LAAS Performance for Terminal Area Navigation*

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BIOGRAPHY

Ronald Braff has been with MITRE since 1970. He was involved in the FAA's Second Generation VORTAC program and aviation application of Loran C. Since the late 1970s he has been involved in the application of GPS to the National Airspace System, and with the LAAS program since its inception in the early 1990s. He was editor of *NAVIGATION* from 1987–97. He received BS (Physics) from Montana State University (1962) and MSEE from New York University (1969).

ABSTRACT

The Local Area Augmentation System (LAAS) Ground Facility (LGF) Performance Type 1 (PT 1) specification has been produced to satisfy GPS augmentation requirements for a local area Category I precision approach system. The objective of the paper is to determine where the PT 1 LGF specification would need modification to enable LAAS to also support terminal area navigation. The body of the paper summarizes the results of integrity risk, continuity risk and availability analyses. These analyses are based on the LGF specification of February 2001. However, the results could change due to future specification revisions. The appendices contain supporting analyses. The results indicate that the subject LGF PT 1 specification would require minor change to meet the terminal area navigation requirements.

INTRODUCTION

The Performance Type 1 (PT 1) Local Area Augmentation System (LAAS) Ground Facility (LGF) specification [1] is for the ground segment of a local area differential GPS system that will provide guidance for Category I precision approaches. The LGF contains reference receivers (RRs), and processors for outputting differential corrections, integrity parameters and navigation information. This output is transmitted to airborne receivers through the VHF Data Broadcast (VDB). In the LGF, there are integrity monitors for both the reference receivers (RRs) and the satellite signals. System descriptions of LAAS are available in the literature [2,3]. When LAAS is used for terminal area (TA) navigation, the required output is position.

The objective of this paper is to determine where the LGF PT 1 specification would need technical modification to enable LAAS to support TA navigation. The analyses are based on the draft LGF specification of February 6, 2001 [1]. However, the results could change due to future specification revisions.

The contents of the paper and their order are:

- Terminal Area Navigational Requirements
- Integrity Risk
- Continuity Risk
- Availability
- Conclusions
- Appendix A: Ranging Source Risk Analysis
- Appendix B: Continuity Risk Analysis
- Appendix C: Availability Analysis

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TERMINAL AREA NAVIGATION REQUIREMENTS

The TA navigation operational requirements are based on draft ICAO Standards and Recommended Practices (SARPS) [4] and the RTCA Minimum Aviation Performance Standards (MASPS) for LAAS [2]. The parameter, *required navigation performance (RNP)*, is used to indicate integrity and continuity navigation performance for a given 95% total accuracy requirement (navigation sensor error + flight technical error). RNP r refers to a 95% total accuracy requirement = r nmi.

Integrity Risk

Integrity risk definition. The integrity risk is the probability that the horizontal position solution error exceeds a horizontal alert limit (HAL) without an alert. HAL can be thought of as a circle about the true position of the aircraft that can be exceeded with some small probability. For TA navigation, the integrity risk allocation for exceeding HAL is 1×10^{-7} for a 1 h exposure time. It is assumed that HAL = r nmi is a sufficient alert limit for meeting RNP r integrity risk due to errors in the position solution. The LAAS integrity algorithms have been designed to provide integrity augmentation for any value of HAL and consequently any value of RNP.

Reference receiver integrity monitoring. Fault-free RR integrity risk is associated with the H0 hypothesis (no RR faults). The H0 integrity monitor provides a check to screen out poorer satellite geometries based on the transmitted standard deviations of the errors in the differential corrections and user to satellites lines-of-sight geometry. This process involves an airborne computation of the H0 horizontal protection level (HPL_H0). HPL_H0 provides a circular error bound on the navigation sensor errors with a probability based on the parameter K_{ffmd}. The integrity monitor equation has the form

$$\begin{aligned} \mathrm{HPL\_H0} = K_{ffmd}\, h_0(&\sigma_{pr\_gnd,n}, \sigma_{air,n}, \\ &\mathrm{Atmosphere\_Terms}_n;\ \mathrm{Satellite\_Geometry}) \le \mathrm{HAL} \end{aligned} \tag{1}$$

K_{ffmd}: fault-free missed detection multiplier corresponding to the tail of a radial distribution, K_{ffmd} =10 was selected

h_0: function for combining ranging errors and satellite geometry for the H0 equation [4]

$\sigma_{pr\_gnd,n}$: broadcast standard deviation of the fault-free errors in the average differential correction for satellite n due to such sources as RR noise and nominal multipath

$\sigma_{air,n}$: standard deviation of the fault-free airborne receiver errors for satellite n

Atmospheric_Terms$_n$: variances of uncorrectable errors due to troposphere and ionosphere delays for satellite n.

A single receiver failure is associated with the H1 hypothesis. HPL_H1 involves a comparison in the aircraft of a horizontal protection level, HPL_H1, to HAL. The HPL_H1 equation contains two terms. The first term is an estimate of the error contribution to the average correction from a single faulted RR. This contribution is estimated by B-value computations in the LGF. The B-value is based on a comparison of the corrections generated by each of the M reference receivers [1]. The second term contains the contributions of the errors due to the non-faulted M – 1 RRs and the rest of the position solution errors. Thus, HPL_H1$_m$ provides a bound on the navigation sensor errors when there is a single failure in RR m. The H1 protection level equation (2) is calculated for each RR and the maximum is compared to HAL.

$$\begin{aligned} \mathrm{HPL\_H1}_m = \left|\sum_{n=1}^{N} S_{xy,n} B_{n,m}\right| + K_{md}\, h_1(&\sigma_{pr\_gnd,n}, \sigma_{pr\_air,n}, \\ &\mathrm{Atmosphere\_Terms}_n;\ \mathrm{Satellite\_Geometry}) \le \mathrm{HAL} \end{aligned} \tag{2}$$

m: index for M RRs

$B_{n,m}$: B-value corresponding to the satellite n measurement by RR m

$S_{xy,n}$: represents the horizontal geometry elements of the position solution corresponding to satellite n.

h_1: function for combining ranging errors and satellite geometry for the H1 equation [4]

K_{md}: multiplier for probability of missed detection, corresponding to a tail probability of a radial error distribution

The metric for HPL in (1,2) is the semi-major axis of the ellipse associated with the bivariate Gaussian distribution [4].

Ranging source integrity monitoring. Ranging source (RS) integrity monitoring refers to the LGF monitors concerned with failures involving the satellite signal and navigation data. There are separate threshold monitors for distorted signal, radio frequency interference (RFI), low signal level, code-carrier divergence, excessive satellite clock acceleration, and ephemeris. The LGF requirements consist of maximum acceptable errors and probability of missed detection [1, 5]. At the time of writing, the ephemeris integrity monitor has been modified to include another airborne equation. This paper does not discuss ephemeris monitoring since it is treated elsewhere and is still not finalized at the time of writing.

Time-to-alarm. The time-to-alarm (TTA) requirement for TA navigation is 3 s, and it is the same as for PT 1 [4].

Continuity Risk

Continuity risk is the probability that LAAS is providing service for a required exposure time, given that service was available at the start of an operation. The TA navigation continuity risk can vary according to operational need. The PT 1 continuity risk requirement is 8×10^{-6} / 15 s. Since the TA navigation exposure time of 1 h is 3600 / 15 = 240 times as long, the resulting TA navigation continuity risk would only be 1.92 _ 10^{-3} / h if there were no continuity augmentation to PT 1. The augmentation assumed in this analysis is receiver autonomous integrity monitoring (RAIM) as a backup in the event of an LGF outage.

Availability

Geometry availability is the probability that at any time the satellite geometry is sufficient for conducting the operation, given the fault free distributions of the errors in the differential corrections and the uncorrectable ephemeris and atmospheric errors. Based on operational need, geometry availability requirements could vary over a wide range; e.g., 0.99 to 0.99999. In PT 1, the K_{ffmd} in (1) is set equal to 5.8. Although increasing K_{ffmd} to 10 for TA navigation decreases integrity risk, the availability decreases. This decrease is due to the increased satellite geometry limiting by the H0 process. An analysis is performed to estimate the extent of the reduction of availability.

INTEGRITY RISK RESULTS

The integrity risk allocation requirements for PT 1 are given in Figure 1 [1]. It is assumed that TA navigation is subject to the same risk categories as PT 1. The TA navigation integrity risks considered are *Fault Free Integrity Risk (H0), Single RR Fault Integrity Risk (H1), Integrity Risk due to Failures in Ranging Sources, VDB Message Corruption, and Failures*. As can be seen in

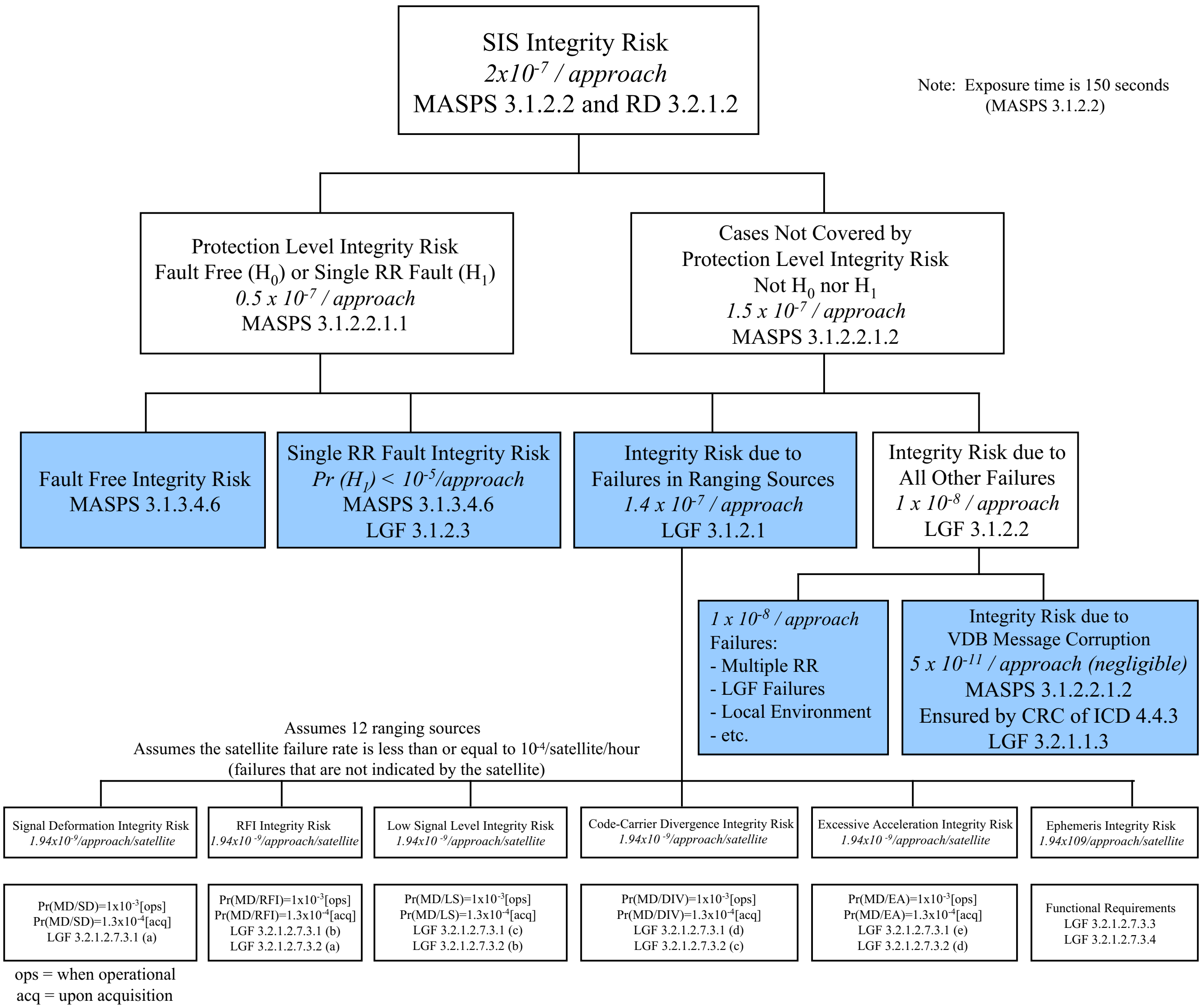

Figure 1. PT 1 Integrity Risk Requirements [1]

Figure 1, these risks span the entire risk diagram. The *Failures* risk encompasses all sources of risk that are not covered by the other risk categories and are concerned with the LGF. Under that category, *Multiple RR* refers to the undetected failure of more than one reference receiver, which is not covered by the H1 test. The results of the TA navigation integrity analysis are presented in Table 1. The calculated probabilities assume the fault-free errors have a Gaussian distribution with zero mean.

The H0 risk is significantly smaller (< 10^{-20}) than the total required maximum risk (10^{-7} / h) because of the K_{ffmd} =10 multiplier [4]. Likewise, the H1 risk is designed to be significantly smaller (~ 10^{-10}) than the total integrity risk by the selection of K_{md} = 5.3 in (2) [4]. The Ranging Source risk, ~ 10^{-15}, is derived in Appendix A.

The *VDB Message Corruption* risk is estimated by extending the exposure time from 150 s to 1 h by 24 _ (5 _ 10^{-11}) ~ 0.01 _ 10^{-7}.

The indicated *Failures* risk (0.99 _ 10^{-7}) is the difference between the total integrity risk requirement (10^{-7}) and the sum of the other integrity risks. From the results in Table 1, only *the VDB Message Corruption* risk need be considered for the subtraction. *Failures* risk is equivalent to (0.99 _ 10^{-7}) _ 150 /3600 = 0.41 _ 10^{-8} /150 s. From Figure 1, the corresponding *Failures* risk for PT 1 is 10^{-8} / 150 s. Therefore, the only recommended change to the LGF PT 1 specification for satisfying the TA navigation integrity risk requirements is the reduction of the *Failures* risk requirement by a factor of 0.4.

Table 1. Estimates of Terminal Area Navigation Integrity Risk

Category of Integrity Risk	Estimated Integrity Risk per h (K_{ffmd} = 10)
Fault Free (H0)	< 10^{-20} (by selection of K_{ffmd} in (1))
Single RR Undetected Failure (H1)	~ 10^{-10} (by selection of K_{md} in (2))
Ranging Source Undetected Failures	~ 10^{-15} (Appendix A)
VDB Message Corruption	~ 10^{-9} (by extension from PT 1)
Failures	0.99 x ~ 10^{-10} (after subtraction of other risks)
Total	10^{-7}

A brief explanation of integrity risk independence from the value of HAL is as follows:

- Fault Free (H0): HPL_H0 bounds the position solution errors and is not a function of HAL; therefore, from (1), pos_error ≤ HPL_H0 → pos_error ≤ HAL
- Single RR Undetected (H1): same reason as H0 since pos_error < HPL_H1 ≤ HAL (2)
- Ranging Source Undetected Failures: although decreasing HAL would seem to indicate that this risk would increase, the increase does not occur because of the increased geometry limiting of the H0 integrity monitor (see Appendix A)
- VDB Message Corruption and Failures: These risks only apply to the LGF so they are independent of HAL

CONTINUITY RISK RESULTS

The continuity risk allocation requirements for PT 1 are given in Figure 2 [1]. It is assumed that TA navigation is subject to the same continuity risk categories as PT 1. The continuity risks considered are *VDB Failure*, *RR Failure & All Other Fault Free Detection*, *sigma pr_gnd Fault Free Detection*, *B-Value Fault Free Detection*, *SV* (satellite) *Loss*, and *No Configuration Change.* These risks span the entire diagram in Figure 2. The results of the analysis are contained in Table 2.

Table 2. Estimates of Terminal Area Navigation Continuity Risks

Category of Continuity Risk	Estimated Continuity Risk per h	Mitigation	Comments
1. VDB Failure	3.5 x 10^{-6}	RAIM	See Appendix B.1
2. RR Failure & All Other FFD	0.08 x 10^{-6}	Use RAIM if RR exclusions cause LGF outage	See Appendix B.2
3. Sigma_pr_gnd FFD	2.2 x 10^{-11}	Use RAIM if RR exclusions cause LGF outage	See Appendix B.3
4. B-Value FFD	~ 0	None needed	See Appendix B.4
Total Due to LGF	**3.58 x 10^{-6}**	**Redundant VDB if needed**	
5. SV Loss	1.02 x 10^{-6}	None	See Appendix B.5
6. No Configuration Change	Assumed ~ 0		HAL = 185 m allows more satellite geometries to be used*
Total	**4.6 x 10^{-6}**		

* For example, an order of magnitude increase in maximum DOP from PT 1

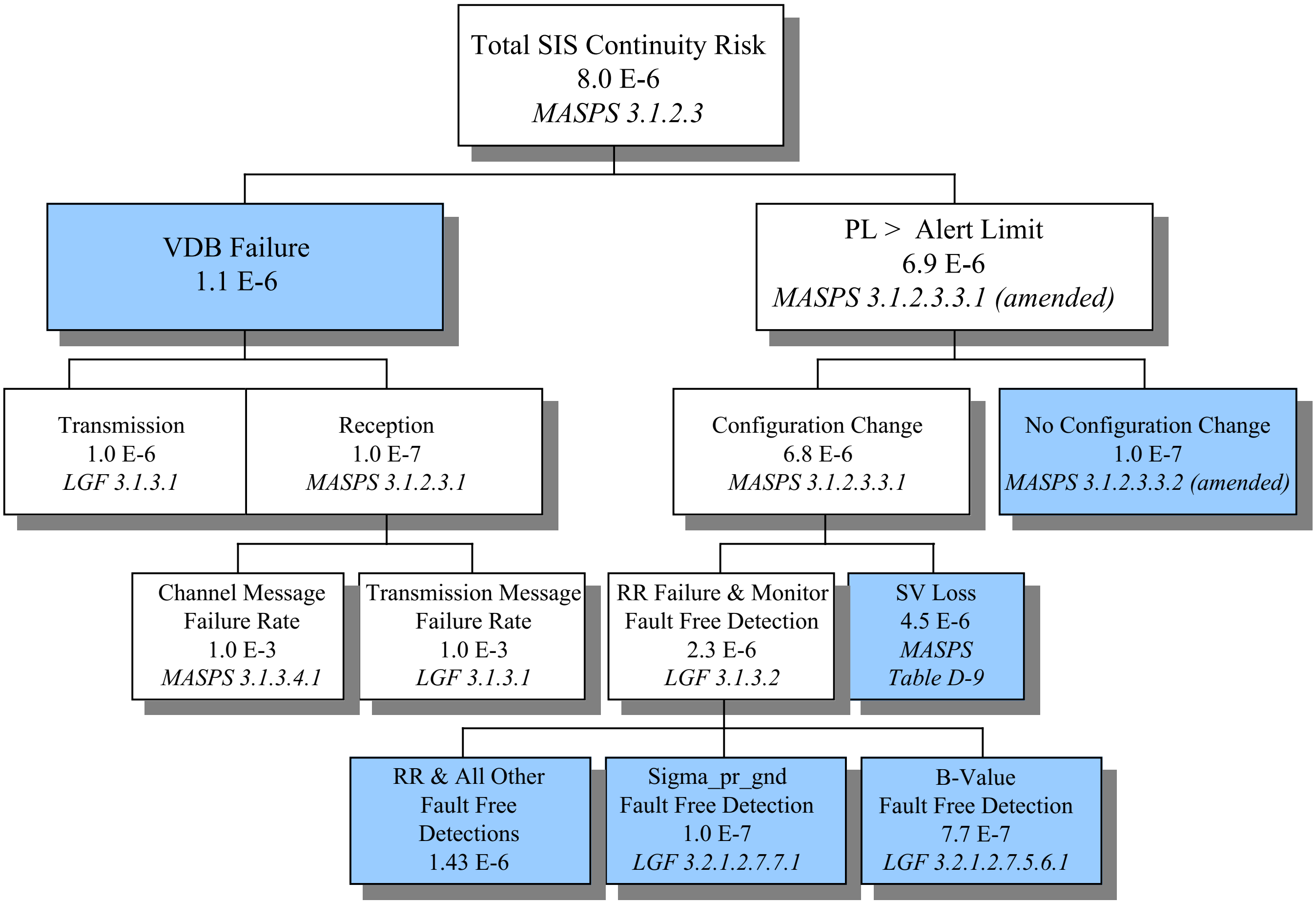

Figure 2. PT 1 Continuity Risk Requirements [1]

The following assumptions were made in the estimations of the continuity risks.

1. If a VDB failure causes an outage of the LGF, RAIM is used as a backup. The availability of RAIM with HAL ≥ 0.1 nmi as estimated from a constellation model is 0.987.
2. When M = 3 RRs, the loss of more than 1 RR produces an outage of the LGF. Then RAIM is used as a backup as described above. These assumptions were applied in 2 and 3 of Table 2.
3. The impact of an RS loss is only considered when 4 or 5 RSs are in view and all of the satellites are assumed to be critical. It is assumed that there are no critical satellites to cause HAL ≥ 0.1 nmi when there are more than 5 in view. This assumption was applied in 2, 4 and 5 in Table 2. Using a GPS constellation model, the assumption was validated by searching for critical satellites that would cause HPL_H0 > 185 m at 23 nmi when N = 6. Of the 25 TAs observed every minute for 24 h, there were only 1770 instances of only 6 satellites in view, and none of these satellites was observed to be critical. To gain further validation, all possible subsets of 6 satellites were investigated, and no critical satellites were found.

The results indicate that continuity risk is of the order of 5 _ 10^{-6} / h. The continuity risk is dominated by VDB failure. Therefore, when terminal area operations require less continuity risk, VDB transmitter redundancy would be needed.

AVAILABILITY ANALYSIS

The availability analysis includes LGF outages and their restoration time, RS outages and their restoration time, and RAIM as a backup for LGF outages. Availability is calculated when the LGF is in its operation state, and is based on HPL_H0 ≤ HAL = 0.1 nmi at a distance of 23 nmi from the LGF (LGF PT 1 coverage volume). The metric for HPL_H0 is the radial error defined by K_{ffmd} × one-sigma semi-major axis of the bivariate error ellipse

[4]. When the LGF is in its outage state, the availability is based on HPL_RAIM ≤ 0.1 nmi. The derivation of Figures 3 through 5 is contained in Appendix C.

Figure 3 shows the results of the availability analysis for K_{ffmd} = 10 when operations are performed with just a GPS constellation and no Inmarsat geostationary satellites (GEOs) are used in the position solution. The estimated availability is 0.9997–0.9998. There is no significant sensitivity to average availability due to LGF corrective maintenance response time.

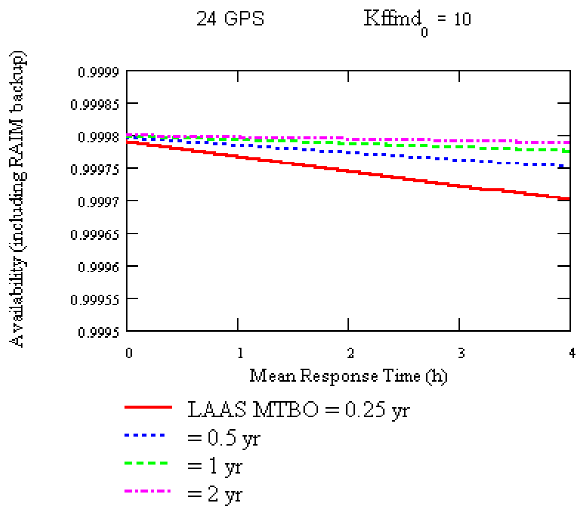

Figure 3. LAAS Terminal Area Navigation Availability: 24 GPS with RAIM Backup for LGF Outages (RNP 0.1 nmi, K_{ffmd} = 10, GAD 3, AAD B)

It is desirable to obtain 0.99999 availability for terminal area operations. To approach this goal, GEOs need to be used also in the position solution. Figure 4 shows the availability estimates with a GPS constellation plus the two Inmarsat satellite transponders that have been leased for WAAS. The improvement in availability is significant, 0.99996–0.999985.

As stated previously, K_{ffmd} = 10 provides increased integrity, but results in a loss of availability. Figure 5 shows that this loss is insignificant (0.001%) when comparing K_{ffmd} = 10 and K_{ffmd} = 6.

The results indicate that near 0.9999 availability is achieved with just the GPS constellation and RAIM. To achieve near 0.99999 availability, the position solution needs to include the two Inmarsat geostationary satellites. Figure 6 indicates the order-of-magnitude decrease in geometry unavailability when the Inmarsat satellites are included in the position solution.

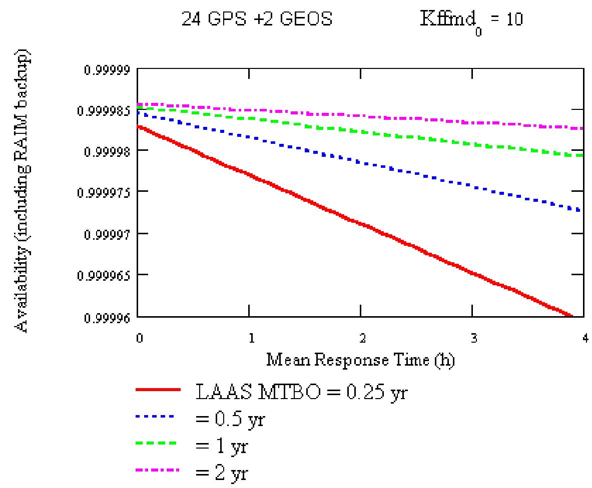

Figure 4. LAAS Terminal Area Navigation Availability: 24 GPS + 2 Inmarsat GEOs with RAIM Backup for LGF Outages (RNP = 0.1 nmi, K_{ffmd} = 10, GAD 3 AAD B)

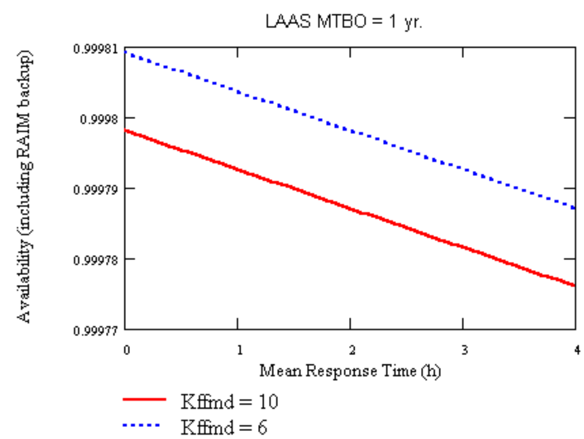

Figure 5. Comparison of Availability Between K_{ffmd} = 10 and 6 (LGF MTBO = 1 Year)

SUMMARY

The definitions of the terminal area (TA) navigation performance parameters and their requirements were reviewed. The parameters analyzed were integrity risk, continuity risk and availability with the objective of identifying needed changes to the LAAS Performance Type 1 specification for accommodating TA navigation. The results of the analysis, which was based on the analysis of the draft specification of February 2001, are:

1. The PT 1 integrity specification would require only a minor change to accommodate TA navigation.
2. It was shown that the LAAS integrity monitors are designed to provide the same integrity risk for any

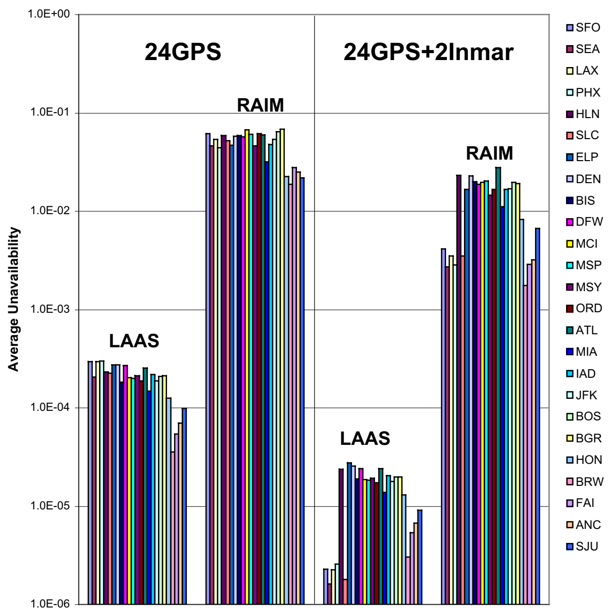

Figure 6. Unavailability at 25 Major Terminal Areas

value of required navigation performance (RNP) as long as their K-multipliers are kept constant.

3. The continuity risk analysis assumes that receiver autonomous integrity monitoring (RAIM) is employed as the backup for LGF outages. For RNP 0.1 nmi, it was estimated that the achievable TA navigation continuity risk is ~ 5×10^{-6} with the VHF Data Broadcast (VDB) accounting for 76% of the risk. The other major component of the risk is satellite loss, which accounts for 22%. Therefore, if required, employing redundant VDB transmitters would decrease the continuity risk.
4. The availability analysis also assumes that RAIM is employed as the backup for LGF outages. Accounting for LAAS Ground Facility (LGF), GPS and RAIM availability, it was estimated that the RNP 0.1 nmi achievable availability at 23 nmi is 0.9997–0.9998 when the position solution contains only GPS satellites. The achievable availability increases to 0.99996–0.99998 when the two existing WAAS geostationary satellites (GEOs) are also included in the position solution. Therefore, GEOs would be needed to approach 0.99999 availability.
5. The fault free missed detection multiplier (K_{ffmd}) = 5.8 for Category I precision approach operations. It was increased to K_{ffmd} = 10 for TA navigation operation. Increasing this multiplier decreases integrity risk, but also decreases availability. It was estimated that the resulting decrease in availability is not significant (0.001%).

ACKNOWLEDGMENTS

Bruce DeCleene (FAA) helped define the analysis. Curt Shively (MITRE/CAASD) provided key suggestions and is the principal reviewer of this paper. Tom Hsiao (MITRE/CAASD) provided the availability and critical satellite verification analyses.

APPENDIX A: RANGING SOURCE UNDETECTED FAILURES RISK ANALYSIS

The objective of the analysis is to show that the specified PT 1 RS integrity monitors can provide low enough integrity risk for TA navigation operations. Referring to Figure 1, there are 6 classes of RS failures. The LGF RS integrity requirements are based on an assumed upper-bound latent failure rate of 1 _ 10^{-4} / h per satellite. These failures are not indicated by the satellite code or navigation message. For simplicity, the failure rate is evenly divided between the 6 classes such that each is assigned a failure rate = 1 _ 10^{-4} h / 6 = 1.67 _ 10^{-5} / h. However, the PT 1 missed-detection probability has been adjusted to account for unequal class failure rates [5]. There are two failure cases considered. The acquisition case occurs when the satellite is first acquired by the LGF and it is in a failed state. Assuming a 1 h average notification time, the probability of a latent failure appearing at acquisition time is estimated to be Notification_Time / Mean-Time-Between-Failures = 1.67 _ 10^{-5}. The PT 1 LGF integrity monitor requirements are specified for an exposure time of 150 s. Therefore, the probability of an RS failure during a PT 1 operation is (150 / 3600) _ 1.67 _ 10^{-5} / h = 6.96 _ 10^{-7} per failure class per 150 s. At the end, the analysis will be adjusted for the TA navigation exposure time of 1 h.

The ranging source risk for failure class i per satellite is given by

$$\text{RS Failure\_Risk}_i = \text{Prob}\{\text{RS Failure }\}_i \;\_\; \text{Prob}\{\text{Missed\_Detection }\} \quad \text{(A-1)}$$

$$\text{Prob}\{\text{RS Failure }\}_i = 1.67 \;\_\; 10^{-5}, \text{ acquisition}$$
$$= 6.96 \;\_\; 10^{-7} / 150 \text{ s, operation}$$

Prob{Missed_Detection } for TA navigation is assumed generic for RS integrity monitoring, and is derived as follows. The LGF generic PT 1 RS integrity monitor model for determining the greatest upper bound of the test threshold is [5].

$$K_T \sigma_{test} \leq (\text{MERR} - K_{md}\, \sigma_{test}) \quad \text{(A-2)}$$

MERR: maximum tolerable error due to a failed RS

K_T: RS monitor threshold multiplier that must satisfy the required probability of false detection, which is based on continuity risk (continuity risk places the lower bound on K_T).

K_{md}: missed detection multiplier, K_{md} = 3.7 for PT 1 acquisition (equivalent to 1.1 _ 10^{-4} missed detection probability) and = 3.1 (1 _ 10^{-3}) for operations [1].

σ_{test}: standard deviation of an integrity monitor's test errors due to noise

In PT 1, MERR is derived from the geometry limiting of the H0 protection level [6]. It is given by

$$MERR = K_{merr}\sigma_{pr\_gnd} \quad \text{(A-3)}$$

From (A-2 and 3), σ_{test} has to satisfy the following inequality

$$\sigma_{test} \leq K_{merr}\sigma_{pr\_gnd} / (K_T + K_{md}) \quad \text{(A-4)}$$

In PT 1, K_{merr} = 4.9, which is based on K_{ffmd} = 5.8 and taking into account all of the other errors in the HPL-H0 equation (1) [6]. K_T is selected to achieve the continuity risk allocation. For this analysis, this allocation is assumed to be 10^{-8}, which is equivalent to K_T = 5.6. Substituting these parameter values into (A-4), the result for PT 1 is $\sigma_{test} \leq 0.527\sigma_{pr\_gnd}$ (acquisition) and $\leq 0.563\sigma_{pr\_gnd}$ (operation). The PT 1 equation for missed detection is

$$\text{Prob}\{\text{Missed\_Detection}\} = \Phi((K_{merr}\sigma_{pr\_gnd} / \sigma_{test} - K_T) \quad \text{(A-5)}$$

where $\Phi(X) = \int_{-\infty}^{X} \frac{1}{\sqrt{2\pi}} \exp(-x^2/2)dx$.

In (A-5) note as K_{merr} becomes greater, the buffer between the threshold and MERR also becomes greater; therefore, the missed detection probability decreases. This feature can be taken advantage of for TA navigation since the K_{merr} of interest is larger than the 4.9 value of PT 1. It is straightforward to apply the one-dimensional derivation of K_{merr} given in [6] in terms of $dRMS = \sqrt{d_{major}^2 + d_{minor}^2}$, where d_{major} and d_{minor} are the one-sigma components of the Gaussian bivariate error ellipse axes. However, $10d_{major}$ is used as the metric for radial error in the TA navigation HPL_H0 equation [4]. To take this into account it is noted that dRMS / √2 ≤ d_{major}. Therefore, setting K_{merr} = 10 / √2 accounts conservatively for d_{major}-axis usage in TA navigation. Substituting the parameters into (A-5) yields

$$\text{Prob}\{\text{Missed\_Detection}\} = \Phi(7.82) = 2.6\text{x } 10^{-15}\text{, acquisition}$$

$$\text{Prob}\{\text{Missed\_Detection}\} = \Phi(6.95) = 1.7 \_ 10^{-12}\text{, operation}$$

From (A-1),

$$\begin{aligned} \text{RS Integrity\_Risk}_i &= 1.67 \_ 10^{-5} \_ (2.6 \_ 10^{-15}) \\ &= 4.3 \_ 10^{-20}\text{, acquisition} \\ &= 6.96 \_ 10^{-7} \_ (1.7 \_ 10^{-12}) \\ &= 1.2 \_ 10^{-18} / 150\text{ s, operation} \end{aligned}$$

For acquisition, there are 6 failure classes per RS: RS_Risk_{acq} = 6 _ 4.3 _ 10^{-20} = 2.6 _ 10^{-19}. For operation there are also 3600 / 150 = 24 PT 1 exposure times to be accounted for RS_Risk_{op}= 24 _ 6 _ 1.2 _ 10^{-18} = 1.7 _10^{-16}. Assuming all RSs are acquired during the exposure time, the combined risk estimate is the sum of the two risks, which is 1.7 _ 10^{-16}. Assuming 12 RSs in view, the total risk estimate is 2 _ 10^{-15}.

Since the integrity risk depends on the value of K_{merr} and consequently on K_{ffmd}, the integrity risk estimated here is the same for any value of HAL.

APPENDIX B: CONTINUITY RISK ANALYSIS

VDB Failure Risk

From Figure 2, the PT 1 VDB continuity risk allocation is 1.1 _ 10^{-6} / 15 s. Extending this risk to 1 h yields 2.64 _ 10^{-4} / h. Since loss of the VDB eliminates LAAS as a navigation aid, RAIM can provide integrity monitoring as a backup. The effectiveness of RAIM in this role is analyzed. Since the PT 1 specification includes provision for providing WAAS GEO corrections, it is assumed the user receivers are equipped to track these GEOs and incorporate them in their RAIM. The RAIM fault detection (FD) and fault detection / exclusion (FDE) availability when HAL = 185 m was derived from a 24-satellite constellation model described in [7] to be $RAIM_{avail}$ = 0.987. The RAIM_FDE availability was derived to be $RAIM_{avail\_fde}$ = 0.822. Therefore, $RAIM_{avail\_fd}$ = 0.987 – 0.822 = 0.165.

Two events encompass the continuity risk. In Event A, RAIM_FD and RAIM_FDE capabilities are not available when there is a VDB outage. Event B assumes when there is a VDB outage, RAIM is available in either the FD or FDE mode. The FDE mode is not considered a continuity risk since navigation can continue if there is a detected latent satellite failure. However, with only FD, the latent failure of a satellite needs to be considered. The continuity risks (CR) are given by

CR_A = VDB_CR _ (1 - $RAIM_{avail}$)
= 2.64 _ 10^{-4} / h _ 0.013 = 3.4 _ 10^{-6} / h (B-1)

CR_B = VDB_CR _ $RAIM_{avail\_fd}$ _
Prob{Failed Satellite} _ N (B-2)

N: number of satellites in view is assumed to be 12. In [2], a conservative estimate of mean-time-between-outages (MTBO) of a GPS satellite is given as 5540 h. Thus the 1 h failure probability is Prob{Failed Satellite} = 1 / 5540 = 1.8 _ 10^{-4}. Substituting into (B2) yields CR_B = 0.97 _ 10^{-7} / h.

The total VDB failure risk = CR_A + CR_B = 3.5 _ 10^{-6} / h. (This risk is dominated by CR_A).

RR & All Other Fault Free Detection

From Figure 2, the failure risk is 1.43 _ 10^{-6} / 15 s. Extending this risk to 1 h yields 3.43 _ 10^{-4} / h. This category of continuity risk is analyzed with respect to both RR failures and fault-free RS exclusions. It is shown that by taking advantage of RAIM and HAL = 185 m, the actual continuity risk is smaller than 3.43 _ 10^{-4} / h.

RR failure risk. The RR reliability is conservatively assumed to be MTBF ~ 10^4 h; therefore, the probability of a RR failure over 1 h is $P_{rr} = 10^{-4}$. For M = 3 RRs, when more than 1 RR fails there is an LGF outage. Using the i = 2 term of the binomial distribution as an approximation, the continuity risk is

$$P\{>1\,\text{Fail}\}_{rr} \approx \frac{M(M-1)}{2} P_{rr}^2 (1 - RAIM_{avail}) \quad \text{(B-3)}$$

Substituting the parameters into (B3) yields $P\{> 1\ \text{Fail}\}_{rr}$ = 3.9 _ 10^{-10}

Exclusion of Critical RS. The analysis assumes simultaneous RS fault free detection is negligible. The continuity risk is given by

CR_RS = $P\{1\ FFD\}_{rs}$ _ P{RS is Critical Satellite } (B-4)

$P\{1\ FFD\}_{rs}$ = N _ C _ P{ RS Monitor_FFD} _ E (B-5)

N: number of RSs in view.
C: number of RS failure classes = 6
P{ RS Monitor_FFD} = 10^{-8} (same value as assumed in Appendix A)
E: number of continuity events in 1 h = 3600 /15 = 240

Substituting these parameter values in (B5) yields $P\{1\ FFD\}_{rs}$ = 1.44N _ 10^{-5}.

Due to HAL = 185 m, it is assumed when N > 5 there are no critical satellites. In the fault free case, loss of a critical satellite leads to poor geometry so that HAL in (1) or (2) is exceeded. Therefore, the analysis is concerned with N = 4 or 5 where it is assumed that all satellites are critical. Then P{RS is Critical Satellite } = P{4 Satellites in View} + P{5 Satellites in View}

An estimate of the probability of the number of satellites operating in the GPS constellation was obtained from [2]. This estimate is based on actual performance between 1995–1997. The conditional probability of 4 or 5 satellites in view was estimated from a constellation model. These parameters are presented in Table B-1.

Table B-1 Parameters for Estimating P {RS is Critical Satellite }

No. of Operational Satellites in Constellation (NOS)	P{NOS}	P{5 in View │ NOS}	P{4 in View │ NOS}
24–26	0.9373	0	0
23	0.0618	0.0172	0
22	0.0009	0.0747	0.0101
21	0	0.1096	0.0179
20	0	0.1725	0.0413

From Table B-1:
P{4 Satellites in View} = 0.0009 _ 0.0101 = 9.09 _ 10^{-6}
P{5 Satellites in View} = 0.0009 _ 0.0747 + 0.0618 _ 0.0172 = 1.13 _ 10^{-3}

Substituting into (B4) yields:
CR_RS = 1.44 _ 10^{-5} CSF
Where CSF = 4 _ 9.09 _ 10^{-6} + 5 _ 1.13 _ 10^{-3}
= 5.69 _ 10^{-3} (critical satellite factor)
CR_RS = 8.2 _ 10^{-8}.

RR & All Other Fault Free Detection. The total risk is the sum of (B3) and (B4):

3.9 _ 10^{-10} + 8.2 _ 10^{-8} = 8.2 _ 10^{-8}.

Sigma_pr_gnd FFD

The sigma_pr_gnd FFD risk involves the exclusion of RRs that could cause an outage. From Figure 2, the PT 1 risk allocation is 1 _ 10^{-7}. However, it is stated in [1] that the probability of a false RR exclusion due to the sigma monitor is less than 1 _ 10^{-7} during any 15 s interval. The following analysis estimates the 1 h exposure time risk based on LGF parameters and RAIM back up with HAL = 185 m. The extended allocation for false exclusion of an RR is 240 _ 1 _ 10^{-7} = 2.4 _ 10^{-5} / h. Given there are M = 3

RRs, an LGF outage would occur if two or more RRs are excluded. This probability is given by

$$\text{Prob}\{>1\ \text{RR Excluded}\}_{\sigma\_rr} \approx \frac{M(M-1)}{2} P_{fd\_rr}^2 \qquad \text{(B-6)}$$

P_{fd_rr}: probability of false detection of a RR = 2.4 _ 10^{-5} / M per h

From (B6), Prob{>1 RR Excluded}$_{\sigma\_rr}$ = 1.73 _ 10^{-9}.

Using RAIM as the backup for the LGF outage, the resulting risk is (1.73 _ 10^{-9}) _ 0.013 = 2.2 _ 10^{-11}.

B-Value Fault Free Detection

From Figure 2, the PT 1 risk allocation is 7.7x 10^{-7} / 15 s. Extending this risk to 1 h yields 1.85 _ 10^{-4} / h. The B-value FFD risk involves the exclusion of RRs or a critical satellite that could cause an outage. The following analysis estimates the 1 h exposure time risk based on LGF parameters and RAIM backup for multiple RR exclusions leading to LGF outage, and HAL = 185 m.

Exclusion of RRs. The B-value threshold is set at = 5.6σ_B. Then the probability of false detection of a B-value is P_{b_rs} = 2Φ(5.6) = 2.14 _ 10^{-8}. The B-values are derived from carrier smoothed code with time constant = 100 s. Therefore, it is assumed that there are 36 independent events during 1 h. It is assumed an RR is excluded if more than 1 of its B-values are exceeding the threshold. An estimate of the probability of RR exclusion over the 1 h exposure time is

$$P\{\text{RR Excluded}\}_{b\_rr} \approx 36 \frac{N(N-1)}{2} P_{b\_rs}^2 \qquad \text{(B-7)}$$

With N = 12, P{RR Excluded}$_{b\_rr}$ = 1.09 _ 10^{-12} / h. There would be an LGF outage when more than 1 RR is excluded. For M = 3, the estimate of this risk is 3 _ (1.09 _ $10^{-12})^2$ ~10^{-24}, a value that can be neglected.

Exclusion of Critical RS. For M = 3, assume a RS is lost if it is excluded from more than 1 RR. An estimate of the probability of this event is

$$P\{\text{RS\_Excluded}\}_{b\_rs} \approx 36 \times 3\, P_{b\_rs}^2 \times \text{CSF} \sim 10^{-16} N \quad \text{(B-8)}$$

Where it is assumed that there are no critical satellites when N > 5. This risk is also negligible.

SV Loss

The risk of satellite loss is given by

$$\text{CR\_Loss} = P\{\text{Satellite Outage}\}\_\text{CSF} \qquad \text{(B-9)}$$

Where it is assumed that there are no critical satellites when N > 5. In [2] a conservative estimate of a GPS satellite mean-time-between-outages is given as MTBO = 5550 h. Thus P{Satellite Outage} = 1 /5550 = 1.8 _ 10^{-4} over the 1 h exposure time. Substituting into (B-9) yields CR_Loss = 1.8 _ 10^{-4} _ 5.69 _ 10^{-3} = 1.02 _ 10^{-6}.

APPENDIX C: AVAILABILITY ANALYSIS

This analysis combines RS geometry availability, LGF availability, and RAIM availability as a backup for LGF outages. RAIM includes both fault detection and exclusion (FDE) and only fault detection (FD). Two availability events are considered:

1. Normal LGF operation: LGF is available and satellite geometry is available for HPL_H0 (K_{ffmd} =10) ≤ HAL = 0.1 nmi, where HPL_H0 is K_{ffmd} _ one-sigma semi-major axis of the bivariate error ellipse.
2. LGF outage: LGF not available and RAIM geometry is available for HPL ≤ 0.1 nmi

The LGF availability is given by the well-known equation

$$\text{Avail}_{lgf} = \text{MTBO} / (\text{MTBO} + \text{MTTR}) \qquad \text{(C-1)}$$
$$\text{MTTR} = \text{MURT} + \text{MRT}$$

MTBO: mean-time-between-LGF outages
MTTR: mean-time-to-restore LGF outage
MURT: mean unscheduled maintenance response time (queue and travel time)
MRT: mean repair time

In the analysis, MTBO is a parameter that varies from 0.25 yr. to 3 yr. The assumed maximum value of MURT is 4 h, and MRT = 0.5 h [1].

The LAAS and RAIM unavailabilities (1 – availability) are given in Figure 6 for a number of major terminal areas that span the country. These availabilities were calculated using a model of the GPS constellation that is given in the WAAS Minimum Operational Performance Standards (MOPS) [7]. The availability calculations are based on the LAAS error budget that was in use during March 2001. The budget components encompass the ground, air, uncorrectable troposphere and uncorrectable ionosphere errors. Uncorrectable ephemeris errors are not considered. The constellation model includes satellite outages and restoration. The assumed LGF configuration is ground accuracy designator (GAD) C3 and the airborne receiver is air accuracy designator (AAD) B. GAD C3 represents 3 RRs having an accuracy equivalent to a multipath-limiting antenna and narrow correlator receiver. AAD B is an airborne receiver having an accuracy equivalent to a narrow correlator receiver. Since the troposphere and ionosphere errors are functions of the distance from the aircraft to the LGF, the LAAS availability is calculated at the edge of the LAAS coverage volume (23 nmi). The RAIM unavailabilities are based on an assumed one-sigma ranging accuracy = 8 m and HAL = 0.1 nmi.

The working equations for calculating availability for the two events are

$$A_1 = Avail_{lgf} \_ Avail_{geom\_laas} \qquad \text{(C-2)}$$

A_1: availability of Event 1
Avail $_{geom\_laas}$: availability of LAAS when LGF is in normal state
= 1 – average LAAS unavailability from Figure 6
= 0.999801 (24 GPS), 0.999986 (24 GPS + 2 GEOs)

$$A_2 = (1 - Avail_{lgf}) \_ Avail_{geom\_raim} \qquad \text{(C-3)}$$

A_2: availability of Event 2
Avail $_{geom\_raim}$: availability of RAIM
= 1 – average RAIM unavailability from Figure 6
= 0.95132 (24 GPS), 0.98705 (24 GPS + 2 GEOs)

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