Effect of Partial-Band Interference on Receiver Estimation of C/N0: Theory

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BIOGRAPHY

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ABSTRACT

Testing the response of C/A code receivers to partial-band interference (continuous Gaussian interference whose power is concentrated in part of the front-end bandwidth) has included examining how receivers measure and report the effect of such interference. This paper considers two fundamentally different ways that receivers measure and report the effect of interference on signal quality, demonstrating that in non-white noise the measures are not the same. While the *effective* C/N_0 reliably measures the effect of interference on a receiver, the *precorrelation* C/N_0 is not reliable. Specifically, precorrelation estimation of N_0 does not properly account for the spectrum of the interference. Depending on the spectrum of the interference, precorrelation estimates may be accurate, or may overestimate the degradation caused by interference, or may underestimate the degradation caused by interference. Theoretical and numerical results are provided and compared to some measured data.

INTRODUCTION

Many receivers report received signal quality as carrier-power-to-noise density ratio, denoted C/N_0. Testing reported in a companion paper [1] has found that different receivers report different C/N_0 when presented with the same non-white interference. These different responses cannot be explained solely by differences in front-end bandwidths, discriminator designs, or other receiver characteristics. This paper suggests how such curious results may occur, identifies an objectively correct measure of interference effects on signal quality, shows which estimate of C/N_0 corresponds to the correct measure, and provides theoretical predictions of the response to different interference spectra.

It has long been recognized that spread spectrum receivers mitigate narrowband interference—the term "processing gain" has been used to describe and quantify this capability. Analysis describing this behavior is available in many standard references. The underlying mathematical development in [2] defines two different but related quantities: coherent output signal-to-noise-plus-interference ratio (SNIR), and noncoherent SNIR. These output SNIRs are also known as postcorrelation SNIRs, since they are defined at the output of the crosscorrelation between reference signal and received signal. Postcorrelation quantities occur after the entire correlation operation (both multiplication by the reference and integration by a time much longer than the chip period).

Coherent output SNIR is defined under the assumption that the phase of the reference signal is aligned with that of the received signal. The estimates of carrier phase and frequency used for carrier tracking and message demodulation are derived from the sequence of inphase parts of the crosscorrelation, computed using time segments of reference and received signal. Thus, the coherent output SNIR characterizes how well the receiver tracks carrier frequency and phase, and demodulates the data message that is phase shift key modulated onto the spread carrier.

Noncoherent output SNIR is defined under the assumption that the phase of the reference signal has unknown alignment with that of the received signal. The test statistic used in acquisition is derived from the sum of squared inphase and quadrature parts of the crosscorrelation. Thus, the noncoherent output SNIR characterizes how well the receiver acquires the signal.

Neither output SNIR measure predicts code tracking performance in non-white interference. A separate measure, presented in [3, 4], describes code tracking performance.

Although the coherent output SNIR and noncoherent SNIR are distinct quantities, [2] shows that there is a single fundamental quantity, termed the effective C/N_0, that is related to both of them. When there is no non-white interference, effective C/N_0 is simply the conventional C/N_0. When non-white interference is present, effective C/N_0 reliably describes how the combination of partial-band interference and noise affect both coherent output SNIR and noncoherent output SNIR, hence how it affects the receiver functions of carrier tracking, data demodulation, and acquisition. It is thus a uniquely relevant descriptor of how interference affects signal quality.

The analysis in this paper focuses on the essence of C/N_0 estimation under idealized conditions. Multiple access interference, especially crosscorrelations between signals, are not considered. Neither are channel effects such as multipath. Finally, the bandwidth of the interference is assumed to be much wider than the reciprocal of the integration time used in the correlator, and the integration time is much greater than the period of a spreading symbol, or chip.

Many receivers estimate and report some measure of the received signal quality for each signal being tracked, typically calling this signal quality estimate a measurement of C/N_0. Based in part on [5], this paper describes two distinct ways that receivers may estimate C/N_0. It shows that both estimates are equivalent when the interference has a flat spectrum over the receiver front-end, but that they differ when non-white interference is present. In non-white interference, one of the estimates provides a valid measure of effective C/N_0, while the other cannot be related to effective C/N_0. In fact, this latter measure may either grossly overestimate or grossly underestimate the effective C/N_0 and thus the received signal quality, depending on the spectrum of the interference.

The next section summarizes theoretical results from [2] that are the foundation of this paper. The following section describes two different estimates of C/N_0 that may be used in receivers. Next, the behavior of these estimates in the presence of non-white interference is analyzed, and expressions are derived for some limiting cases. Numerical results are used to contrast the two estimates, and predictions are compared to measured behavior of actual receivers. Finally, results and conclusions are summarized.

THEORETICAL BACKGROUND

This analysis assumes a (complex-valued) baseband signal $s(t)$ that is known (except for unknown delay and perhaps phase) at the receiver. While it is assumed that the frequency of arrival is known perfectly, as long as the frequency of arrival is known to within a fraction of the reciprocal of the coherent integration time, this assumption applies. The received data is then the sum of the signal and noise plus interference, $w(t)$. The received data is observed over a long time interval T_{obs}: $x(t) = \exp\{i\theta\}s(t-t_0) + w(t), \quad 0 \le t \le T_{\mathrm{obs}}$, where the unknown delay, or time of arrival, t_0, is defined relative to an arbitrary origin. The received noise plus interference is statistically independent of the signal.

Consistent with most analyses of code tracking accuracy, it is assumed that the time of arrival, t_0, is fixed, so no dynamics are considered explicitly. However, the results hold as long as any change in t_0 is somewhat slower than the reciprocal bandwidth of the code-tracking loop. Without loss of generality, t_0 is set to zero.

The signal of interest (henceforth called the signal) is $s(t)$; all signals not of interest, plus thermal noise, are grouped into $w(t)$. The analysis in this paper assumes that $w(t)$ is continuous, Gaussian, stationary, and circularly symmetric.

It is assumed that any automatic gain control (AGC) in the receiver responds slowly compared to the bandwidths of signal, noise, and interference, so the gain over the time of interest is approximated by a constant γ. The front end of the receiver, including mixing and quantization, is modeled as linear. Also, the transfer function of the receiver chain is approximated by a constant within complex bandwidth β_{r}, and zero outside that bandwidth.

The signal has power spectrum $CG_s(f)$, where $G_s(f)$ is the power spectral density normalized to unit area over infinite limits, and C is the received power of the signal. The thermal noise has power spectral density N_0, and the interference has power spectral density $C_\iota G_\iota(f)$ with $\int_{-\beta_{\mathrm{r}}/2}^{\beta_{\mathrm{r}}/2} G_\iota(f)df = 1$. Assume that the gain in the receiver processing chain is known, and let the integration time for correlating received data against reference signal be T.

[2] derives the mean and variance of the real part of the crosscorrelation between the received data and the reference signal. Coherent output SNIR is the squared mean when the reference signal is aligned in delay, frequency, and phase, divided by the variance, (which is shown in [2] to be independent of the delay):

$$\rho_{\mathrm{c}} = \frac{2T\frac{C}{N_0}\left[\int_{-\beta_{\mathrm{r}}/2}^{\beta_{\mathrm{r}}/2} G_s(f)df\right]^2}{\int_{-\beta_{\mathrm{r}}/2}^{\beta_{\mathrm{r}}/2} G_s(f)df + \frac{C_\iota}{N_0}\int_{-\beta_{\mathrm{r}}/2}^{\beta_{\mathrm{r}}/2} G_\iota(f)G_s(f)df}. \tag{1}$$

When the front-end bandwidth is wide enough to contain essentially all of the signal power, (1) becomes

$$\rho_c = \frac{2T\frac{C}{N_0}}{1+\frac{C_\iota}{N_0}\int_{-\beta_r/2}^{\beta_r/2} G_\iota(f)G_s(f)df}, \quad \int_{-\beta_r/2}^{\beta_r/2} G_s(f)df = 1. \quad (2)$$

Similarly, [2] derives the mean and variance of the quantity resulting from taking the magnitude squared of the crosscorrelation between the received data and the reference signal. Noncoherent output SNIR is the magnitude-squared mean of this quantity when the reference signal is matched in delay and frequency, but with arbitrary phase alignment, divided by its variance, (which is shown in [2] to be independent of the delay):

$$\rho_n = \frac{T\frac{C}{N_0}\left[\int_{-\beta_r/2}^{\beta_r/2} G_s(f)df\right]^2}{\int_{-\beta_r/2}^{\beta_r/2} G_s(f)df + \frac{C_\iota}{N_0}\int_{-\beta_r/2}^{\beta_r/2} G_\iota(f)G_s(f)df} + 1. \quad (3)$$

When the front-end bandwidth is wide enough to contain essentially all of the signal power, (3) becomes

$$\rho_n = \frac{T\frac{C}{N_0}}{1+\frac{C_\iota}{N_0}\int_{-\beta_r/2}^{\beta_r/2} G_\iota(f)G_s(f)df} + 1, \quad \int_{-\beta_r/2}^{\beta_r/2} G_s(f)df = 1. \quad (4)$$

The effective C/N_0 is defined as

$$\left(\frac{C}{N_0}\right)_{\text{eff}} = \frac{C\int_{-\beta_r/2}^{\beta_r/2} G_s(f)df}{N_0\int_{-\beta_r/2}^{\beta_r/2} G_s(f)df + C_\iota\int_{-\beta_r/2}^{\beta_r/2} G_\iota(f)G_s(f)df}$$

$$= \frac{\frac{C}{N_0}\int_{-\beta_r/2}^{\beta_r/2} G_s(f)df}{\int_{-\beta_r/2}^{\beta_r/2} G_s(f)df + \frac{C_\iota}{N_0}\int_{-\beta_r/2}^{\beta_r/2} G_\iota(f)G_s(f)df} \quad (5)$$

$$= \frac{C}{N_0}\left[1+\frac{C_\iota}{N_0}\frac{\int_{-\beta_r/2}^{\beta_r/2} G_\iota(f)G_s(f)df}{\int_{-\beta_r/2}^{\beta_r/2} G_s(f)df}\right]^{-1}.$$

When the front-end bandwidth is wide enough to contain essentially all of the signal power, (5) becomes

$$\left(\frac{C}{N_0}\right)_{\text{eff}} = \frac{C}{N_0 + C_\iota\int_{-\beta_r/2}^{\beta_r/2} G_\iota(f)G_s(f)df} \quad (6)$$

$$= \frac{C}{N_0}\left[1+\frac{C_\iota}{N_0}\int_{-\beta_r/2}^{\beta_r/2} G_\iota(f)G_s(f)df\right]^{-1}, \quad \int_{-\beta_r/2}^{\beta_r/2} G_s(f)df = 1.$$

(5) and (6) show how interference affects the effective C/N_0. As expected, only interference passed by the receiver front end has any effect. However, multiplication of the interference spectrum by the signal spectrum within the integral shows that the interference also is effectively filtered with magnitude-squared transfer function the power spectrum of the signal. Thus, if the signal has much narrower bandwidth than the front-end bandwidth, interference outside the signal bandwidth but within the front-end bandwidth has substantially less effect on effective C/N_0 than interference having the same power near band center.

Comparing (5) with (1) shows that the coherent output SNIR is

$$\rho_c = 2T\left(\frac{C}{N_0}\right)_{\text{eff}}\int_{-\beta/2}^{\beta/2} G_s(f)df, \quad (7)$$

or, when the front-end bandwidth contains essentially all the signal power,

$$\rho_c = 2T\left(\frac{C}{N_0}\right)_{\text{eff}}, \quad \int_{-\beta/2}^{\beta/2} G_s(f)df = 1. \quad (8)$$

Similarly, comparing (5) with (3) yields the noncoherent output SNIR

$$\rho_n = T\left(\frac{C}{N_0}\right)_{\text{eff}}\int_{-\beta_r/2}^{\beta_r/2} G_s(f)df + 1, \quad (9)$$

or, when the front-end bandwidth is wide enough to contain essentially all of the signal power,

$$\rho_n = T\left(\frac{C}{N_0}\right)_{\text{eff}} + 1, \quad \int_{-\beta_r/2}^{\beta_r/2} G_s(f)df = 1. \quad (10)$$

Consequently, the effective C/N_0 is sufficient for describing how interference affects both the coherent and noncoherent output SNIRs, hence carrier tracking, data message demodulation, and acquisition.

[2] also shows that the variance of the coherent correlator output at any lag value is proportional to a quantity that will be called the postcorrelation noise density:

$$(N_0)_{\text{post}} = N_0\int_{-\beta_r/2}^{\beta_r/2} G_s(f)df + C_\iota\int_{-\beta_r/2}^{\beta_r/2} G_\iota(f)G_s(f)df. \quad (11)$$

The numerator in the first line of (5) is the received signal power passed through the front end (since the received signal power C is defined over infinite bandwidth, and the signal's power spectrum $G_s(f)$ is normalized to unit area over infinite bandwidth). Further, the postcorrelation noise density (11) is the same as the denominator in the first line of (5), indicating that the postcorrelation noise density is the noise density that affects the effective C/N_0. In (11), the interference is effectively filtered by the signal spectrum, as described in the discussion after (6).

When the front-end bandwidth is wide enough to contain essentially all of the signal power, (11) becomes

$$\left(N_0\right)_{\text{post}} = N_0 + C_\iota \int_{-\beta_r/2}^{\beta_r/2} G_\iota(f) G_s(f) df, \qquad \int_{-\beta_r/2}^{\beta_r/2} G_s(f) df = 1 . \tag{12}$$

Consider the precorrelation noise density:

$$\begin{aligned}\left(N_0\right)_{\text{pre}} &= \frac{N_0}{\beta_r} \int_{-\beta_r/2}^{\beta_r/2} df + \frac{C_\iota}{\beta_r} \int_{-\beta_r/2}^{\beta_r/2} G_\iota(f) df \\ &= N_0 + \frac{C_\iota}{\beta_r} \int_{-\beta_r/2}^{\beta_r/2} G_\iota(f) df .\end{aligned} \tag{13}$$

While this quantity does not occur in any of the expressions that describe how interference affects receiver performance, it arises in the next section. (13) is similar to (11), except that in (13), the noise and interference spectra are not multiplied by the signal spectrum. When all of interference power is within the front-end bandwidth, (13) simplifies to

$$\left(N_0\right)_{\text{pre}} = N_0 + \frac{C_\iota}{\beta_r}, \qquad \int_{-\beta_r/2}^{\beta_r/2} G_\iota(f) df = 1 , \tag{14}$$

showing that the interference power is reduced by the front-end bandwidth, independent of interference spectral shape.

RECEIVER ESTIMATION OF INTERFERENCE EFFECTS

This section describes two possible approaches for estimating the effect of interference in a receiver. While many ad hoc approaches could be defined, the previous section demonstrates that the effective C/N_0 defined in (3) uniquely describes the effect of interference from the point of acquisition performance, carrier tracking performance, and message demodulation performance. Thus, an estimate provides a meaningful measure of interference effects if it approximates the effective C/N_0.

Since the measure should apply when the signal carrier power is much less than the power in the noise plus interference at the front end, the estimate of signal power must be based on the correlation output, so that the signal power is enhanced relative to the noise and interference. This estimation of signal power is described in the first subsection. Two different approaches for estimating the effective noise density, however, are considered in subsequent subsections. While different specific algorithms may be employed in receivers, it is expected that their behavior is consistent with one of these two approaches.

Estimation of Signal Power

Denote the received data after front-end filtering by $x_1(t) = \exp\{i\theta\} s_1(t) + w_1(t)$, and assume that the front-end bandwidth is large so that $\int_{-\beta_r/2}^{\beta_r/2} G_s(f) df \cong 1$. When the reference signal $r(t)$ is aligned with the carrier phase and frequency, the kth correlator output is

$$c_k(\tau) = \frac{1}{T} \int_{kT}^{(k+1)T} x_1(t) r^*(t-\tau) e^{-i\theta} dt, \tag{15}$$

and the average of the real part of K of them, each with the references aligned in time,

$$\hat{C} = \frac{1}{K}\left[\sum_{k=1}^{K} \Re\left\{c_k(0)\right\}\right], \tag{16}$$

approaches the signal power when K is large.

Estimation of Postcorrelation Noise Density

As seen in the analysis in [2], the variance of $c_k(\tau)$ is twice the variance of either $\Re\{c_k(\tau)\}$ or $\Im\{c_k(\tau)\}$, because of the assumption that noise plus interference is Gaussian and circularly symmetric. None of these variances depend on τ. The sample variance of $\Re\{c_k(\tau)\}$ is then $\frac{1}{K}\left[\sum_{k=1}^{K}\left(\Re\{c_k(\tau)\} - \frac{1}{K}\sum_{\ell=1}^{K}\Re\{c_\ell(\tau)\}\right)^2\right]$. The estimation process can be simplified by choosing a value of τ at least several chip periods away from the correlation peak, so that $\frac{1}{K}\left[\sum_{k=1}^{K}\Re\{c_k(\tau)\}\right] \cong 0$ and the sample variance is then approximated by the average squared value $\frac{1}{K}\left[\sum_{k=1}^{K}\left(\Re\{c_k(\tau)\}\right)^2\right]$ when K is large. Since the average value of $\Im\{c_k(\tau)\}$ is zero for all values of τ, an alternate estimate of the sample variance is $\frac{1}{K}\left[\sum_{k=1}^{K}\left(\Im\{c_k(\tau)\}\right)^2\right]$ for any value of τ. In fact, variance estimation can employ the prompt correlator output at the orthogonal phase: $\frac{1}{K}\left[\sum_{k=1}^{K}\left(\Im\{c_k(0)\}\right)^2\right]$, again avoiding the need to subtract the average squared value, and also using correlator taps that are already available in a typical code tracking loop. Note that these estimates use only the real part or only the imaginary part of the correlator outputs, implying that the receiver is accurately tracking carrier phase.

When K is large and ergodicity applies, these estimates approach the variance of $\Re\{c_k(\tau)\}$, which [2] shows is

$$Var\left\{\Re\{c_k(\tau)\}\right\} = \frac{C}{2T}\left[N_0 + C_\iota \int_{-\beta_r/2}^{\beta_r/2} G_\iota(f) G_s(f) df\right]. \tag{17}$$

Thus, there are several equivalent estimates of the postcorrelation noise density

$$\left(\hat{N}_0\right)_{\text{post}} = \frac{2T}{\hat{C}}\frac{1}{K}\left[\sum_{k=1}^{K}\left(\Re\{c_k(\tau)\} - \sum_{\ell=1}^{K}\Re\{c_\ell(\tau)\}\right)^2\right], \quad \text{any } \tau$$

$$= \frac{2T}{\hat{C}}\frac{1}{K}\left[\sum_{k=1}^{K}\left(\Re\{c_k(\tau)\}\right)^2\right], \qquad \text{any } \tau \text{ away from peak}$$

$$= \frac{2T}{\hat{C}}\frac{1}{K}\left[\sum_{k=1}^{K}\left(\Im\{c_k(\tau)\}\right)^2\right], \qquad \text{any } \tau, \text{ including } \tau = 0. \tag{18}$$

As long as K is large, and assuming that C is known, the postcorrelation noise density estimate approaches its mean

$$E\left\{\left(\hat{N}_0\right)_{\text{post}}\right\} = N_0 + C_\iota \int_{-\beta_r/2}^{\beta_r/2} G_\iota(f) G_s(f) df. \tag{19}$$

As long as K is large, dividing (16) by (18) yields the postcorrelation C/N_0, which is a close approximation to the effective C/N_0.

Estimation of Precorrelation Noise Density

Suppose instead that the receiver estimates the power in the received data after front-end filtering by $\frac{1}{LT}\int_0^{LT}|x_1(t)|^2 dt$, then forms the precorrelation noise density estimate by dividing this power estimate by the front-end bandwidth

$$\left(\hat{N}_0\right)_{\text{pre}} = \frac{1}{\beta_r LT}\int_0^{LT}|x_1(t)|^2 dt. \tag{20}$$

In (20), L is the number of correlation integration times used to form the estimate. The mean of $|x_1(t)|^2$ is

$$E\left\{|x_1(t)|^2\right\} = E\left\{|s_1(t)|^2 + |w_1(t)|^2\right\}$$
$$= C\int_{-\beta_r/2}^{\beta_r/2} G_s(f)df + \int_{-\beta_r/2}^{\beta_r/2}\left[N_0 + C_\iota G_\iota(f)\right]df. \tag{21}$$

As long as the signal power at the input is much less than the combined power of noise and interference at the input, (21) is approximated by

$$E\left\{|x_1(t)|^2\right\} \cong \int_{-\beta_r/2}^{\beta_r/2}\left[N_0 + C_\iota G_\iota(f)\right]df, \text{ low input SNIR.} \tag{22}$$

As long as L is large, the precorrelation noise density estimate approaches its mean

$$E\left\{\left(\hat{N}_0\right)_{\text{pre}}\right\} = \frac{1}{\beta_r LT}\int_0^{LT} E\left\{|x_1(t)|^2\right\}dt$$
$$\cong \frac{1}{\beta_r}\int_{-\beta_r/2}^{\beta_r/2}\left[N_0 + C_\iota G_\iota(f)\right]df \tag{23}$$
$$= N_0 + \frac{C_\iota}{\beta_r}\int_{-\beta_r/2}^{\beta_r/2} G_\iota(f)df, \quad \text{low input SNIR.}$$

Observe that (23) is the same as the precorrelation noise density defined in (13).

Dividing (16) by (23) yields an estimate of the precorrelation C/N_0.

$$\left(\frac{C}{N_0}\right)_{\text{pre}} = \frac{C}{N_0}\left[\frac{\int_{-\beta_r/2}^{\beta_r/2} G_s(f)df}{1 + \frac{C_\iota}{N_0}\frac{1}{\beta_r}\int_{-\beta_r/2}^{\beta_r/2} G_\iota(f)df}\right], \text{low input SNIR.} \tag{24}$$

When the front-end bandwidth is wide enough to contain essentially all of the signal power,

$$\left(\frac{C}{N_0}\right)_{\text{pre}} = \frac{C}{N_0}\left[1 + \frac{C_\iota}{N_0}\frac{1}{\beta_r}\int_{-\beta_r/2}^{\beta_r/2} G_\iota(f)df\right]^{-1}, \tag{25}$$
$$\text{low input SNIR, } \int_{-\beta/2}^{\beta/2} G_s(f)df = 1.$$

While there are many ways of estimating precorrelation noise density, one pointed out in [5] merits special consideration. While estimating the precorrelation noise density can be performed directly using power measurement circuitry or an equivalent algorithm on digital samples, a simpler alternative may exist using AGC circuitry. AGC circuitry monitors the input power (for example, by counting the fraction of time that the most significant bit is triggered) to control the RF gain. Since this feedback signal is related to input power, it can be used (with suitable calibration) to estimate the precorrelation noise density.

Comparison of Estimates

Observe that both the estimate of postcorrelation C/N_0 and the estimate of precorrelation C/N_0 use the same postcorrelation estimate of C. The essential difference between them is the estimate of N_0 used—the postcorrelation estimate of N_0 has the same form as the denominator in effective C/N_0, with the signal's spectrum filtering the interference spectrum, while the precorrelation estimate of N_0 fundamentally differs from any known expression for receiver performance, since the spectral shape of the interference within the front-end bandwidth has no effect on the estimate.

Since the postcorrelation C/N_0 is a close approximation to the effective C/N_0, the remaining discussion treats the two as identical, and calls both quantities the effective C/N_0. The precorrelation C/N_0 is different, so it is considered separately.

Since the expressions are simpler when the front-end bandwidth is wide enough to contain essentially all of the signal power and all of the interference power, this is assumed throughout the remainder of this section. However, the results would be qualitatively similar if this assumption were not applied.

If the interference has flat spectrum over the front-end bandwidth,

$$G_\iota(f) = \begin{cases} \dfrac{1}{\beta_{\mathrm{r}}}, & |f| \le \dfrac{\beta_{\mathrm{r}}}{2} \\ 0, & \text{otherwise,} \end{cases} \tag{26}$$

with total power C_ι, the effective C/N_0 with this interference is obtained by substituting (26) into (6), yielding

$$\left(\frac{C}{N_0}\right)_{\mathrm{eff}} = \frac{C}{N_0}\left[1 + \frac{C_\iota}{N_0}\frac{1}{\beta_{\mathrm{r}}}\right]^{-1}, \quad \int_{-\beta_{\mathrm{r}}/2}^{\beta_{\mathrm{r}}/2} G_s(f)df = 1. \tag{27}$$

Similarly, the precorrelation C/N_0 is obtained by substituting (26) into (25), yielding

$$\left(\frac{C}{N_0}\right)_{\mathrm{pre}} = \frac{C}{N_0}\left[1 + \frac{C_\iota}{N_0}\frac{1}{\beta_{\mathrm{r}}}\right]^{-1}, \quad \text{low input SNIR,} \int_{-\beta/2}^{\beta/2} G_s(f)df = 1. \tag{28}$$

(27) and (28) show that precorrelation C/N_0 is the same as effective C/N_0 for interference with flat spectrum.

When the interference occupies a narrow bandwidth (much less than the chip rate but much larger than any periodicity in the spreading sequence), both the effective C/N_0 and precorrelation C/N_0 are approximately of the form

$$\left(\frac{C}{N_0}\right) = \frac{C}{N_0}\left[1 + \frac{C_\iota}{N_0}\Gamma\right]^{-1}, \quad \int_{-\beta/2}^{\beta/2} G_s(f)df = 1 \tag{29}$$

where Γ is a constant that turns out to differ for the effective C/N_0 and precorrelation C/N_0.

Assume that the interference occupies a narrow bandwidth directly in band center, for example,

$$G_\iota(f) = \begin{cases} \dfrac{1}{\beta_\iota}, & |f| \le \dfrac{\beta_\iota}{2} \\ 0, & \text{otherwise,} \end{cases} \tag{30}$$

with total power C_ι and β_ι much less than the chip rate of the signal so that $G_s(f)$ is approximately constant over β_ι. Then the effective C/N_0 for interference at band center is obtained by substituting (30) into (6), yielding

$$\begin{aligned} \left(\frac{C}{N_0}\right)_{\mathrm{eff}} &= \frac{C}{N_0}\left[1 + \frac{C_\iota}{N_0}\frac{1}{\beta_\iota}\int_{-\beta_\iota/2}^{\beta_\iota/2} G_s(f)df\right]^{-1} \\ &\cong \frac{C}{N_0}\left[1 + \frac{C_\iota}{N_0}G_s(0)\right]^{-1}, \quad \int_{-\beta_{\mathrm{r}}/2}^{\beta_{\mathrm{r}}/2} G_s(f)df = 1. \end{aligned} \tag{31}$$

In contrast, substituting (30) into (25) yields the precorrelation C/N_0 for this case:

$$\begin{aligned} \left(\frac{C}{N_0}\right)_{\mathrm{pre}} &= \frac{C}{N_0}\left[1 + \frac{C_\iota}{N_0}\frac{1}{\beta_{\mathrm{r}}\beta_\iota}\int_{-\beta_\iota/2}^{\beta_\iota/2} df\right]^{-1} \\ &= \frac{C}{N_0}\left[1 + \frac{C_\iota}{N_0}\frac{1}{\beta_{\mathrm{r}}}\right]^{-1}, \quad \text{low input SNIR,} \int_{-\beta/2}^{\beta/2} G_s(f)df = 1. \end{aligned} \tag{32}$$

Observe that while the expressions for effective C/N_0 and precorrelation C/N_0 have similar form (29), the constants multiplying the interference power in the denominator are not the same. To explore this difference further, let the signal have a conventional BPSK modulation with chip period T_{c} the reciprocal of the chip rate f_{c},

$$G_s(f) = T_{\mathrm{c}}\mathrm{sinc}^2\left[\pi f T_{\mathrm{c}}\right]. \tag{33}$$

This analysis neglects any spectral lines arising from periodic spreading codes. As long as the interference bandwidth is much greater than any such period, this is a valid approximation.

Then $G_s(0) = T_{\mathrm{c}} = 1/f_{\mathrm{c}}$, and (31) becomes

$$\left(\frac{C}{N_0}\right)_{\mathrm{eff}} \cong \frac{C}{N_0}\left[1 + \frac{C_\iota}{N_0}\frac{1}{f_{\mathrm{c}}}\right]^{-1}, \quad \int_{-\beta_{\mathrm{r}}/2}^{\beta_{\mathrm{r}}/2} G_s(f)df = 1. \tag{34}$$

This shows that the interference power degrades effective C/N_0 diminished by the processing gain of the signal, where the processing gain is equal to the chip rate $f_{\mathrm{c}} = 1/T_{\mathrm{c}}$. This is a well-known result that describes the performance of a spread spectrum receiver in the presence of narrowband interference at band center.

For $\int_{-\beta_{\mathrm{r}}/2}^{\beta_{\mathrm{r}}/2} G_s(f)df = 1$ to be a valid approximation, the front-end bandwidth must be much greater than the chip rate, so that $\beta_{\mathrm{r}}T_{\mathrm{c}} >> 1$. Then the interference power is multiplied by a much smaller number in the precorrelation C/N_0 than in effective C/N_0, meaning that, as interference power increases, its effect is not fully reported by precorrelation C/N_0.

For example, suppose that the signal spectrum is given by (33), and $\beta_{\mathrm{r}} = 8f_c$, equivalent to a front-end bandwidth of approximately 8 MHz for a C/A code receiver. Then the precorrelation C/N_0 reports less degradation of effective C/N_0 than actually occurs—here it underestimates the effect of the interference by approximately 9 dB. The receiver is actually degraded as predicted by effective C/N_0, but the precorrelation C/N_0 does not accurately reflect this degradation.

Now suppose the interference power is concentrated near the edge of the front-end bandwidth. As long as the signal spectrum is symmetric, it makes no difference if the interference spectrum is symmetric, with power split between both band edges, or if all the interference power is near one band edge. For example, let

$$G_\iota(f) = \begin{cases} \frac{1}{\beta_\iota}, & |f - f_\iota| \le \frac{\beta_\iota}{2} \\ 0, & \text{otherwise,} \end{cases} \quad (35)$$

with total power C_ι and $-\beta_r/2 \le f_\iota - \beta_\iota/2 < f_\iota + \beta_\iota/2 \le \beta_r/2$. The effective C/N_0 with interference concentrated near f_ι is obtained by substituting (35) into (6), yielding

$$\begin{aligned} \left(\frac{C}{N_0}\right)_{\text{eff}} &= \frac{C}{N_0}\left[1 + \frac{C_\iota}{N_0}\frac{1}{\beta_\iota}\int_{f_\iota-\beta_\iota/2}^{f_\iota+\beta_\iota/2} G_s(f)df\right]^{-1} \\ &\cong \frac{C}{N_0}\left[1 + \frac{C_\iota}{N_0}G_s(f_\iota)\right]^{-1}, \quad \int_{-\beta_r/2}^{\beta_r/2} G_s(f)df = 1. \end{aligned} \quad (36)$$

In contrast, substituting (35) into (26) yields

$$\left(\frac{C}{N_0}\right)_{\text{pre}} = \frac{C}{N_0}\left[1 + \frac{C_\iota}{N_0}\frac{1}{\beta_r}\right]^{-1}, \quad \text{low input SNIR}, \int_{-\beta/2}^{\beta/2} G_s(f)df = 1. \quad (37)$$

Effective C/N_0 depends on the signal spectrum evaluated at the center frequency of the interference, while the precorrelation C/N_0 depends only on the front-end bandwidth, and not on the signal spectrum nor on the spectrum of the interference. In fact, (37) is identical to the result (32) obtained for interference at band center.

If the signal spectrum is given by (33), $\beta_r = 8f_c$ and $f_\iota = 3.5f_c$, interference is near band edge so (36) becomes

$$\left(\frac{C}{N_0}\right)_{\text{eff}} \cong \frac{C}{N_0}\left[1 + \frac{C_\iota}{N_0}\frac{1}{(3.5\pi)^2 f_c}\right]^{-1}, \quad \int_{-\beta_r/2}^{\beta_r/2} G_s(f)df = 1, \quad (38)$$

while (37) becomes

$$\left(\frac{C}{N_0}\right)_{\text{pre}} = \frac{C}{N_0}\left[1 + \frac{C_\iota}{N_0}\frac{1}{8f_c}\right]^{-1}, \quad \text{low input SNIR}, \int_{-\beta/2}^{\beta/2} G_s(f)df = 1. \quad (39)$$

Here, the precorrelation C/N_0 reports more degradation of effective C/N_0 than actually occurs—for this example it overestimates the effect of the interference by approximately 11.8 dB. The receiver is actually only degraded as predicted by effective C/N_0, but the precorrelation C/N_0 indicates much worse degradation.

Since effective C/N_0 relates directly to receiver performance in areas such as acquisition, carrier tracking, and data demodulation, these results show that precorrelation C/N_0 does not report reliable information about these aspects of receiver performance when the interference spectrum is not flat.

NUMERICAL RESULTS

For computation of numerical results, consider a C/A code receiver with thermal noise density –203 dBW/Hz and received signal power of –158 dBW, for a nominal received C/N_0 of 45 dB-Hz, neglecting implementation losses.

The first results consider interference at band center with a bandwidth of 10 kHz. Figure 1 compares effective C/N_0 and precorrelation C/N_0 for a receiver front-end bandwidth of 8 MHz. While effective C/N_0 relates the true effect of interference on acquisition, carrier tracking, and data message demodulation, precorrelation C/N_0 does not indicate the full degrading effect of this interference, underestimating the effect of the interference by approximately 9 dB as predicted in the previous section.

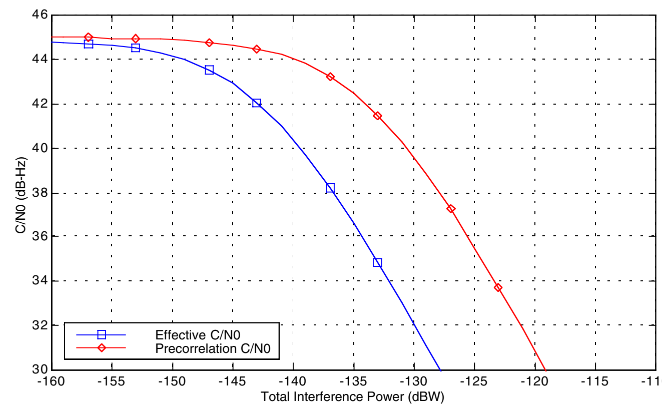

Figure 1. Theoretical Results for Narrowband Interference at Band Center, Receiver Front-End Bandwidth 8 MHz

Figure 2 shows the same quantities as in Figure 1, but for a receiver with 16 MHz front-end bandwidth. For this wider-bandwidth receiver, precorrelation C/N_0 underestimates the effect of the interference by approximately 12 dB. While the receiver is significantly degraded as the interference power exceeds –135 dBW, the precorrelation C/N_0 only reports mild degradation.

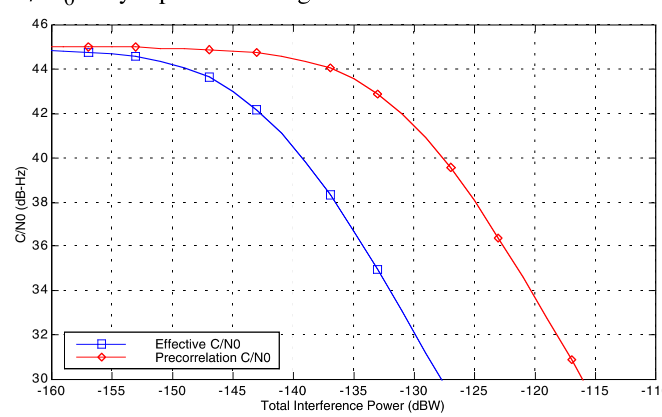

Figure 2. Theoretical Results for Narrowband Interference at Band Center, Receiver Front-End Bandwidth 16 MHz

Consider next narrowband interference offset from center frequency by 3.5 times the chip rate (3.5805 MHz for C/A code). Figure 3 compares effective C/N_0 and precorrelation C/N_0 for a receiver with 8 MHz front-end bandwidth. In contrast to Figures 1 and 2, the precorrelation C/N_0 indicates that this interference degrades receiver performance much worse than it actually does. As predicted analytically in the previous section, precorrelation C/N_0 overestimates the effect of the interference by approximately 12 dB. For example, while the precorrelation C/N_0 reports significant degradation at interference power of –125 dBW, the actual degradation of effective C/N_0 is quite mild.

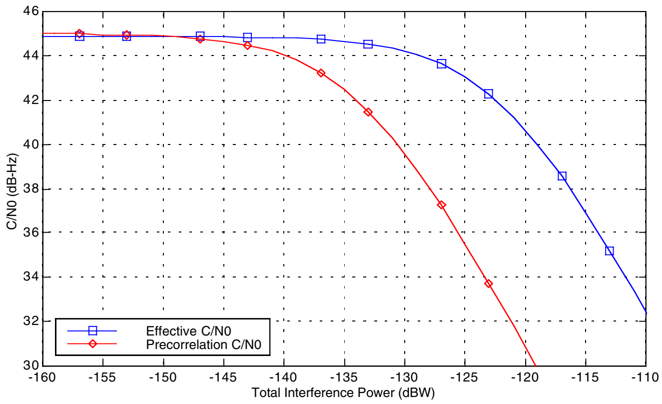

Figure 3. Theoretical Results for Narrowband Interference Offset from Band Center by 3.5 Times the Chip Rate, Receiver Front-End Bandwidth 8 MHz

Figure 4 shows the same quantities as in Figure 3 for a receiver with 16 MHz front-end bandwidth. For this wider-bandwidth receiver, precorrelation C/N_0 underestimates the effect of the interference by approximately 9 dB. The effective C/N_0 shows that this interference has almost the same effect for front-end bandwidths of 8 MHz and 16 MHz, but the precorrelation C/N_0 changes by 3 dB.

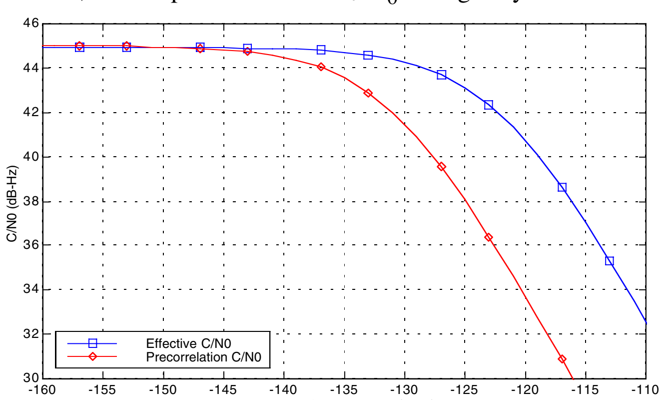

Figure 4. Theoretical Results for Narrowband Interference Offset from Band Center by 3.5 Times the Chip Rate, Receiver Front-End Bandwidth 16 MHz

Consider next narrowband interference offset from center frequency by 7.5 times the chip rate (7.6725 MHz for C/A code). Figure 5 compares effective C/N_0 and precorrelation C/N_0 for a receiver with 16 MHz front-end bandwidth. While precorrelation C/N_0 understates the effect of interference near band center, it overstates the effect of interference near band edge.

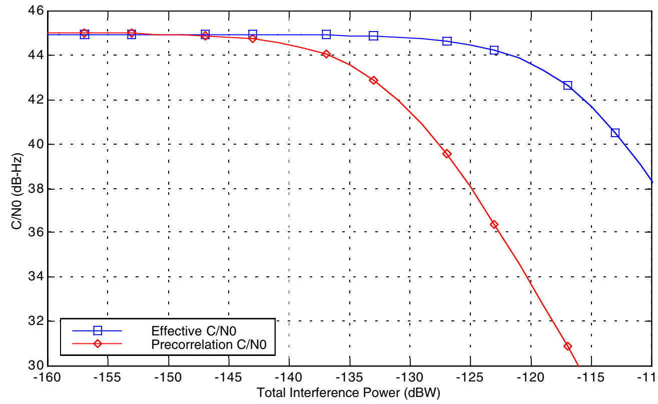

Figure 5. Theoretical Results for Narrowband Interference Offset from Band Center by 7.5 Times the Chip Rate, Receiver Front-End Bandwidth 16 MHz

Figure 6 compares effective C/N_0 and precorrelation C/N_0 for a receiver with 8 MHz front-end bandwidth, and interference whose spectrum is centered at band center, with constant interference power corresponding to C_ι/N_0 of 75 dB-Hz and varying bandwidth. The effective C/N_0 shows that the receiver's ability to acquire, track carrier, and demodulate data is degraded most when the interference bandwidth is small, and that the same power interference has nearly 8 dB less effect when the its bandwidth is the same as the front-end bandwidth. Precorrelation C/N_0 does not indicate the greater degradation caused by narrowband interference at band center, but indicates that C/N_0 is better than it actually is.

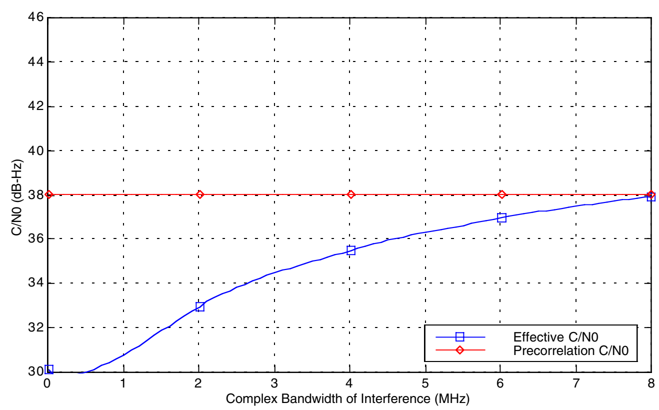

Figure 6. Theoretical Results for Interference Whose Spectrum is Centered in the Band, with C_ι/N_0 of 75 dB-Hz, Receiver Front-End Bandwidth 8 MHz

Figure 7 shows the same results under the same conditions as Figure 6, except with receiver front-end bandwidth of 16 MHz. Here again, the precorrelation C/N_0 does not report the greater degradation caused by narrowband interference at band center, and instead reports less degradation than actually occurs.

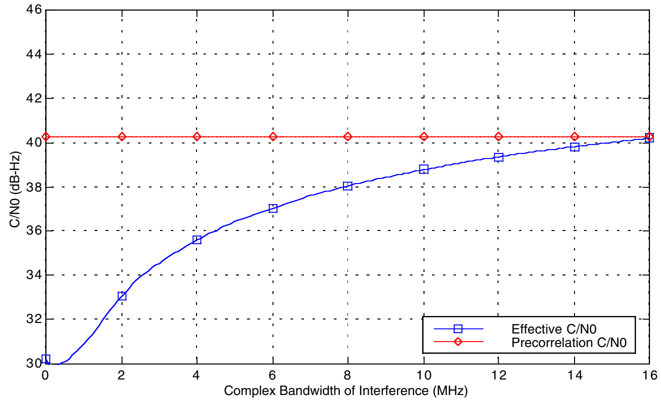

Figure 7. Theoretical Results for Interference Whose Spectrum is Centered in the Band, with C_t / N_0 of 75 dB-Hz, Receiver Front-End Bandwidth 16 MHz

Figure 8 compares effective C / N_0 and precorrelation C / N_0 for a receiver with 8 MHz front-end bandwidth, and interference with 10 kHz bandwidth, having a constant interference power corresponding to C_t / N_0 of 75 dB-Hz, but varying center frequencies. The effective C / N_0 shows that the receiver's ability to acquire, track carrier, and demodulate data is degraded most when the interference is concentrated near band center, and that the same power interference has up to 15 dB less effect when the interference bandwidth is far from band center, but still within the front-end bandwidth. The precorrelation C / N_0 does not report the greater degradation caused by narrowband interference at band center. Instead it reports less degradation than actually occurs for interference near band center, and greater degradation than actually occurs for interference far from band center.

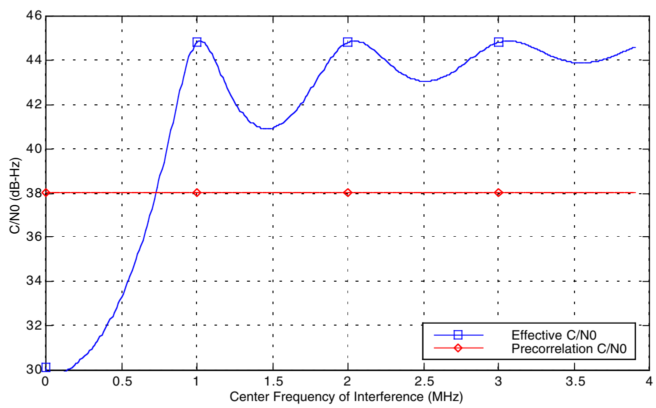

Figure 8. Theoretical Results for Narrowband Interference with C_t / N_0 of 75 dB-Hz, Receiver Front-End Bandwidth 8 MHz

Figure 9 shows the same results under the same conditions as Figure 8, except with receiver front-end bandwidth of 16 MHz. Precorrelation C / N_0 reports less degradation than actually occurs for interference near band center, and greater degradation than actually occurs for interference far from band center.

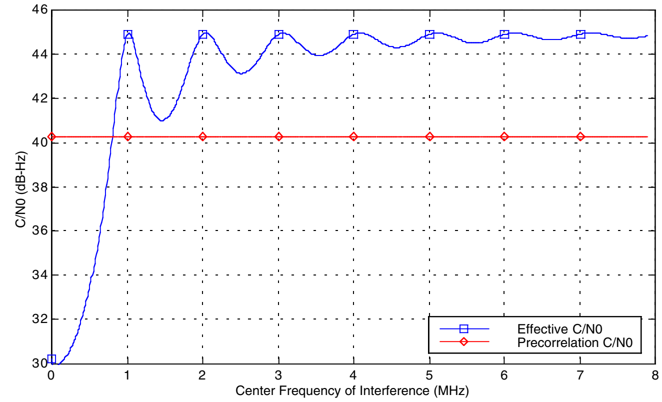

Figure 9. Theoretical Results for Narrowband Interference with C_t / N_0 of 75 dB-Hz, Receiver Front-End Bandwidth 16 MHz

Figure 10 compares predicted effective C / N_0 and precorrelation C / N_0 for a receiver with 8 MHz front-end bandwidth, and partial-band interference near band edge, at different levels of received interference power. Also shown are C / N_0 values reported by a commercial receiver referred to as receiver Model A. To apply the theory, the receiver's front-end bandwidth, the received signal power, and the thermal noise level at the receiver must all be known. Measurements using the methodology described in [1] show that the front-end bandwidth of this receiver is approximately 8 MHz, and that the receiver reports C / N_0 based on postcorrelation measurement of the noise density. The measurements used a received signal power level of –157 dBW and noise density of –200 dBW/Hz. The theoretical curves in Figure 10 show that precorrelation C / N_0 indicates degradation at much lower power levels than actually disturb the effective C / N_0. In fact, effective C / N_0 is degraded to 40 dB-Hz only when the received interference power exceeds –111 dBW. The C / N_0 values reported by the receiver closely match those theoretically predicted for effective C / N_0, substantiating the theory and the indications that the receiver reports C / N_0 based on postcorrelation measurement of the noise density.

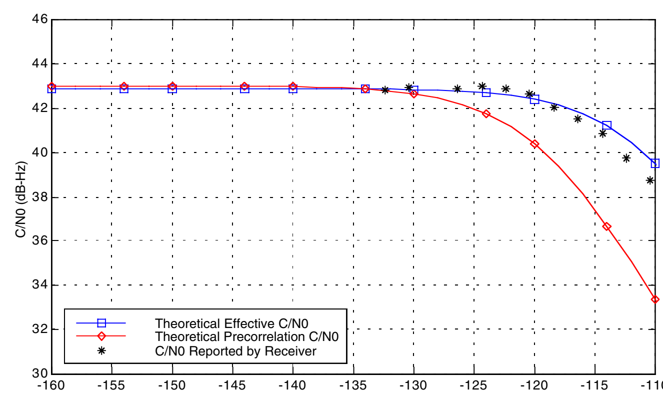

Figure 10. Results for Band-Edge Interference, Theory Uses Receiver Front-End Bandwidth 8 MHz, Model A Receiver

To examine the sensitivity of the results in Figure 10 to receiver front-end bandwidth, Figure 11 shows the same results as in Figure 10, only with the theory evaluated assuming a receiver front-end bandwidth of 16 MHz rather than the 8 MHz that was measured. As expected, the theoretical effective C/N_0 changes little, and still matches the measured data closely. The reported C/N_0 does not match the precorrelation C/N_0, which reports much greater degradation than actually occurs.

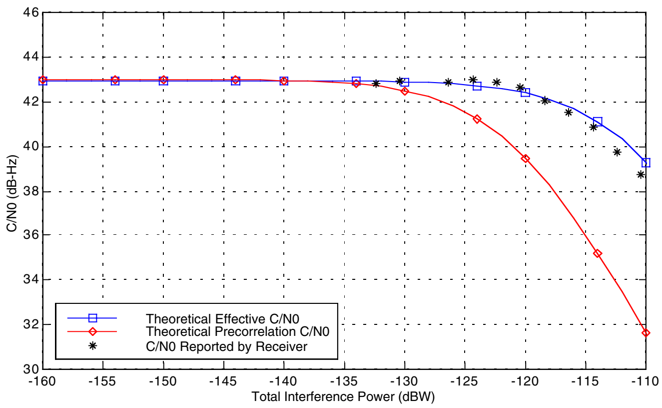

Figure 11. Results for Band-Edge Interference, Theory Uses Receiver Front-End Bandwidth 16 MHz, Model A Receiver

Figure 12 compares effective C/N_0 and precorrelation C/N_0 for a receiver with 16 MHz front-end bandwidth, and band-edge interference at different levels of received interference power. Also shown are C/N_0 values reported by a commercially available receiver referred to as Model B. Measurements using the approach described in [1] show that the front-end bandwidth of this receiver is approximately 16 MHz, and that the receiver reports C/N_0 based on precorrelation measurement of the noise density. The measurements used a received signal power level of –155 dBW and noise density of –200 dBW/Hz. The C/N_0 values reported by the receiver closely match those theoretically predicted for precorrelation C/N_0, substantiating the theory and the indications that the receiver reports C/N_0 based on precorrelation measurement of the noise density. These results show that the receiver reports much greater degradation than actually occurs for interference near band edge; other measurements have shown that this receiver reports much less degradation than actually occurs for interference near band center.

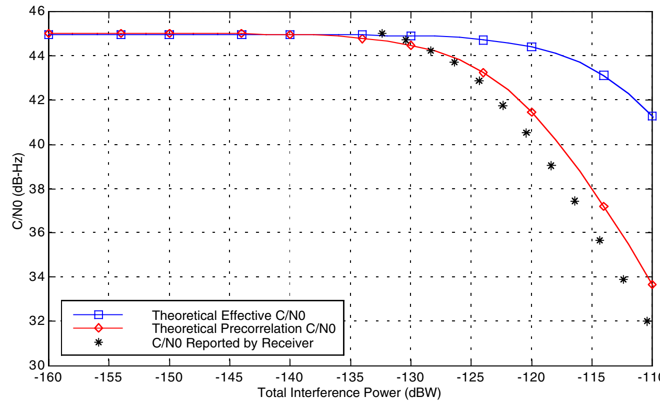

Figure 12. Results for Band-Edge Interference, Theory Uses Receiver Front-End Bandwidth 16 MHz, Model B Receiver

Theory shows that precorrelation C/N_0 is very sensitive to the front-end bandwidth used, especially when the interference power is concentrated at the filter rolloff, as occurs here. Figure 13 shows that using 18 MHz front-end bandwidth produces theoretical predictions of precorrelation C/N_0 that closely match the experimental results. An alternative approach would be to extend the theory to model the actual filter rolloff rather than approximating it as a rectangle.

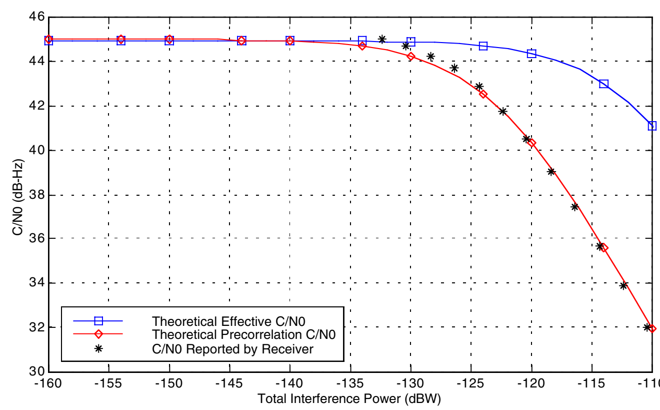

Figure 13. Results for Band-Edge Interference, Theory Uses Receiver Front-End Bandwidth 18 MHz, Model B Receiver

To examine the sensitivity of the results in Figure 12 and Figure 13 to receiver front-end bandwidth, Figure 14 shows the same results as in these figures, only with the theory evaluated assuming a receiver front-end bandwidth of 8 MHz rather than the 16 MHz that was measured in [1]. As expected, the theoretical effective C/N_0 shows very little change with this change. The data does not match the precorrelation C/N_0, demonstrating that this quantity is very sensitive to front-end bandwidth, unlike the effective C/N_0 that actually describes the degradation.

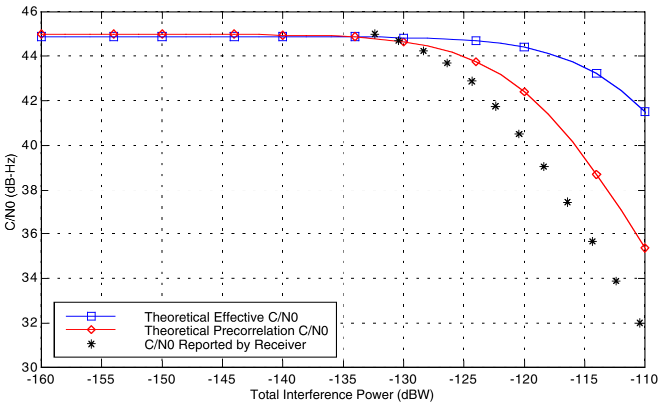

Figure 14. Results for Band-Edge Interference, Theory Uses Receiver Front-End Bandwidth 8 MHz, Model B Receiver

Figure 15 compares effective C / N_0 and precorrelation C / N_0 for a receiver with 24 MHz front-end bandwidth, and band-edge interference at different levels of received interference power. The effective C / N_0 changes very little with the difference in receiver front-end bandwidth, while the precorrelation C / N_0 has greater discrepancy. A receiver that reports precorrelation C / N_0 would report degradation at almost 15 dB lower interference power than degradation actually occurs.

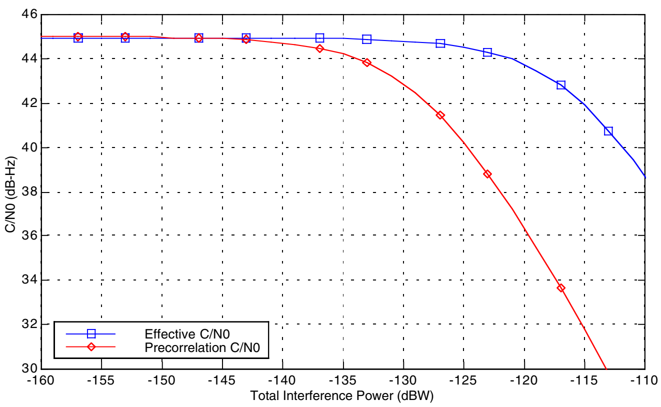

Figure 15. Theoretical Results for Band-Edge Interference, Receiver Front-End Bandwidth 24 MHz

CONCLUSIONS

Theory and experimental results have been presented to consider some ways that receivers estimate the effect of interference on C / N_0, under the assumptions that the interference is continuous, Gaussian, stationary, and circularly symmetric. Also, the results here assume that the receiver front end responds linearly to the noise and interference. At some point, the interference power can be high enough so that functions like mixing and quantization are affected in ways not considered here. Nonetheless, this theory appears to apply for the receivers considered here.

Effective C / N_0 is not an arbitrary descriptor of signal quality, but instead is directly and uniquely related to the performance of essential receiver functions such as acquisition, carrier tracking, and data demodulation.

Some GPS receiver designs may attempt to estimate C / N_0 from precorrelation measurements. While this estimation approach can be reliable in noise and interference that is flat over the front-end bandwidth, it is unreliable for interference whose spectrum is not flat over that bandwidth. When the interference power is concentrated near band center, the precorrelation C / N_0 is higher than the effective C / N_0, so that the reported C / N_0 does not indicate receiver degradation that actually occurs. Conversely, when the interference power is concentrated near band edge, the estimated C / N_0 from the precorrelation C / N_0 is lower than the true effective C / N_0, so the reported C / N_0 indicates that the receiver is degraded more than it actually is. Further, precorrelation C / N_0 is very sensitive to the receiver front-end bandwidth, making it difficult to interpret and to predict theoretically.

The consequences of reporting precorrelation C / N_0 depend on how the receiver uses this erroneous information. If the information is merely reported, then the reliability of the reported C / N_0 may only be misleading, and have little other consequence. Conversely, if the receiver adjusts its state or mode based on the reported C / N_0 (e.g., it does not track a signal with too low a reported C / N_0), then the consequences of erroneously estimating C / N_0 can be more significant. In particular, the receiver may be unreliable when exposed to interference that does not have a flat spectrum.

Furthermore, using reported C / N_0 is not a reliable way to assess the effect of interference on a receiver. Unless it is determined that the receiver uses reliable algorithms for reporting the effect of interference (i.e., that the receiver reliably reports the effective C / N_0), curious results may be obtained, yielding misleading conclusions.

Based on this work, it is recommended that receivers report C / N_0 based on postcorrelation quantities, or an equivalent technique that approximates effective C / N_0. Further, C / N_0 reported by receivers should be used only after determination that the reported C / N_0 is a reliable indicator of receiver performance—that it corresponds to effective C / N_0.

ACKNOWLEDGMENTS

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