + T$ can be decomposed so that information is developed in the following sequence, developing probabilistic information first about Z_1, then Z_2, and finally Z itself:

$Z_1 = X — Y$

$Z_2 = Z_1 \div S$

$Z = Z_2 + T$

4.7.1.3 Overview of CAPE s Fractile and Beta Methods

MITRE s CAPE modelers have used two methods to calculate improved information about functions of random variables.[17] They are the Fractile Method and the Beta Method. Both methods use the pairwise decomposition strategy just described. Consistent with the second principle of CAPE modeling, both methods also use analytic approximations that achieve improved results while minimizing the calculations required. The Beta Method is especially appropriate when one has the statistics of historical data: mean, variance, maximum, minimum. The Fractile Method is especially appropriate when one has the historical data itself, not just the statistics that represent the data.

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[17] See Henry Neimeier, Nonlinear Multiattribute Risk Simulation, unpublished paper, ca. January 1996, where both methods are defined and described.

With the Fractile Method, one uses discrete random variables to approximate continuous random variables, selecting values for each discrete variable that match equally-probable intervals in the continuous variable s domain. While a continuous random variable can take on an infinite number of possible values, its approximation, a discrete random variable, is constructed to take on only a few values. This makes it feasible to compute the function of two variables for all possible (discrete) combinations of the variables. For instance, two discrete variables with five equally-probable values each for miles traveled and fuel consumed can easily be used to compute 25 separate, equally probable values for miles per gallon. The resulting 25 values can be used to find the average miles per gallon, or they can be used in other probabilistic calculations.

The Beta Method is somewhat simpler to implement than the Fractile Method. It applies when the function of the random variables is arithmetical, involving only addition, subtraction, multiplication, and division. Instead of approximating the random variables with distributions, CAPE modelers use standard probability formulas to combine the means and variances of the random variables to approximate the mean and variance of the arithmetical function of the variables. They also keep track of the two bounds of the arithmetical function, which are its greatest value and its least value. Then they fit the bounds and the approximate mean and variance to a beta probability density function for the function.

The Beta Method, unlike the Fractile Method, requires that the domains of the random variables and of the function of the variables be bounded finitely. That is, one must be able to reasonably approximate each distribution with a finite domain, specifying a maximum value and a minimum value.

With both the Fractile Method and Beta Method, one ends up with a probability distribution or density function for the function of the random variables. This result may then be used to calculate better information about yet another function of random variables, to implement the threshold method (p. 34), or to find the mean of the function of the random variables.

4.7.2 The Fractile Method

4.7.2.1 Introduction

Let us say that two independent continuous random variables, X and Y are to be combined by the function $f(X,Y)$ into a third random variable Z. So, $Z = f(X,Y)$. Let us also say that X, Y, and Z may take on values within bounded or unbounded sets of the real numbers (for instance, from zero to twenty, or from minus infinity to twenty). Finally, let us say that a continuous, monotonically increasing cumulative distribution function $G(x)$ is defined for X, and another $H(y)$ is defined for Y.[18]

In the Fractile Method, a discrete random variable, X , with equally probable values of X , is selected to approximate the random variable X. Another, Y , is selected to approximate the random variable Y. All combinations of X and Y are combined with the function $f(X,Y)$. From the result, a set of equally probable Z values is selected to represent the random variable Z.

4.7.2.2 Constructing discrete approximations to the random variables

The possible values of X are selected from the domain of X. Likewise the possible values of Y are selected from the domain of Y. For instance, if X (or Y) is defined as a real number varying between 15 and 35, then the possible values of X (or Y) might be chosen to be 17, 21, 25, 29, and 33. The method for selecting values of X is described next in terms of X and $G(x)$. The same method is used to select values of Y using $H(y)$ instead of $G(X)$. The method has been called the Bracket Median method. [19]

To select the values of X , we imagine that the domain of X is divided into a number (e.g., 3, 5, 7, 9) of equally probable intervals. The intervals are not equal in terms of X itself, but in terms of $G(x)$. For instance, if x_a and x_b divide the domain of X into three equally probable intervals, where $x_a < x_b$, then $G(x_a) = 1/3$ and $G(x_b) = 2/3$. So the interval $(-\infty, x_a)$ has probability 1/3, the interval (x_a, x_b) has probability 1/3, and the interval (x_b, ∞) has probability 1/3.

After the domain of X is divided into equally probable intervals, we find one value of X to represent each interval. We could select X to be the average value of X within the interval, or — if the interval is bounded — the midpoint of the interval. Instead, we select X as the median of X in the interval. Thus X is the value of X such that the conditional probability that X is greater than X , given X is in the interval, is 0.5.

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[18] The cumulative distribution function (CDF) for an independent random variable X is a function $G(x)$ that gives at any value x of X the probability that X is less than or equal to x, i.e., $G(x) = \text{Prob}(X \le x)$. The derivative of $G(x)$ with respect to x is the probability density function for X, $g(x)$. The fractiles of the random variable X are defined in terms of the CDF. The p fractile is defined as the value of X, x_p, for which $G(x_p) = p$. For instance, the 0.8 fractile is the value of x for which $G(x) = 0.8$.

[19] The Bracket Median method for approximating a continuous probability distribution is described in Robert T. Clemen, *Making Hard Decisions; An Introduction to Decision Analysis* (Boston, PWS-Kent Publishing Company, 1991), p. 220.

When considered from the standpoint of $G(x)$, not from the standpoint of X, the medians defining the possible values of X occur at the midpoints of the equally probable intervals. Suppose, as described above, that x_a and x_b divide the domain of X into three equally probable intervals, so that $G(x_a) = 1/3$ and $G(x_b) = 2/3$. Let us call the medians of X in these three intervals x_1, x_2, and x_3. The medians are the values of X that make $G(x_1) = 1/6$, $G(x_2) =1/2$, and $G(x_3) = 5/6$. These are the midpoints of the three intervals (0 to 1/3), (1/3 to 2/3), and (2/3 to 1).

Expressed in a formula, the medians x_i for N equally probable intervals are chosen at the points where $G(x_i) = (i - 0.5)/N$, for i ranging from 1 to N. The selected values for N equally-probable intervals are all considered equally probable, with probability $1/N$.

4.7.2.3 Constructing an approximation of the function of the random variables

To select values of Z to represent Z, first we use the Bracket Median method to construct X and Y. We construct X and Y both to have the same number of possible values, N, and we make sure the number of possible values is odd (e.g., 3, 5, 7, 9). Then we calculate N^2 candidate values of Z by finding values of $Z = f(X, Y)$ for all N^2 possible combinations of X and Y. The candidate values are all equally probable; each has probability $1/N^2$.

These N^2 values of Z make a fine discrete approximation of Z, except that there are too many of them if Z is going to be combined in a pairwise decomposition process with more random variables. Accordingly, the N^2 values of Z are condensed to N values, as follows. First the values are sorted in order from least to greatest. Then, beginning at position $(N+1)/2$, every Nth value is selected. (The start at position $(N+1)/2$ discards an equal number of values at the start and end of the sorted set of values.) The resulting N values are all considered equally probable, with probability $1 \div N$. They are the Fractile Method s discrete approximation of random variable Z.

The reason for condensing the N^2 values back to N values is to keep the calculations at a manageable level when several other functions of random variables will be computed. For instance, if we are interested in a function of four different random variables, and we approximate each with a 9-element discrete variable, we would in principle have to compute $9\times9\times9\times9 = 6{,}561$ combinations of the discrete variables. But if we decompose the problem into operations on two variables at a time, and condense back to nine elements after each operation, then we have to compute only $9\times9 + 9\times9 + 9\times9 = 243$ combinations. In general, to generate a discrete function of M independent random variables with the Fractile Method, condensing at each stage, one computes the values of only $(M-1)N^2$ combinations. Otherwise one would compute the values of N^M combinations.

The quality of the Fractile Method s results increases with N. That is, nine equally probable intervals give more accuracy than three intervals. Henry Neimeier has compared the accuracy of the results when various values of N are used with various functions $f(X,Y)$.[20]

4.7.2.4 Procedural Summary of the Fractile Method

The Fractile Method has the following steps for calculating N equally probable representative values of $Z = f(X,Y)$:

1. Choose an odd number, N (e.g., 3, 5, 7, or 9).
2. Calculate values $x_1, x_2, \ , x_{N-1}, x_N$ to represent X in using $f(X,Y)$; calculate them as values of x (i.e., fractiles) corresponding to

   $x_i = G^{-1}(F_i)$ for $i = 1$ to N
   where $F_i = (i - 0.5)/N$ and
   where inverse function $G^{-1}(u)$ gives the value of x for which $G(x) = u$
3. Calculate values $y_1, y_2, \ , y_{N-1}, y_N$ to represent Y in using $f(X,Y)$; calculate them as values of Y (i.e., fractiles) corresponding to

   $y_i = H^{-1}(F_i)$ for $i = 1$ to N
   where $F_i = (i - 0.5)/N$ and
   where inverse function $H^{-1}(u)$ gives the value of y for which $H(y) = u$
4. Using $Z = f(X,Y)$, calculate values of Z for all N^2 combinations of the representative values of X and Y. (These N^2 values are all equally probable since they represent equally probable combinations of X and Y.)
5. Sort the N^2 values of Z in increasing order.
6. Select N of the N^2 values of Z as follows: Start with the value in position $(N+1)/2$ down the list; then select every N^{th} item down the list from there. Discard the rest of the N^2-N values of Z.

The values of Z selected in Step 6 are the N points for a discrete random variable Z that approximates the random variable Z. The N points are equally probable, with probability equal to $1 \div N$.

4.7.2.5 Using the Fractile Method with Dependent Variables

The Fractile Method s procedure, as described above, applies only to functions of independent random variables. However, the method can be extended easily for functions of

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[20] Henry Neimeier, Nonlinear Multiattribute Risk Simulation, unpublished paper, ca. January 1996.

two dependent variables. Letting X and Y be dependent random variables, we now define $G(x)$ to be the marginal distribution function for X, and we define $H(y|x)$ to be the distribution function for Y conditioned on knowing that the value of X is x.[21] As before, we use $G(x)$ to calculate N representative values of X. Then we use $H(y|x)$ to calculate N equally probable representative values of Y for each of the representative values of X. That is, for each representative X value, calculate a different set of N representative Y values. A total of $N+N^2$ fractiles must be calculated. The reason is that Y now depends probabilistically on X, so that there is a different distribution function of Y for each value of X.

Though the two random variables are dependent, there are still only N^2 combinations of X and Y to consider for computing the Z values with $f(X,Y)$. It is just that the representative Y values have to be recalculated for each different representative value of X. Otherwise, the calculation for Y proceeds as described in the basic and Bracket-Median processes above.

The Fractile Method for dependent variables is easy to apply when there are only two random variables. However, the process is more complex for more than two variables, and the amount of calculation increases exponentially with the number of variables. The increase in calculation for dependent variables comes in the number of fractiles computed, not in the number of combinations of x and y. Whether the variables are dependent or not, one has to compute only $(M-1)N^2$ combinations. In general, however, for M dependent variables, one has to calculate $N(N^M-1)/(N-1)$ fractiles. For instance, using $N = 9$ and $M = 4$, one would have to calculate 7,380 fractiles to find a function of four dependent random variables. By contrast, for M independent variables, one has to compute only $M \times N$ fractiles, or $4 \times 9 = 36$ fractiles in the example.

4.7.3 The Beta Method

4.7.3.1 Introduction

The most typical functions of random variables in CAPE models are addition, subtraction, multiplication, and division. For instance,

- One adds the number of targets in each range band to find the total number of targets in the theater.

[21] Let us define $G(x,y)$ to be the joint distribution function for X and Y, where $G(x,y)$ is the joint probability that X is less than or equal to x *and* that Y is less than or equal to y. Then the *marginal distribution function* for X is simply $G(x) = G(x,\infty)$. The *conditional distribution* for Y, $H(y/x)$, is the ratio of the partial derivatives of $G(x,y)$ and $G(x)$, both derivatives taken with respect to X: $H(y|x) = G_x(x,y)/G_x(x)$. (Be careful in calculating $H(y/x)$ if $G(x,y)$ is not continuously differentiable for all combinations of x and y.)

- One divides the number of images awaiting exploitation by an exploitation rate to find the time needed to exploit all the images.
- One multiplies a sensor s collection rate by the number of hours it collects images to find the total number of images collected.

For such arithmetic functions of random variables there exist standard formulas for calculating the mean and variance of the function from the means and variances of the random variables. These formulas for arithmetic functions are often easier to use than the Fractile Method since one does not need to calculate the fractiles for each random variable.

However, the mean and variance alone for a function of random variables may be insufficient for our needs in CAPE modeling. We may also need a complete probability density function (PDF) or cumulative distribution function (CDF) for the function of the random variables. How does one generate a PDF for the arithmetic functions?

The beta distribution is known to be a robust PDF that one can apply, using its four parameters, to approximate the shapes of many different PDFs. The four parameters of the beta PDF can be calculated from four statistics: the maximum, minimum, mean, and variance of its random variate. Hence, if we calculate the mean and variance for a function of two random variables with the standard formulas, and if we simultaneously calculate the maximum and minimum of the function, we can use the beta distribution to approximate the PDF for the function of random variables.[22]

In the Beta Method, the independent random variables are represented by only four statistics — the mean, variance, maximum, and minimum — instead of by the N fractiles of the Fractile Method. Using the decomposition strategy, whenever two variables are combined into a subfunction, the eight (2×4) statistics for the two independent variables are used to generate the corresponding four statistics for the function of the random variables. Only after all the subfunctions are processed, so that one has the four statistics for the complete function, does one use the final four statistics to calculate the parameters for the corresponding beta distribution. While the Beta Method s name comes from the fitting of a beta probability distribution, most of the work is in the calculation of the four statistics, which do not involve the beta distribution at all.

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22 If either of the two random variables is unbounded (can become infinitely large), then the beta distribution s maximum and minimum probably cannot be calculated. Instead, to use the beta distribution, it is necessary to say that for all practical purposes the random variables are bounded and to specify the bounds.

4.7.3.2 Calculating the Four Statistics

For two random variables, X and Y, and one arithmetic function, $Z = f(X,Y)$, formulas are provided below to calculate the statistics of Z in terms of the statistics of X and Y. In the formulas, the mean, variance, maximum, and minimum, are represented by the symbols μ, V, *Max*, and *Min*, respectively, with appropriate subscripts to designate X, Y, and Z. All the formulas assume that X and Y are independent random variables. The formulas for *Min* and *Max* assume that both X and Y are non-negative real numbers, which is typical for CAPE analyses. (Other formulas, more complex, are needed if either X or Y can be negative or if they are dependent random variables.)

Table 1-1. Arithmetic Functions of Two Independent Variables

Sum, $Z = X + Y$

$\mu_z = \mu_x + \mu_y$ and $V_z = V_x + V_y$ (exact, not approximate)

$Min_z = Min_x + Min_y$

$Max_z = Max_x + Max_y$

Difference, $Z = X — Y$

$\mu_z = \mu_x - \mu_y$ and $V_z = V_x + V_y$ (exact, not approximate)

$Min_z = Min_x - Max_y$

$Max_z = Max_x - Min_y$

Product, $Z = XY$

$\mu_z = \mu_x\mu_y$ and $V_z = V_xV_y + V_x\mu_y^2 + V_y\mu_x^2$ (exact, not approximate)

$Min_z = Min_xMin_y$

$Max_z = Max_xMax_y$

Division, $Z = X \div Y$

$\mu_z = (\mu_x \div \mu_y)[1 + (V_y \div \mu_y^2)]$ (approximate)

$V_z = (\mu_x \div \mu_y)^2[(V_x \div \mu_x^2) + (V_y \div \mu_y^2)]$ (approximate)

$Min_z = Min_x \div Max_y$

$Max_z = Max_x \div Min_y$

4.7.3.3 Defining the Beta Distribution

CAPE modelers use the beta distribution to approximate other PDFs because it can represent a skewed mono-modal distribution for which the mean and variance can be selected separately.[23] The normal distribution is less preferred because it cannot represent a skewed distribution, though the mean and variance can be chosen separately. A triangular distribution can be skewed, but its variance cannot be selected separately from its mode.

The beta distribution is most naturally applied to random variables that are uncertain proportions: What fraction of American registered voters will vote this year? What fraction of customers will return for a second visit to the restaurant? Zero at the bottom and 1.0 at the top bound such random variables. In CAPE analyses, many of the random variables are also bounded by practical considerations, though they are not proportions between zero and 1.0. Typical bounded random variables for CAPE analyses are sorties per aircraft per day and images exploited per day per analyst.

The formula for the beta probability density function $b(w)$ for random variable *W,* where *W* must be between a minimum of 0.0 and a maximum of 1.0, is as follows:

$b(w) = k\, w^{r-1}(1-w)^{n-r-1}$

Where
n, r, and w are real numbers (not necessarily integers);
$n > r > 0$; $1 \geq w \geq 0$; and
k is a normalizing constant.

In the formula, note that when $n = 2r$, $b(w)$ is symmetric about the value $w = 0.5$. Therefore the ratio $n/2r$ governs the skew of the distribution to the left or right of 0.5.

The mean value of *W* is $\mu_W = r/n$, and the variance is $V_W = r(n-r)/n^2(n+1)$. One may also calculate the variance as $V_W = \mu_W(1-\mu_W)/(1+n)$. Therefore, for a given mean value μ_W, the value of n determines the variance of the distribution.

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23 The skew of the distribution is determined practically by the relative position assigned to the mean between the maximum and minimum. Suppose the minimum and maximum estimated for an uncertain random variable are 100 and 200. Someone might also estimate a mean of 120. This causes a skew to the left.

4.7.3.4 Fitting the Beta Distribution to the Four Statistics

For an arbitrary bounded random variable Z, if we are given the four statistics of minimum, maximum, mean, and variance, we fit them to a beta distribution as follows:

1. Let the linear function $W = (Z — Min_z)/(Max_z — Min_z)$ transform the random variable Z to the range of a beta variate W, namely 0.0 to 1.0. Likewise the inverse function $Z = W \times (Max_z - Min_z) + Min_z$ converts beta variates to the domain of Z.

2. Given the linear function, calculate the mean and variance of W from the four parameters of Z as follows:

   $\mu_W = (\mu_z — Min_z)/(Max_z — Min_z)$

   $V_W = V_z/(Max_z — Min_z)^2$

3. Calculate the beta parameters, n and r, as follows:

   $n = [\mu_W(1-\mu_W)/V_W] — 1$

   $r = \mu_W n.$

Caveat. The beta distribution with parameters n and r will have the same mean and variance, appropriately transformed, as the random variable Z. The shape of the beta distribution will usually be a good match to the shape of the random variable s distribution. However, there are cases where it is not a good match, such as when Z s distribution function is lognormal. Though it is usually a good approximation, it is up to the analyst to judge the suitability of the approximation in each case.

Appendices and Glossary

Appendix A

Justification of the CAPE Principles

The three principles of the CAPE methodology produce C4ISR models that differ greatly from many existing models used for military operations research. Such existing models are used to statistically estimate the performance of systems by repeated next-event simulations of the systems operation. MITRE staff believe that the simplifications of the CAPE methodology, which avoids statistical sampling, are more appropriate to the class of problems they address than the other models. There are several reasons, as follow:

First, the class of problems to which CAPE models are applied stretch the imagination into areas of C4ISR not previously modeled and for which definitive information for modeling often does not exist. The CAPE methodology supports an experimental development of conceptualizations of the study problem throughout the course of the study, using sensitivity analysis to discover major effects. Generally, a discrete-event simulation is a poor vehicle for trying out new broad conceptualizations or for conducting sensitivity analyses:

- A discrete-event simulation takes much time and computational resources to produce statistically valid results. This inhibits broad reformulation of the models, and it inhibits sensitivity analysis. Often, only a few iterations of the simulation can be generated, when hundreds are required for statistically significant results.
- In discrete-event simulation, the causal chain linking random variables to the MOEs is obscured through the use of random number generation and statistical sampling. That is, the sampling process introduces noise that makes it hard to measure the sensitivity of MOEs to changes in model parameters. In CAPE models, where there is no sampling, the sensitivity effects are easy to measure precisely.

Second, the class of studies to which CAPE models are applied involves the quick exploration of a large, complex C4ISR system, the identification of key factors affecting performance, and the comparison of alternative hypothetical combinations of new ISR capabilities in terms of mission effectiveness. Discrete-event simulation models are generally unsatisfactory for such work. They take too much time for model-building, data-collection, verification, and validation.

Third, discrete-event simulation models have higher fidelity than CAPE models, because they represent individual events that actually occur in life, and they model a time-line with state histories. However, such higher fidelity does not of itself guarantee higher accuracy or validity. In fact, it may guarantee the opposite. If a phenomenon is well understood at the aggregate level, for instance, but is poorly understood at the event level, then an analysis based on an event-driven simulation model will lead to less certain results than one based on an aggregate model. In the class of studies to which CAPE is applied, the C4ISR phenomena generally are not understood well at either the aggregate level or the event level, and the questions under study are framed at the aggregate level. Therefore MITRE s strategy is to model and analyze questions at the aggregate level with the CAPE methodology.

Fourth, since there is so much unknown in the C4ISR field, it is very difficult to validate the parametric inputs to any model. To develop high fidelity, event-level models, one must generally specify much more parametric input than one would specify for use in aggregate CAPE models. Event-level models therefore increase the task of validation. If time and knowledge are short, an aggregate CAPE model with validated inputs can be more believable than a complex event-driven model that has too many inputs to be well validated.

Appendix B

Deriving a Model of Target Movement

by Kenneth P. Kuskey, Ph.D.
with contributions by Brian K. Schmidt, Ph.D.[24]

March 2000

B.1 Introduction

For many sensor-to-shooter (STS) systems, significant delay occurs between sensing a mobile ground target and engaging it with a weapon system. Such delay is typical for STS systems based on still-image sensors such as electro-optical, infrared, and radar. (They are less typical of continuous-coverage systems such as real-time video and moving-target sensors.) During the delay, the target may move so far that it cannot be found again and successfully engaged by the weapon system sent to look for it. One major purpose of new investment in such still-image STS systems is to shorten the delay, so that the mobile target is more likely to be found and engaged successfully.

Target movement involves the interaction of complex probabilistic phenomena. Because of the limited time available for completing MITRE s initial CAPE analyses of STS systems in the C4ISR Mission Assessment of 1997, MITRE staff modeled target movement quickly with heuristic methods instead of deriving models from fundamental considerations.[25] Later, during 1998 and 1999, a probabilistic model of target movement was derived with basic probability calculus from explicit assumptions. An overview of the new target movement model follows, describing its assumptions and derivations.

B.2 General Assumptions

In CAPE analyses, target-movement models represent broad target classes instead of individual targets. Each target class may include several different kinds of actual targets.

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24 The author gratefully acknowledges MITRE staff member Dr. Brian K. Schmidt s contributions to this work. Brian pointed out the bias one experiences in observing target movement. This stimulated the author to model uncertainty in pause time as a function of sensor revisit time. Brian developed alternatives, often shorter and deeper, for many derivations the author reports here. He provided key equations (see Section B.8) to provide an exact analytic solution where the author had been content with an approximation.

25 CAPE is an acronym for C4ISR Analytic Performance Evaluation. C4ISR is an acronym for Command, Control, Communications, Computers, Intelligence, Surveillance, and Reconnaissance.

Typical target classes are short mobile column and large long-dwell target. The CAPE modeler s basic assumptions about the movements of such targets are as follows:

- Mobile ground targets alternate between moving and pausing.
- A target s alternating move times T_m and pause times T_p are independent random variables. (Actual values of the random variables are signified by t_m and t_p.)
- A target s location is considered to be a geometric point, even though some targets, such as a mobile column, may be large.
- A target s velocity V in its moves is a random variable, independent of move times and pause times. (Actual values of the velocity are symbolized by v.)
- During move times, the target moves a distance $X = T_m \times V$. The distance is a function of the two random variables T_m and V. (Actual values of distance are symbolized by x.)
- The uncertainties in T_m, T_p , and V for each target class are represented by triangular probability density functions (PDFs): $f_m(t_m)$, $f_p(t_p)$, and $f_v(v)$. These PDFs typically are *conditional* PDFs, differing for each combination of range band and phase of the war. The PDFs average over all possible reasons for pauses and moves of the target class in the various range bands and phases of the war.

B.3 Pausing-Target Model

For imagery-based STS systems, the image may be recorded when the target is moving or when it is pausing. Arguably, the most important case to analyze is the image made of a target that one can tell was pausing. There is potential to strike the target before it moves. By contrast, the moving target is certain to move from its observed location, and it is likely to move too far to be found by a strike sortie (assuming that a pilot cannot find a target that has moved more than about 5 km).

For the target observed pausing, one generally will not know exactly when the pause began nor when it will end. One only knows when the observation was made. The pause may have begun just before the observation, or it may have ended just after the observation. To represent this state of knowledge, the CAPE modeler makes the following assumptions:

- Surveillance and reconnaisance, which are activities to observe targets, occur on a regular cycle. The time between attempted observations is the *revisit time, t_r.*
- One can tell whether a target was pausing at the time it was observed, but the collected information does not indicate when the pause began, nor does it indicate when the pause will end.

- One can determine from past records of observations whether a currently observed target was pausing in the same location at the previous observation. This assumption simplifies the analysis of situations where the revisit time is shorter than the maximum length of time a target pauses. It is not necessary when the revisit time is longer than the maximum pause time.
- For our purposes, the only observations of interest are those in which a target is observed pausing that was not observed pausing in the same location at the previous observation. We assume the decision to attack a target is based on its initial observation. Later observations might locate the target more precisely, or help to call off a strike if the target should move. However, these are taken to be second order considerations as compared to the main action of planning and launching the strike.

Given the assumptions, the modeler s first question about the pausing target is How long will the pause continue after the target is observed? The following derivation produces a cumulative distribution function (CDF) $F_{rp}(t_{rp}/t_r)$ for the target s remaining pause time, T_{rp}, given the revisit time, t_r. The results match the statistics of simulated observations of remaining pause times, where the simulations are based on the assumptions above.

B.3.1 Summary Derivation of the CDF for Remaining Pause Time

1. Consider the current observation and the most immediate prior observation. Let us say that a pause began at some time after the prior observation. That pause may still be in progress at the current observation, or it may have already completed.
2. Let T_s stand for how long it was *before* the current observation that the pause started, a random variable. (So t_s = 1 means the pause started 1 hour before the observation.)
3. Represent uncertainty in T_s as a uniform PDF $f_s(t_s)$ over the domain (0, t_r), where t_r is the deterministic time between revisits of the sensor system. T_s cannot be greater than t_r, since we postulated that the pause began after the prior observation. We are also assuming, by use of the uniform distribution, that pauses will start with equal likelihood at all times between observations.
4. Define the remaining pause time T_{rp} to be a function of the two random variables, T_p and T_s, namely $T_{rp} = T_p — T_s$. Note that T_p may be smaller than T_s, so that T_{rp} may be negative. It is negative in those cases where the pause ends before the current observation. Though unobserved, such pauses are important for this derivation.
5. Derive a CDF $G_{rp}(t_{rp}/t_r)$ for T_{rp} using $f_s(t_s)$ and $f_p(t_p)$, assuming that T_p and T_s are probabilistically independent. In general, the CDF for T_{rp} will include both positive and negative values of T_{rp}, corresponding to observed and unobserved pauses.
6. Only the non-negative values of T_{rp} apply in our situation, since we know that the pause was in fact observed. Therefore,

7. Using the CDF just derived, $G_{rp}(t_{rp}/t_r)$, derive a new conditional CDF for T_{rp} given T_{rp} is non-negative. Call this CDF $F_{rp}(t_{rp}/t_r)$.

The derived CDF, $F_{rp}(t_{rp}/t_r)$, answers the modeler s first question, How long will the pause continue after a target is observed? Interestingly, the answer depends on the revisit time t_r. The shorter the revisit time, the more likely one is to observe the target near the beginning of its pause. Therefore, the shorter the revisit time, the longer one can expect to see the target pause after it is observed. The equations will be developed below.

B.3.2 Detailed Derivation of the CDF for Remaining Pause Time

Steps 5, 6, and 7 of the summary derivation are illustrated here so that readers may understand the basis of the final equations for $F_{rp}(t_{rp}/t_r)$. A derivation from first principles is used to illustrate the relationship between T_{rp}, T_p, and T_s.

B.3.2.1 Framing the Situation

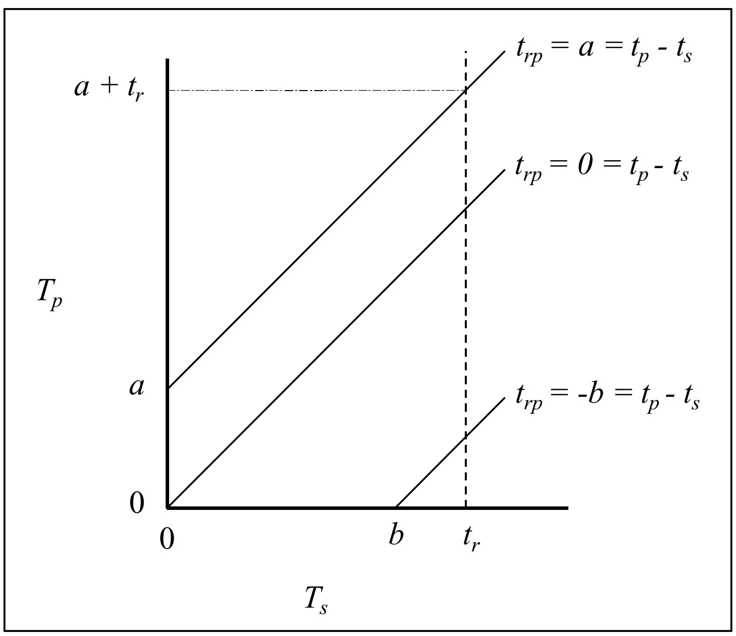

Figure B-1. Calculating the Probability that $T_{rp} \leq t_{rp}$

Consider the graph in Figure B-1. In the graph, the possible values of T_s range from 0 to t_r. The possible values of T_p are shown ranging from zero upward without limit. The three 45¡ lines represent the parametric equation $t_{rp} = t_p$ —$_s t$for three different representative values of t_{rp}, which are a, 0, and $-b$. There is one line for each different value of t_{rp}. Note that the higher the 45¡ line is on the graph, the larger its value of t_{rp}. Also note that the line for t_{rp}

= 0 runs through the origin of the graph. Any 45¡ line below the one running through the origin represents a negative value of t_{rp}.

For any specific value of t_{rp}, the line representing it divides the graph into two parts. Below the line, in the shaded region, are all combinations of T_p and T_s for which T_{rp} is less than t_{rp}. Above the line are all combinations of T_p and T_s for which T_{rp} is greater than t_{rp}.

B.3.2.2 The Mathematical Problem

The probability that T_{rp} is less than or equal to a specific value t_{rp} is equal to the probability that a random combination of T_s and T_p is in the shaded region of the graph that falls below the line corresponding to the value t_{rp}. Because we assume T_s and T_p are probabilistically independent, the calculation amounts to integrating $f_p(t_p)$ and $f_s(t_s)$ over the region below the line to find the joint probability that both T_s and T_p are in the region.

To carry out the integration there are two cases to consider: $t_{rp} > 0$ and $t_{rp} \leq 0$. The integration result differs somewhat in the two cases. However, as it turns out, the more complex result for the case for $t_{rp} > 0$ can be applied in the second case as well. Certain terms in the complex result are always zero in the second case. Consequently, we will document only the first case.

In the figure, consider the line intersecting the T_p axis at a. The region for which $T_{rp} \leq a$ is the trapezoid shown shaded on the graph. It is bounded on the right by t_r since T_s can be no larger than t_r. It is bounded on the top by $t_p = a + t_r$ since that is the largest possible value of t_p consistent with keeping T_r less than or equal to t_r and having $t_{rp} = a$. We want to calculate the probability that random samples of T_p and T_r will both be in the trapezoid.

B.3.2.3 The Mathematical Solution

The shaded trapezoid may be divided into two non-overlapping parts as shown in Figure B-2: (1) a rectangle in which all values of T_r are equally likely, regardless of the value of T_p; and (2) a triangle in which the value of T_p constrains the range for T_s. Because the rectangle and triangle do not overlap, the probability that a joint sample (T_s, T_p) is in the trapezoid may be calculated as the sum of the two probabilities that the sample is in the two parts of the trapezoid:

$$\Pr\left((T_s, T_p) \subset Trapezoid\right) = \Pr\left((T_s, T_p) \subset Rectangle\right) + \Pr\left((T_s, T_p) \subset Triangle\right)$$

We first consider the rectangle, which has values of T_p that are less than or equal to a. In the rectangle, all values of T_s are equally likely for any value of T_p. The probability that a random pair (T_p, T_s) is in the rectangle is calculated by integrating the joint density function $f_p(t_p) \times f_s(t_s)$ over the rectangular region, as follows:

$$\Pr\left((T_x, T_p) \subset Rectangle\right) = \int_0^a dt_p \int_0^{t_r} dt_s f_p(t_p) f_s(t_s)$$

Because all values of T_r are equally likely for all the values of T_p in the rectangle, this equation may be written simply as

$$\Pr\left((T_x, T_p) \subset Rectangle\right) = \int_0^a dt_p f_p(t_p) = F_p(a)$$

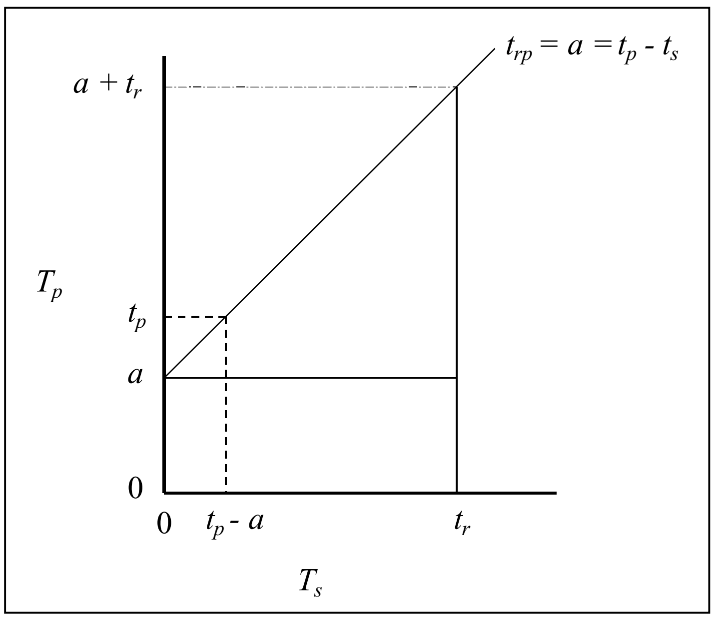

Figure B-2. Two Regions for Calculating Probabilities

Second we treat values of T_p that are greater than a. The region where T_p is greater than a is the triangle that sits on top of the rectangle. In the triangle, only some values of T_s are within the trapezoid, not all possible values. As shown on the graph, with the dashed lines, the allowable range of T_s for a given value of t_p is the interval from t_p—a to t_r The probability that a random pair (T_p, T_s) is in the triangle is calculated by integrating the joint density function $f_p(t_p) \times f_s(t_s)$ over the triangular region, as follows:

$$\Pr\left((T_p, T_s) \subset Triangle\right) = \int_{a}^{a+t_r} dt_p f_p(t_p) \int_{t_p - a}^{t_r} dt_s f_s(t_s)$$

Because $f_s(t_s)$ is a uniform distribution on the domain $(0, t_r)$, the inner integral may be simplified as follows:

$$\Pr\left((T_p, T_s) \subset Triangle\right) = \int_{a}^{a+t_r} dt_p f_p(t_p) \frac{t_r - (t_p - a)}{t_r}$$

This equation may be separated into two integrals:

$$\Pr\left((T_p, T_s) \subset Triangle\right) = \left[(1 + a/t_r) \int_{a}^{a+t_r} dt_p f_p(t_p)\right] - \left[(1/t_r) \int_{a}^{a+t_r} dt_p f_p(t_p) t_p\right]$$

The first integral can be written simply in terms of $F_p(a + t_r) - F_p(a)$. The second integral is more interesting. Let us define a CDF for experienced pause time, T_{ep}, to be $F_{ep}(t_{ep})$, calculated from $f_p(t_p)$ in the following manner:[26]

$$F_{ep}(t_{ep}) = \frac{1}{\bar{t}_p} \int_0^{t_{ep}} t \cdot f_p(t) \cdot dt$$

where the denominator is the average pause time with respect to $f_p(t_p)$, calculated as

$$\bar{t}_p = \int_0^{\infty} t \cdot f_p(t) \cdot dt$$

Let us now introduce $F_p(t_p)$ and $F_{ep}(t_{ep})$ into the probability we were calculating for the triangle:

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26 For an interpretation of experienced pause time, please see the end of this subsection, after the derivation.

$$\Pr\left((T_p, T_s) \subset Triangle\right) = \left(1 + \frac{a}{t_r}\right)\left[F_p(a + t_r) - F_p(a)\right] - \frac{\bar{t}_p}{t_r}\left[F_{ep}(a + t_r) - F_{ep}(a)\right]$$

Adding the two probabilities that (T_p, T_s) is either in the triangle or the rectangle, we obtain the probability that (T_p, T_s) is in the trapezoid:

$$\Pr\left((T_p, T_s) \subset Trapezoid\right) = \left(1 + \frac{a}{t_r}\right)F_p(a + t_r) - \frac{a}{t_r}F_p(a) - \frac{\bar{t}_p}{t_r}\left[F_{ep}(a + t_r) - F_{ep}(a)\right]$$

This equation has been derived with the understanding that a is greater than or equal to zero. In fact, if one derives the probability that (T_p, T_s) is within the trapezoid when a is negative, then the above equation applies, but it is simplified: When a is less than zero, the two terms $F_p(a)$ and $F_{ep}(a)$ are both zero and drop out because the terms are only defined for non-negative values. From the simplified equation, we may note that whenever a is less than $-t_r$, both $F_p(a + t_r)$ and $F_{ep}(a + t_r)$ are zero. So the probability (T_p, T_s) is within the trapezoid is zero. This means that the smallest allowable value of a, without knowing anything further about the possible limits of $F_p(t_p)$, is $-t_r$.

Now we are in a position to complete step 5 of the summary derivation, writing the CDF $G_{rp}(t_{rp}/t_r)$ for T_{rp}, as follows, where t_{rp} has been substituted for a in the equation above:

$$G_{rp}(t_{rp}|t_r) = \left(1 + \frac{t_{rp}}{t_r}\right)F_p(t_{rp} + t_r) - \frac{t_{rp}}{t_r}F_p(t_{rp}) - \frac{\bar{t}_p}{t_r}\left[F_{ep}(t_{rp} + t_r) - F_{ep}(t_{rp})\right] \quad \text{for } t_{rp} \geq -t_r$$

This equation may be simplified in form through the use of an interesting auxiliary function, which we will label $FF_p(t_p)$.[27] It is based on $f_p(t_p)$, as follows:

$$FF_p(t_p) = \int_0^{t_p} dt F_p(t) = t_p F_p(t_p) - \bar{t}_p F_{ep}(t_p)$$

In terms of the auxiliary function, we may write the equation for $G_{rp}(t_{rp}/t_r)$ as follows:

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27 Brian Schmidt brought this auxiliary function to the author s attention.

$$G_{rp}(t_{rp} \mid t_r) = \frac{1}{t_r}\left(FF_p(t_{rp} + t_r) - FF_p(t_{rp})\right)$$

Finally, given $G_{rp}(t_{rp}/t_r)$, we complete step 7 of the summary derivation by calculating $F_{rp}(t_{rp}/t_r)$ as the conditional CDF for T_{rp} when T_{rp} is known to be greater than zero:

$$F_{rp}(t_{rp} \mid t_r) = \frac{G_{rp}(t_{rp} \mid t_r) - G_{rp}(0 \mid t_r)}{1 - G_{rp}(0 \mid t_r)} \qquad \mathit{for}\ t_{rp} \geq 0$$

Note that $G_{rp}(0/t_r)$ has the following form and is easy to evaluate:

$$G_{rp}(0 \mid t_r) = F_p(t_r) - \frac{\bar{t}_p}{t_r} F_{ep}(t_r)$$

When $f_p(t_p)$ is a triangular, uniform, or exponential probability density function, the calculation of $F_{rp}(t_{rp}/t_r)$ is purely analytical, involving no numerical integration or table-lookup procedures.

B.3.2.4 Special Cases: Constant and Once-Only Target Observation

Two special cases of $F_{rp}(t_{rp}/t_r)$ merit consideration. In one case, the revisit time, t_r, gets very small and approaches zero, corresponding to constant observation of the target. In the other case, the revisit time gets very large and approaches infinity, corresponding to once-only observation.

To consider the two cases, we assume that $f_p(t_p)$ is a triangular PDF with three parameters: *min*, *mode*, and *max*.[28] Then $f_p(t_p)$ is zero for values of t_p less than *min* and also for values of t_p greater than *max*. It is positive at *mode* and at all other values of t_p between *min* and *max*. Similarly, $F_p(t_p)$ and $F_{ep}(t_p)$ are zero for values of t_p less than *min*, and 1.0 for values of t_p greater than *max*. In what follows, let us assume without true loss of generality that *min* is greater than zero.

When t_r is smaller than *min*, the factor $G_{rp}(0/t_r)$ becomes zero. That is, interpreting the meaning of $G_{rp}(0/t_r)$, it is impossible for a pause to both start and end between two

[28] Brian Schmidt has provided the author a more general analysis of the two cases in which he assumes only that $f_p(t_p) = 0$ for all t_p greater than some value *max*. He does not need to assume, as the author does, that $f_p(t_p)$ has any particular form, such as the triangular PDF, to obtain the same results developed here.

successive observations when the time between observations is smaller than *min*. So every pause will be observed if t_r is smaller than *min*. In this situation, $F_{rp}(t_{rp}/t_r) = G_{rp}(t_{rp}/t_r)$, and it may be written as follows:

$$F_{rp}(t_{rp} \mid t_r < min) = F_p(t_r + t_{rp}) + \frac{t_{rp}}{t_r}\left[F_p(t_r + t_{rp}) - F_p(t_{rp})\right] - \frac{\bar{t}_p}{t_r}\left[F_{ep}(t_r + t_{rp}) - F_{ep}(t_{rp})\right]$$

From this formula, it may be shown with L H spital s rule that when t_r is zero, so that the target is constantly observed, then $F_{rp}(t_{rp}|t_r = 0) = F_p(t_{rp})$. That is, the PDF of the target s observed pause times is the basic pause time distribution, $f_p(t_p)$.

Now consider the second case, when t_r becomes large. If t_r is larger than *max*, then the formula for $G_{rp}(0/t_r)$ is simplified as follows:

$$G_{rp}(0 \mid t_r > max) = 1 - \frac{\bar{t}_p}{t_r}$$

When this is substituted into the formula for $F_{rp}(t_{rp}|t_r)$, and when we recognize that the two terms $F_p(t_{rp}+ t_r)$ and $F_{ep}(t_{rp}+ t_r)$ both equal 1.0 when t_r is greater than *max*, then all the terms with t_r fall away, leaving the following result that is no longer a function of t_r:

$$F_{rp}(t_{rp} \mid t_r > max) = \frac{t_{rp}}{\bar{t}_p}\left[1 - F_p(t_{rp})\right] + F_{ep}(t_{rp}) = \frac{1}{\bar{t}_p}\left[t_{rp} - FF_p(t_{rp})\right]$$

Consequently, if t_r exceeds *max*, even approaching infinity, the CDF $F_{rp}(t_{rp}|t_r)$ no longer changes when t_r increases. It is constant with respect to t_r.

B.3.2.5 Interpretation of Experienced Pause Time

The PDF $F_{ep}(t_{ep})$ represents the distribution of actual pause times (not just the remaining pause times, but the full pause times) one would *experience* by observation if

- the pauses are generated from a target-movement process based on $f_p(t_p)$;
- observations are made regularly at a fixed time interval (the revisit time t_r); and
- t_r is greater, for practical purposes, than the maximum pause time allowed by $f_p(t_p)$.

The form of $F_{ep}(t_{ep})$ accounts for an observational bias that occurs with large revisit times. With such revisit times, long pause times will cumulatively take up a larger fraction of time in a long history of pauses than will short pauses, as compared to the fraction of long pauses that are generated. Even if short pauses are just as likely to occur when pauses are generated, the process of observation biases the observer toward seeing more of the long pauses than the short pauses.

While $F_{ep}(t_{ep})$ describes the CDF for experienced pause when the revisit time is longer than the maximum pause time, there is a complementary situation in which the revisit time is shorter than the minimum pause time. In that situation, every pause is experienced, short or long, and the appropriate CDF for the experienced pause is $F_p(t_{ep})$, which is the CDF based on $f_p(t_p)$.

B.4 Moving-Target Model

The next question a modeler asks about the mobile ground target is Given it was observed pausing, how far might it have moved from its observed location if we attack it t hours after the observation? To answer the question, we derive a CDF $F_X(x/t,t_r)$ for the distance X the target may have moved in the time t since it was observed, given the revisit time for observations, t_r.

To simplify the derivation of $F_X(x/t,t_r)$, the CAPE modeler makes the following moving-target assumptions:

- During a move, the target moves strictly away from the location of its last pause. This may not always be true, but it simplifies analysis.
- The target starts moving no more than once after it is observed; it may have finished moving and be pausing again at time *t*. This assumption represents an understanding that the first full move after the observed pause will usually move the target such a large distance that it cannot be found and engaged if a strike should arrive after the move is over. If it does not move the target out of range, we are assuming the target will be pausing (for the second time) when the strike arrives.[29]

Given the additional assumptions, it takes two steps to answer the movement question. First, $F_X(x/t,t_r)$ *is* expanded over target velocity v and expressed as the weighted average over velocity of another CDF for X, one that is conditioned on specific values of both t and velocity v. We call the conditional CDF $F_X(x/v,t,t_r)$ and calculate $F_X(x/t,t_r)$ with it as follows:

$$F_X(x \mid t, t_r) = \int_0^{\infty} f_v(v) \cdot F_X(x \mid v, t, t_r) \cdot dv$$

29 Brian Schmidt has sketched a procedure for calculating target movement probabilities in the general case where several moves and pauses are possible during the time *t*.

Second, $F_X(x/v,t,t_r)$ is derived from $F_{rp}(t_{rp}/t_r)$ and $f_m(t_m)$. This derivation requires a bit of backward thinking. The CDF $F_X(x/v,t,t_r)$ is the probability that the distance moved *(X)* is *smaller* than or equal to *x* at time *t* given velocity *v*. We will approach this by deriving the *complementary* CDF, $1 - F_X(x/v,t,t_r)$, which is the probability that the distance moved is *larger* than or equal to *x* at time *t* given velocity *v.*

The complementary CDF is derived through the following line of thinking:

1. For the distance moved to be *larger* than or equal to *x* at time *t,* two events must have occurred:
   - *Event #1:* The pause ended before *t,* leaving enough time (x/v) for the target to move at least as far as *x.* That is, the pause time T_{rp} was less than or equal to $t - x/v$; and
   - *Event #2:* The move starting when the pause ended was at least long enough (x/v) for the target to move the distance *x* before beginning another pause. That is, the move time T_m was greater than or equal to x/v.
2. The two events are probabilistically independent since pause times and move times are probabilistically independent.
3. The probability that both events occur, so that the distance moved at time *t* must be larger than or equal to *x*, is the product of the probabilities of the two independent events.
4. The probability for the first event comes from the cumulative distribution function for remaining pause time, as follows:

$$\mathrm{P}(\mathit{Event\ \#1}) = F_{rp}\left(t - \frac{x}{v} \mid t_r\right)$$

5. The probability of the second event may be expressed in terms of the complement of the CDF for move time, $F_m(t_m)$, which is based on $f_m(t_m)$, as follows:

$$\mathrm{P}(\mathit{Event\ \#2}) = 1 - F_m\left(\frac{x}{v}\right)$$

6. The product of the two probabilities yields the complementary cumulative probability we seek, as follows:

$$1 - F_X(x \mid v,t,t_r) = F_{rp}\left(t - \frac{x}{v} \mid t_r\right) \cdot \left[1 - F_m\left(\frac{x}{v}\right)\right]$$

7. From which we may conclude that

$$F_X(x \mid v,t,t_r) = 1 - F_{rp}\left(t - \frac{x}{v} \mid t_r\right) \cdot \left[1 - F_m\left(\frac{x}{v}\right)\right]$$

8. Finally, substituting this result into the equation that expanded $F_X(x/t,t_r)$ over velocity, we obtain the CDF for the distance X the target may have moved at time t, conditioned on revisit time t_r.

$$F_X(x \mid t,t_r) = 1 - \int_0^\infty f_v(v) \cdot F_{rp}(t - x/v \mid t_r) \cdot [1 - F_m(x/v)] \cdot dv$$

This equation answers the movement question, Given the target was observed pausing, how far might it have moved from its observed location if we attack it t hours after the observation? Again, as for the first question, the answer depends on the revisit time, t_r. The shorter the revisit time, the smaller the distance the target will tend to move, because the remaining pause will tend to be longer. In practice, MITRE staff members have used numerical integration to implement the equation.

B.5 Response-Time Model

The next question the modeler asks about a mobile ground target is Given it was observed pausing, how far might it move from its observed location *by the time we can get a strike there to attack it?* This question brings the response time of the sensor-to-shooter system into our consideration. The analysis expands to include the arrival of the strike, and its time of arrival is another random variable. The arrival occurs an uncertain length of time T after the observation. We will call T the response time of the sensor-to-shooter system. It is the time that occurs *between* the observation and the arrival of the strike at the target s observed location. We will assume that T is probabilistically independent of when the target moves. As the PDF for T, we write $f_T(t)$.

Previously, CAPE modelers have represented the uncertainty in response time with a simple two-part approach: Define the time from observation to launch of the strike as a random variable with a triangular PDF; add to that time a deterministic time that it takes to reach the target after launch. In Dynamic CAPE, for instance, they estimated the mode for the observation-to-launch time. Then they set the maximum to be 50 percent greater than the mode; and they set the minimum to be 50 percent less. Then they added the flyout time to each of these three PDF parameters. The resultant PDF represented their uncertainty about the length of time it takes to perform all the separate tasks of processing, exploitation, dissemination, target analysis, mission planning, mission preparation, and flyout of the strike. In Dynamic CAPE, the parameters for this PDF differ by type of sensor and intelligence/C2 system supporting the sensor, strike platform, range band, and theater.

Given the PDF for response time (the triangular PDF that CAPE modelers have used previously, or another), it may be combined with $F_X(x/t,t_r)$ to produce a CDF for the distance X the target may have moved between its observation and the arrival of the strike:

$$F_X(x \mid t_r) = \int_0^\infty f_T(t) \cdot F_X(x \mid t, t_r) \cdot dt$$

This equation essentially averages $F_X(x/t,t_r)$ over all possible response times.

When the previous equation for $F_X(x/t,t_r)$ is substituted into this integral, we obtain the following equation:

$$F_X(x \mid t_r) = 1 - \int_0^\infty f_v(v) \cdot [1 - F_m(x/v)] \cdot \left[\int_0^\infty f_T(t) \cdot F_{rp}(t - x/v \mid t_r)\ dt \right] dv$$

Mathematically, this equation answers the response-time question. However, the equation leads to a practical problem. How does one calculate the integrals? To do a double integral numerically requires a great amount of calculation, on the order of N^2, where N is the number of intervals used for each integral.

One approach is to calculate the inner integral over response time as an analytic result, under the assumption that both $f_p(t_p)$ and $f_T(t)$ are triangular PDFs. That reduces the amount of numerical integration by a factor of N. MITRE staff member Brian Schmidt has derived equations to do such calculation. His results are sophisticated, elegant, and too complex to describe in detail. They are attached as an addendum at the end of this Appendix.

Another approach, requiring a little less computation, is to approximate $F_{rp}(t\text{-}x/v/t_r)$ with the first three terms of a Taylor Series so that the integral can be approximated analytically in terms of the mean and variance of the response time. This simplified approach is sketched here. It produces results similar to those of the analytic equations in many cases. However, it makes large errors in other cases, especially when the standard deviation of the response time is large relative to the mean.

The mean and variance of the response time T are calculated as follows:

$$\overline{T} = \int_0^\infty t \cdot f_T(t) \cdot dt \qquad\qquad Var(T) = \int_0^\infty (t - \overline{T})^2 \cdot f_T(t) \cdot dt$$

The first three terms of the Taylor Series expansion, written in terms of the mean and variance of response time, are as follows:

$$F_{rp}(t-x/v\,|\,t_r)=F_{rp}(\overline{T}-x/v\,|\,t_r)+(t-\overline{T})\frac{\partial}{\partial t}F_{rp}(\overline{T}-x/v\,|\,t_r)+\tfrac{1}{2}(t-\overline{T})^2\frac{\partial^2}{\partial t^2}F_{rp}(\overline{T}-x/v\,|\,t_r)$$

Integrated over response time *t,* the second term always integrates to zero. The third term integrates to one half the variance of the response time multiplied by the second derivative shown. The second derivative, readily calculated from the equation for $G_{rp}(t_{rp}/t_r)$, has a simple form:

$$\frac{\partial^2}{\partial t^2}F_{rp}(t\leftarrow\overline{T}-x/v\,|\,t_r)=\frac{f_p(t_r+\overline{T}-x/v)-f_p(\overline{T}-x/v)}{t_r\cdot[1-G_{rp}(0\,|\,t_r)]}$$

When the approximation for the integral over response time is substituted into the basic equation for $F_X(x/t_r)$, we obtain the following equation, which involves just one numerical integration. It is an approximation to the answer to the modeler s third question, Given the target was observed pausing, how far might it move from its observed location by the time we can get a strike there to attack it? While the equation may appear formidable, it is straightforward to compute:

$$F_X(x\,|\,t_r)=1-\int_0^{\infty}f_v(v)\cdot[1-F_m(x/v)]\cdot\left[F_{rp}(\overline{T}-x/v)\,|\,t_r)+\tfrac{1}{2}Var(T)\frac{f_p(t_r+\overline{T}-x/v)-f_p(\overline{T}-x/v)}{t_r\cdot[1-G_{rp}(0\,|\,t_r)]}\right]$$

As cautioned above, MITRE staff should analyze the accuracy of this approximation, using the analytic approach, prior to using it in specific analyses. It will tend to be most accurate when the standard deviation of the response time is small relative to the mean. That may not be a typical situation.

B.6 Capture Model

The modeler s response-time question has a corollary: Will the target be within the engagement envelope or acquisition basket of the strike sortie we send, as it arrives? In other words, what is the probability that the target is *captured* by the strike sortie s acquisition system at the time the sortie arrives?

In the simplified concept of target engagement that is part of MITRE s existing high-level STS analyses, it is assumed that if the target is still within some specific distance *s* of its reported first-observed location then it is vulnerable to attack when a sortie arrives, with some probability of success. However, if the target is outside the required distance, it cannot be successfully attacked.

The capture question involves not only the distance the target might have moved, but also the geolocation error of the reported location. That is, the strike will be sent to the coordinates determined from the observation, but they might have been in error to some extent.

To date MITRE staff have not modeled the combination of distance moved and geolocation error to determine the probability that the target is within the distance *s*. Accordingly, we can currently answer the question only as if there is no geolocation error.

Let us define the *engagement envelope* for a specific strike platform/weapon combination as the distance *s*. Let us also say that if the target has moved any distance less than or equal to *s* then it is within the engagement envelope and will be captured by the strike sortie. Then in the special case where there is no geolocation error, the probability P_C that the target is captured is written as follows:

$$P_C = F_X(s \mid t_r)$$

Note that this probability is dependent on t_r. The smaller t_r, the higher the probability will be, in general.

B.7 Summary

For many sensor-to-shooter (STS) systems, significant delay occurs between sensing a mobile ground target and engaging it with a weapon system. One major purpose of new investment in STS systems is to shorten the delay, so that the mobile target is more likely to be found and engaged successfully.

To enable analysis of such investments, MITRE has derived a model of target movement. The model represents mobile ground targets in their race against the imagery-based STS system.

Target movement involves the interaction of complex probabilistic phenomena. Accordingly the target movement model has been derived from explicit probabilistic assumptions using basic probability calculus.

The derived model provides the analyst with probability distributions for

- The pause that remains after a mobile target is observed pausing;
- The distance the target may have moved at any specific time after it is observed; and
- The distance the target may have moved when a strike sortie arrives to attack it, where the time of arrival is itself a random variable.

These probability distributions are calculated from more basic probabilistic models of target movement time, pause time, and velocity, as well as from the revisit time used for managing the surveillance and reconnaissance of targets. The distributions are used to calculate the probability that a target observed pausing will be within engagement range when a sortie, launched on the basis of the observation, arrives to attack the target. This involves a model of the sensor-to-shooter response time.

The derived probability distributions have been implemented in software. They exist as Visual Basic code and as Analytica models.

B.8 Addendum from Brian Schmidt

Analytic Approach to Computing the Probability of Target Escape
by Brian K. Schmidt
6 October 1999

This note presents an analytic approach for taking into account the sense and respond time distribution in the CAPE target movement model. The formulas presented here allow one to compute the probability of target escape *exactly* in the case where the sense and respond time distribution is triangular.

B.8.1 Notation

Given any probability density function $f(t)$, we denote the cumulative distribution function (CDF) by $F(t)$. Iterated cumulative distributions may then by defined as follows:

$$F^{(n)}(t) = \int_{-\infty}^{t} F^{(n-1)}(x)dx$$

where $F^{(1)}(t) = F(t)$.

B.8.2 Properties of the Triangle Distribution

A triangle distribution is specified by giving three parameters:

a = minimum value

m = mode

b = maximum value

From these values, all the properties of the distribution may be calculated. It is useful to define the quantities below:

h = height of the triangle

$$= \frac{2}{b-a}$$

A	=	$m - a$ =	left side of base of triangle
B	=	$b - m$ =	right side of base of triangle
α	=	h/A	
β	=	h/B	
μ	=	mean value of the random variable	
	=	$\dfrac{a+m+b}{3}$	
k_2	=	$\left(\frac{1}{36}\right)\left(A^2 + AB + B^2\right)$	
k_3	=	$\left(\frac{1}{1620}\right)\left(A - B\right)\left(2A^2 + 5AB + 2B^2\right)$	

The quantities k_2 and k_3 can be understood by relating them to the central moments:

μ_n	=	n^{th} central moment	
	=	$\int_{-\infty}^{\infty} (t-\mu)^n f(t)\, dt$	
μ_2	=	$2k_2$	(variance)
μ_3	=	$-6k_3$	(related to skewness)

Using these quantities, we can give formulas for the density and iterated cumulative distribution functions.

Interval	$f(t)$	$F(t)$	$F^{(2)}(t)$
$-\infty < t < a$	0	0	0
$a < t < m$	$\alpha(t - a)$	$\frac{1}{2}\alpha(t-a)^2$	$\frac{1}{6}\alpha(t-a)^3$
$m < t < b$	$\beta(b - t)$	$1 - \frac{1}{2}\beta(b-t)^2$	$(t-\mu) + \frac{1}{6}\beta(b-t)^3$
$b < t < \infty$	0	1	$(t-\mu)$

Interval	$F^{(3)}(t)$	$F^{(4)}(t)$
$-\infty < t < a$	0	0
$a < t < m$	$\frac{1}{24}\alpha(t-a)^4$	$\frac{1}{120}\alpha(t-a)^5$
$m < t < b$	$k_2 + \frac{1}{2}(t-\mu)^2 - \frac{1}{24}\beta(b-t)^4$	$k_3 + k_2(t-\mu) + \frac{1}{6}(t-\mu)^3 + \frac{1}{120}\beta(b-t)^5$
$b < t < \infty$	$k_2 + \frac{1}{2}(t-\mu)^2$	$k_3 + k_2(t-\mu) + \frac{1}{6}(t-\mu)^3$

B.8.3 Remaining Pause Time

In the CAPE target movement model, we are given a probability distribution for the target pause time. The remaining pause time is defined to be the amount of pause time that remains after the target has been detected by the sensor.

Define:

t_r = sensor revisit time

f_p = density function for pause time

f_{rp} = density function for remaining pause time

The properties of f_{rp} can be derived from those of f_p. (The basic formulas are derived in the writeup of the CAPE target movement model.) To summarize the results, it is convenient to introduce the following constants:

$$c_n = F_p^{(n)}(t_r)$$

$$\gamma = \frac{1}{t_r - c_2}$$

Then the iterated cumulative distribution functions for remaining pause time are written as follows, for two cases:

Case 1: When $t > 0$:

$$F_{rp}(t) = \gamma\left(F_p^{(2)}(t+t_r) - F_p^{(2)}(t) - c_2\right)$$

$$F_{rp}^{(2)}(t) = \gamma\left(F_p^{(3)}(t+t_r) - F_p^{(3)}(t) - c_3 - c_2 t\right)$$

$$F_{rp}^{(3)}(t) = \gamma\left(F_p^{(4)}(t+t_r) - F_p^{(4)}(t) - c_4 - c_3 t - \tfrac{1}{2}c_2 t^2\right)$$

Case 2: When $t \le 0$:

$$F_{rp}^{(n)}(t) \quad = \quad 0$$

B.8.4 Probability of Escape

After a target is detected by the sensor, a certain amount of time goes by before a weapon arrives to engage the target. We call this the sense and respond time. If this time is too long, the target may start moving before the weapon arrives and may get far enough away from its initial position that the weapon cannot find it. We call this escape. (This is not the same thing as survival ; the target has a chance of surviving even if it does not escape, because the weapon may engage the target yet fail to destroy it.)

Define:

f_s = density function for sense and respond time

x = distance target must move in order to escape

v = target speed

In these calculations, we assume that x and v are given constants and that, in effect, the target does not pause a second time after it begins moving. (These assumptions can easily be relaxed, as is shown in the complete writeup of the target movement model.) It follows that:

P_{esc} = probability of escape

$$= \int_0^{\infty} f_s(t)\, F_{rp}(t - x/v)\, dt$$

We will now analyze the case where the sense and respond time is given by a triangle distribution. In this case:

$$P_{esc} = \int_0^{\infty} f_s(t)\, F_{rp}(t - x/v)\, dt$$

$$= \int_{a_s}^{m_s} \alpha_s (t - a_s) F_{rp}(t - x/v)\, dt + \int_{m_s}^{b_s} \beta_s (b_s - t) F_{rp}(t - x/v)\, dt$$

To evaluate these integrals, note that, for any constant k:

$$\int (t - k) F_{rp}(t - x/v)\, dt$$

$$= \quad (t - k) F_{rp}^{(2)}(t - x/v) - \int F_{rp}^{(2)}(t - x/v)\, dt \qquad \textit{(integrating by parts)}$$

$$= \quad (t - k) F_{rp}^{(2)}(t - x/v) - F_{rp}^{(3)}(t - x/v)$$

For notational convenience, define:

$$h(t,k) \quad = \quad (t - k) F_{rp}^{(2)}(t - x/v) - F_{rp}^{(3)}(t - x/v)$$

Then:

$$P_{esc} \quad = \quad \alpha_s \left[h(t, a_s)\right]_{a_s}^{m_s} - \beta_s \left[h(t, b_s)\right]_{m_s}^{b_s}$$

$$= \quad \alpha_s \left(h(m_s, a_s) - h(a_s, a_s)\right) - \beta_s \left(h(b_s, b_s) - h(m_s, b_s)\right)$$

This expression may be easily evaluated using the formulas developed in the previous sections. In particular, this expression goes up as high as the third cumulative distribution for the remaining pause time; this may be computed using distributions going up to the fourth cumulative distribution of the pause time. If the pause time also satisfies a triangle distribution, these distributions are given by the formulas presented earlier in this note.

Glossary

Analytic This term denotes that answers to questions about C4ISR systems are developed through the use of algebra and calculus and, with few exceptions, without the use of random number generation and statistical sampling methods. Mathematical equations and tables represent the characteristics (or properties) of the C4ISR system; they make up a *model* of the system.

CAPE An acronym for C4ISR analytic performance evaluation.

C4ISR Denotes an integrated system of command, control, communications, computers, intelligence, surveillance, and reconnaissance. It connotes a set of questions to answer about the total system, not merely about one component of the system. However, in our studies we sometimes focus on a single component of the system (e.g., communications) to determine its effects on the total system s performance.

Dynamic system
A system that is modeled and analyzed in terms of how it changes with time. For a dynamic probabilistic system the modeling and analysis involve the use of probabilities.

Measure of effectiveness (MOE)
Denotes the quantification of a military outcome. The MOE is used to estimate day-to-day or ultimate success in conducting military operations. The outcome could be objective (percentage of enemy targets destroyed) or subjective (strength of the enemy commander s will to fight).

Measure of performance (MOP)
Denotes the quantification of the outcomes of intermediate processes, such as surveillance and targeting, that combine to produce military outcomes. The intermediate outcomes could be objective (photographs taken per hour) or subjective (strength of the enemy s morale).

Model A set of propositions or equations describing in simplified form some aspects of our experience. [30]

Performance evaluation
Connotes that the main questions posed in studies that use CAPE models are questions concerning the performance of C4ISR systems: how the systems interact, what affects their performance, how the systems can be improved, and how different systems would perform against the same criteria.

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[30] Web Dictionary of Cybernetics and Systems

Probabilistic analysis

An analysis that is based in part on probabilities. Such analysis may be wholly in terms of probability calculus, or it may involve the use of random numbers and statistical sampling (e.g., Monte Carlo analysis). A probability is a number used to quantify and communicate one's confidence (belief) that a specific result is or will be true for an uncertain event, or that a specific decision outcome will result from a decision. Low numbers express low confidence, and high numbers express high confidence.

Simulation

An analytical process that constructs a system s state history in chronological order. Like a movie, a state history describes the condition of the system, including the condition of its subsystems, at each of a chronological succession of instants. [31]

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[31] G. W. Evans, II, G. F. Wallace, G. L. Sutherland, *Simulation Using Digital Computers* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1967), p. 5.