Unambiguous Tracker for GPS Binary-Offset-Carrier Signals

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BIOGRAPHY

Ronald Fante is a Fellow of The MITRE Corporation and holds a Ph.D. from Princeton University. He has published 150 journal articles and one textbook. He has been elected a Fellow of the IEEE, the Optical Society of America, and the Institute of Physics. He has received the IEEE Third Millennium Medal, shared the IEEE Schelkunoff Prize (twice), the USAF Marcus O'Day Prize, and the MITRE Best Paper Award (twice). He has served as IEEE Distinguished Lecturer and was Editor in Chief of the IEEE Transactions on Antennas and Propagation.

ABSTRACT

Binary-offset-carrier waveforms have autocorrelation functions containing multiple peaks, thus leading to potential tracking ambiguities. We have developed a discriminator that is monotonic over the entire range of delays where the correlation function has significant support; thus eliminating ambiguities.

The price paid is the need for additional gates, some loss in sensitivity, and some additional computations.

INTRODUCTION

It is now well known that the binary offset-carrier (BOC) waveform proposed [1] for the future Global Positioning System (GPS) has a correlation function containing multiple peaks with magnitudes that are nearly equal to the magnitude of the central peak, as is evident from Figure 1. This means that a signal tracker can lock onto the wrong peak, thus producing a tracking error. Fine and Wilson [2] have devised a novel approach known as "bump-jump" to remove this ambiguity. Their approach compares the outputs in very-early and very-late gates with the prompt gate output to determine if the prompt gate is tracking the largest peak; if not, the tracker jumps to the largest of the other two gates and repeats the procedures until the prompt gate has the largest output. In this report, we will present an alternative approach that is in some sense a variant of the bump-jump method. Our approach uses samples of the correlation function at 2N different times, and then forms an appropriate discriminator that removes all ambiguities. In particular, we are able to derive a discriminator that is monotonic over the entire range of delays where the correlation function has significant support. That is, that discriminator is monotonic over an interval of approximately 336ns.

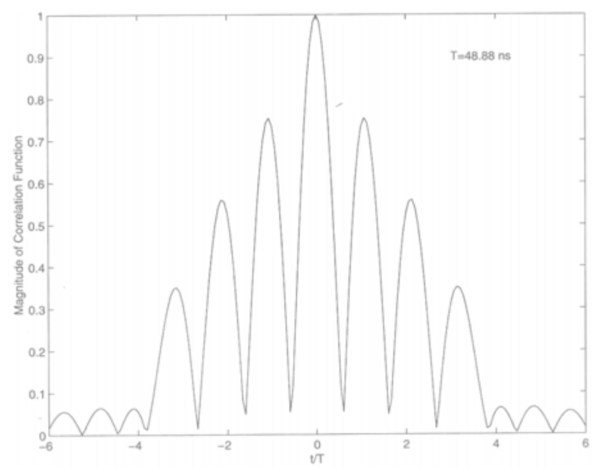

Figure 1. Absolute Value of Filtered BOC Autocorrelation Function

THEORY OF OPERATION

The classic early-late gate discriminator that is used for conventional correlation functions has the error function

$$E_n(\tau) = \left|R\left(\tau + D/2\right)\right|^n - \left|R\left(\tau - D/2\right)\right|^n \qquad (1)$$

where n is usually either 1 or 2, D is the spacing between the early and late gates, (the prompt gate has $D = 0$ and evaluates $|R(\tau)|^n$) and $R(\tau)$ is the cross correlation function. A typical plot of $E_2(\tau)$ in the noise-free limit is shown in Figure 2 for the correlation function shown in Figure 1.

The discriminator in Equation (1) is sensitive to signal amplitudes, and in order to remove this sensitivity the normalized discriminator

$$S_n(\tau) = \frac{\left|R\left(\tau + D/2\right)\right|^n - \left|R\left(\tau - D/2\right)\right|^n}{\left|R\left(\tau + D/2\right)\right|^n + \left|R\left(\tau - D/2\right)\right|^n} \qquad (2)$$

is often used, but this again has multiple ambiguities for the BOC.

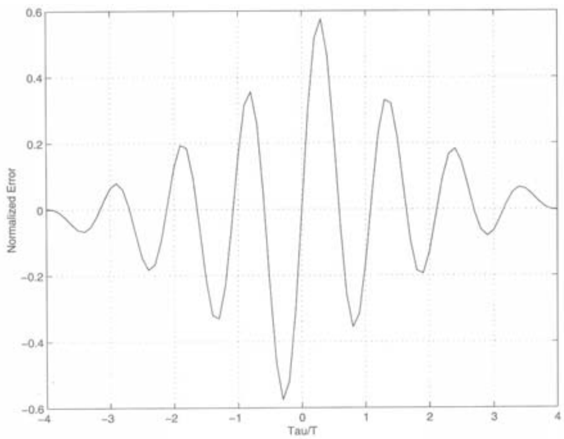

Figure 2. E_2 Error Function for Two Tap Discriminator

The method we propose to eliminate the multiple ambiguities caused by the BOC correlation function is a variant of the classical approach presented in Equations (1) and (2). We propose to use more than two samples of the cross correlation function. In particular, we replace Equation (1) by

$$\mathrm{E}_n(\tau) = \sum_{m=1}^{2N} a(m)\left|R\left[\left(m - N - \frac{1}{2}\right)D + \tau\right]\right|^n \qquad (3)$$

and Equation (2) is replaced by

$$S_n(\tau) = \frac{\sum_{m=1}^{2N} a(m)\left|R\left[\left(m - N - \frac{1}{2}\right)D + \tau\right]\right|^n}{\sum_{m=1}^{2N} |a(m)|\left|R\left[\left(m - N - \frac{1}{2}\right)D + \tau\right]\right|^n}. \qquad (4)$$

Our goal is to choose the coefficients $a(m)$ so as to obtain an error function that is linear with the form $E_n(\tau) = \alpha\tau$ or $S_n(\tau) = \alpha\tau$ for $-T_1 < \tau < T_1$, where α is an arbitrary constant and $2T_1$ is the region of support of the correlation function $R(\tau)$. From Figure 1 we see that $T_1 \approx 3.5T$.

Let us first concentrate on the $E_n(\tau)$ error function. We can derive the desired coefficients by minimizing the mean square error between $E_n(\tau)$ and $\alpha\tau$. That is, we minimize

$$\delta = \frac{1}{2T_1}\int_{-T_1}^{T_1} d\tau \left|E_n(\tau) - \alpha\tau\right|^2 \qquad (5)$$

where we can choose α such that

$$\sum_{m=1}^{2N} |a(m)|^2 = 1 \qquad (6)$$

Let us define the vectors

$$\mathrm{a}^{\mathrm{T}} = [\mathrm{a}(1)\ \mathrm{a}(2) \ldots\ \mathrm{a}(2\mathrm{N})] \qquad (7)$$

$$\mathrm{R}_{\mathrm{n}}^{\mathrm{T}} = \left[\ \left|\mathrm{R}\left(\left(-\mathrm{N} + \tfrac{1}{2}\right)\mathrm{D} + \tau\right)\right|^{\mathrm{n}}\ \ldots\ \left|\mathrm{R}\left(\left(\mathrm{N} - \tfrac{1}{2}\right)\mathrm{D} + \tau\right)\right|^{\mathrm{n}}\ \right] \qquad (8)$$

Then, it is readily shown that δ is minimized if the coefficient vector a satisfies

$$a = \frac{1}{\gamma}\Phi^{-1}g \qquad (9)$$

where

$$\Phi = \frac{1}{2T_1}\int_{-T_1}^{T_1} R_n R_n^T d\tau \qquad (10)$$

$$g = \frac{1}{2T_1}\int_{-T_1}^{T_1} \tau R_n(\tau) d\tau \qquad (11)$$

and

$$\gamma^{-2} = \left(\Phi^{-1}g\right)^T\left(\Phi^{-1}g\right) \qquad . \qquad (12)$$

Using the coefficients obtained in Equation (9), one can readily calculate the error function $E_n(\tau)$ for any n, N, and D.

In Figure 3, we show the E_2 error function using 16 taps for three different tap spacings. A similar response was achieved using 14 taps, but 12 taps were not sufficient to achieve the goal of a linear, monotonic response for $|\tau| < 3.5\mathrm{T}$.

We would like to achieve a monotonic response with fewer that 14 or 16 taps. Therefore, we have also considered the S_n discriminator defined in Equation (4).

Unfortunately, there is no simple method to determine the coefficients, $a(n)$, that produce the minimum mean square error between $S_n(\tau)$ and $\alpha\,\tau$, so we choose the $a(m)$ coefficients as

$$a(m) = 1 + \beta\,(m - N)/N\,, \quad m = 1 \text{ to } N$$
$$= -1 + \beta(m - N - 1)/N\,, \quad m = N + 1 \text{ to } 2N \quad (13)$$

where β is an arbitrary constant. The values of $a(m)$ in Equation (13) are asymmetric and give $S_n(0) = 0$, as required. In Figure 4, we show the S_2 error function for $\beta = -2$ when 10 and 12 taps are used. Note that using 12 taps for and $\beta = -2$ gives a monotonic error function over the entire range $|\tau| \le 3.5T$ where $R(\tau)$ has significant support (A similar conclusion is true for $\beta = -1$. Thus, the S_2 discriminator can achieve our goal using only 12 taps.

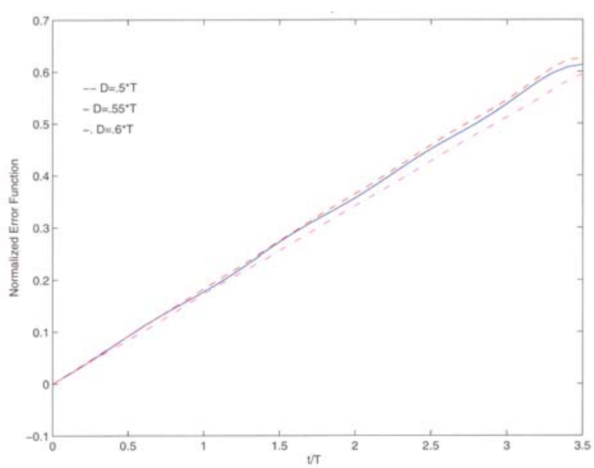

Figure 3. E_2 Error Function when 16 Taps are Used

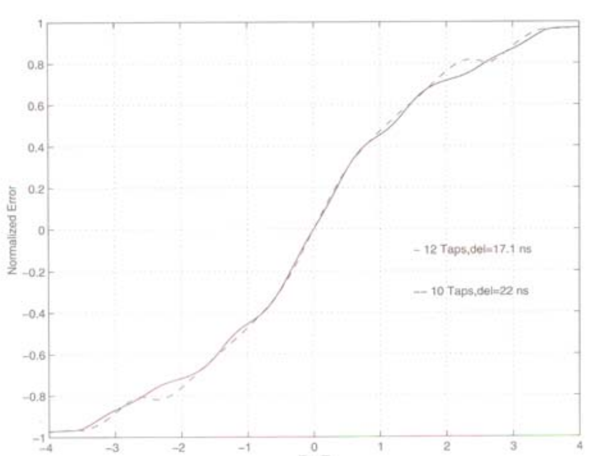

Figure 4. S_2 Error Function when 10 or 12 Taps are Used and $\beta = -2$

Thus, we have shown that a 12 tap S_2 discriminator or a 14 tap E_2 discriminator can achieve our goals in the absence of noise. In the next section, we consider how noise affects the estimate of delay.

SENSITIVITY LOSS

Although we have derived a monotonic error function, we now demonstrate that we pay a price in loss of sensitivity, relative to the classical two-tap discriminator. If we include the effects of noise, it is possible to show [3] that the standard deviation σ of the time-of-arrival error is

$$\sigma = \frac{KT}{(S/N)^{1/2}} \qquad (14)$$

where S/N is the signal-to-noise ratio at the correlator output, T=48.88ns and values of K are presented in Table 1.

Table 1. Values of the S_2 and E_2 Sensitivity Coefficients

Type	Number of Taps	K
S_2	2	0.35
S_2	12	1.32
E_2	14	3.5

From the results in Table 1, we see that the 12 tap S_2 discriminator is 3.77 times less sensitive than the 2 tap S_2 discriminator, and the 14 tap E_2) discriminator is even less sensitive. This is the price paid for the monotonic behavior of the 12 tap S_2 and 14 tap E_2 discriminators. This suggests we could combine the 12 tap S_2 discriminator (or the 14 tap E_2)) with a 2 tap S_2 (or E_2) discriminator to get the best of both worlds: unambiguous behavior plus good sensitivity at $\tau = 0$ (at the prompt gate). For example, we could use the 12 tap S_2 discriminator to get us to the correct correlation peak and the 2 tap E_2 or S_2 discriminator for the "fine tuning". For this procedure the gates at $t/T = \pm 0.175$ are the conventional early and late gates, whereas the gates at $t/T = \pm 0.525, \pm.825, \pm 1.225, \pm 1.575$ and ± 1.925 are multiple very early and very late gates.

It should be noted that the relatively large value of K in Table 1 for 14 tap E_2 discriminator suggests that a better design procedure (than requiring that $E_2(\tau) = \alpha\tau$) would be to require in Equation (5) that $E_2(\tau) = \alpha_1|\tau|$ for $|\tau| < T$ and $E_2(\tau) = \alpha_2|\tau|$ for $|\tau| > T$, where $\alpha_1 > \alpha_2$. The results of this design are shown in Figure 5. This design reduces the sensitivity factor K from 3.5 to 2.5.

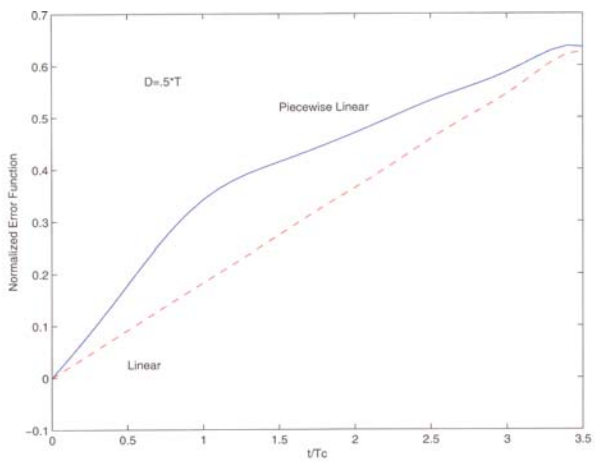

Figure 5. Comparison of Linear and Piecewise Linear E_2 Errors

We next need to determine the signal-to-noise ratio one can expect, and if it is sufficient to avoid the aforementioned errors. The carrier-to-noise ratio in dB-Hz is given by

$$\frac{C}{N_o} = P_s + G - 10\log_{10}(kT_o) - F - L \quad (15)$$

where $P_s =$ received power (dBW), $G =$ receive antenna gain in satellite direction, $k =$ Boltzmann's constant, $T = 293^\circ$ K, $F =$ receiver noise figure and $L =$ loss. Suppose the minimum value of received power for the BOC at L1 is $-157dBW$. If the receiver antenna has a cosine power pattern, then its average gain over the region from 15° above the horizon to zenith is approximately 1dBi. If the receiver noise figure is 4dB and the losses are 2dB, then from Equation (15) we see that the minimum value of $C/N_o = 42$ dB-Hz.

The signal-to-noise ratio after one integrate-and-dump cycle is CT_o/N_o, where $T_o =$ integration interval. The phase lock loop integrates $M = (B_L T_o)^{-1}$ of these cycles, so the output signal-to-noise ratio is

$$\frac{S}{N} = \frac{CT_o}{N_o}\ \frac{1}{T_o B_L} = \frac{C}{B_L N_o} \quad (16)$$

where B_L loop bandwidth

If Equation (16) is used in (14), we obtain the results in Table 2 for the tracking accuracy for some typical loop bandwidths when C/N_0 = 42 dB.

Table 2. Tracking Accuracy of 12-TAP S_2 and 14 Tap E_2 Discriminators

$B_L(Hz)$	σ_τ (ns) for S_2	$\sigma_\tau(ns)$ for linear E_2	$\sigma_\tau(ns)$for piecewise – linear E_2
0.8	0.46	1.2	0.85
3	0.89	2.3	1.6
5	1.15	3	2.1
8	1.45	3.5	2.7

Thus, the 12-tap S_2 tracker or the 14-tap E_2 tracker can typically achieve accuracies on the order of 1 to 3 feet. If smaller accuracies are required then a 2-tap adjunct discriminator must be used.

SENSITIVITY TO CODE DISTORTION

We have evaluated the effect of uncorrected dispersive errors on the performance of the E_2 and S_2 trackers developed here. The details are omitted, but the results are summarized in Table 3. Based on the results in Table 3, we require that the group delay in the receiver be equalized, so that the maximum group delay errors are less than 3 ns across the operating band.

We also examined the effect of ionospheric dispersion on the trackers. The ionosphere has a negligible impact.

Table 3. Effect of Dispersive Errors on E_2 Error Function

Type	Effect on Slope	Effect on Null Location
Amplitude – Linear	Increase	None
Amplitude – Quadratic	Decrease	None
Amplitude – Cubic	None	None
Amplitude – Sinusoidal Ripple	None	None
Group Delay – Linear	None	None
Group Delay – Quadratic	None	May be Significant unless Error <3ns
Group Delay – Cubic	None	None
Group Delay – Sinusoidal Ripple	None	May be Significant unless Error <3ns

SUMMARY

We have demonstrated that by a judicious choice of the number of taps, tap spacing and tap coefficients we can synthesize a tracker that is monotonic over the entire range of delays where the BOC autocorrelation function has significant support. This allows one to track BOC Autocorrelation maximum to an accuracy of order of 0.5 to 3 nanoseconds (depending on loop bandwidth) and without fear of the tracker moving in the wrong direction or stalling in a local minimum. If finer accuracy is needed, 2-taps at $t/T = \pm\delta$, where $\delta \leq 0.25$ can be added.

LIST OF REFERENCES

1. J. W. Betz, "Binary Offset Carrier Modulation for Radionavigation", Navigation, Volume 48, pp 227-246, Winter 2001-2002.

2. P. Fine & W. Wilson, "Tracking Algorithm for GPS Offset Carrier Signals", Proceedings of the Institute of Navigation National Technical Meeting, January 1999.

3. R. Fante, "Unambiguous Tracker for GPS Offset-Carrier Signals", MITRE Technical Report MTR02B0000055, December 2002.