AIAA Guidance, Navigation, and Control Conference
2 - 5 August 2010, Toronto, Ontario Canada

AIAA 2010-7581

Multivehicle Spacing Along Smooth Curvilinear Paths

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This paper presents an approach to spacing vehicles along an arbitrary path. Previous research by the authors has shown the development of formation control laws assuming constant distances between vehicles as viewed by inertial observers. However, that approach is not advantageous for spacing vehicles along an arbitrary reference trajectory. By defining along-path and perpendicular-to-path spacing errors, ideal trajectories can be found for each vehicle in the formation. A sliding-mode control law is used to track the ideal trajectories, which provides better performance than a simpler control form. Simulation results demonstrate feasibility of the approach for a five-vehicle formation.

I. Introduction

Decentralized, cooperative control concepts for multivehicle systems have received much interest in the research community due to a multitude of applications including formation control. Formation control research has often focused on control for unmanned systems or robotics to enable autonomous applications such as border surveillance, environmental monitoring, and a wide-range of defense applications. Aircraft spacing in future air-traffic-management operations is a primary motivating application for the research in this paper. Applications, such as Interval Management, using avionics that aid in aircraft spacing are being worked on actively in the international standards arena and in the US as part of Federal Aviation Administration (FAA) development of Automatic Dependent Surveillance Broadcast (ADS-B), which forms a critical component of the FAA's planned Next Generation (NextGen) system. The Surveillance and Broadcast Services (SBS) Program within the FAA, in particular, is charged with developing the Interval Management concept including research in a variety of applications that will derive efficiency and safety benefits from improved aircraft spacing and standardization of Interval Management equipment used onboard aircraft to aid in spacing operations.

Research approaches for the decentralized control of multivehicle systems have been varied, with the approach often developed to suit a desired application. Some research has investigated heading-only control laws for vehicle formations moving at a constant speed.[1,2] Researchers have also investigated "leader-follower" formation control, where vehicles maintain a desired distance relative to a preceding vehicle in the formation.[3–5] References [3] and [4] showed that a kinematic model for planar vehicle motion can be decoupled allowing spacing control laws to be developed for motion in orthogonal directions in an inertial frame; and, control laws were designed to drive vehicles to a desired formation as defined by constant inertial distances between neighboring vehicles. Swaroop et al. have investigated string stability, a measure of how disturbances are propagated through a string of vehicles coupled through their control laws, for a number of cooperative, spacing control laws.[5,6]

A separate body of research has investigated formation control for vehicles tracking an arbitrary path.[7–11] The work in reference [9] investigates using a formation of mobile sensors to generate contour plots in a plane. In reference [10], a sliding-mode controller is applied to track a reference trajectory or a reference vehicle to achieve the desired formation. Similarly, Kang et al. develop formation control laws where the lead vehicle tracks a sinusoidal reference trajectory.[11]

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Research in spacing for future air traffic systems has focused on the development of control laws that longitudinally space aircraft along a path.[12,13] Some research has focused on a state-based control methodology, where the spacing aircraft calculate control inputs from their immediately preceding aircraft's state information alone.[12,14] In contrast, a trajectory-based approach developed in reference [13], has similarities to the spacing approach presented in this paper. String-stability analysis has been investigated for the aircraft spacing problem, which illustrates common challenges.[14–17]

In this paper, we investigate spacing vehicles, or formation control, along an arbitrary path. Whereas previous work by the authors has focused on spacing vehicles for constant distances in an inertial frame, spacing along an arbitrary path cannot be described using constant distances in a fixed frame.[4,18] The approach presented here assumes that a reference trajectory can be described by its along-path distance, or arclength. Vehicle motion in proximity to the path can then be described by a position along the path and a perpendicular distance to the path. Each vehicle in the formation tracks an ideal trajectory, generated from the position of its lead vehicle and knowledge of the reference trajectory. Spacing along an arbitrary path however, may introduce string instabilities to the formation using certain control laws. A sliding-mode control law, developed in reference [5], is implemented to eliminate string instabilities.

This paper is organized as follows. In Section II, background is presented on the vehicle model from which the cooperative control laws are designed. Control laws for spacing vehicles with constant distances in an inertial frame are developed, and a motivating example is used to illustrate the limitations of spacing in an inertial frame. The main results are presented in Section III. Arclength spacing is presented to motivate the spacing approach presented in this paper; the methodology to generate an ideal trajectory for the ith vehicle in the formation is shown; and lastly, the sliding-mode control law is presented. Simulation results are presented in Section IV. A smoothing algorithm is used to generate a reference trajectory for a five-vehicle formation. Simulation results compare the performance of two control laws to highlight the differences in using string-unstable and string-stable control forms to space vehicles along an arbitrary path.

II. Background

A. Vehicle Model

The development of the cooperative control laws presented in this paper are based upon a commonly-used nonlinear vehicle model. This model represents vehicles with negligible velocity in the direction perpendicular to the vehicle's heading; e.g., UAVs and differentially-driven robots.

$$\dot{x} = v\cos\theta; \qquad \dot{y} = v\sin\theta; \qquad \dot{\theta} = \omega \tag{1}$$

Here, x and y are the inertial position of the vehicle, and θ is the heading angle with respect to the x axis. The control inputs to the vehicle are the velocity, v, and angular turn rate, ω.

This vehicle model is differentially flat with flat outputs x and y, which allows the vehicle states and controls to be expressed as functions of the flat outputs and their higher derivatives.[4,19]

$$\theta = \tan^{-1}\left(\frac{\dot{y}}{\dot{x}}\right); \qquad v = \sqrt{\dot{x}^2 + \dot{y}^2}; \qquad \omega = \frac{\ddot{y}\dot{x} - \dot{y}\ddot{x}}{v^2} \tag{2}$$

Second derivatives of the flat outputs are the highest derivatives that appear in ω, and a time derivative of the velocity reveals the second derivatives of the flat outputs.

$$\dot{v} = \frac{\dot{x}\ddot{x} + \dot{y}\ddot{y}}{v} \tag{3}$$

Therefore, new control inputs can be defined as $(\ddot{x}, \ddot{y}) = (u, w)$, and the nonlinear system in Equation (1) can be represented as uncoupled double integrators.

B. Spacing with Constant Distances in the Inertial Frame

Previous research has investigated the development of decentralized, cooperative control laws exploiting the decoupled, double-integrator model.[4] Control laws are designed by defining spacing errors between vehicles in the x and y directions. Derivatives of the spacing-error equations reveal the selection of control inputs such that the vehicles converge to the desired formation.

Consider the definition of the spacing errors in a leader-follower formation, where the error of the ith vehicle is defined relative to the (i-1)th, or the immediately preceding, vehicle. Developments presented here are shown in the x direction only (the development in the y direction is identical).

$$e_i = x_{i-1} - x_i - d_i; \qquad i = 1, ..., n \tag{4}$$

Here, x_i is the inertial position of the ith vehicle, and d_i is the constant desired distance between the ith and (i-1)th vehicles. The first vehicle tracks a reference trajectory described by x_0.

A control law of the following form yields the desired formation for appropriately selected gains.

$$\begin{aligned} u_i &= k_i e_i + c_i \dot{e}_i + \bar{k}_i \left(e_1 + e_2 + ... + e_i \right) + \bar{c}_i \left(\dot{e}_1 + \dot{e}_2 + ... + \dot{e}_i \right) + \ddot{x}_0 \\ &= k_i \left(x_{i-1} - x_i - d_i \right) + c_i \left(\dot{x}_{i-1} - \dot{x}_i \right) + \bar{k}_i \left(x_0 - x_i - \sum_{j=1}^{i} d_j \right) + \bar{c}_i \left(\dot{x}_0 - \dot{x}_i \right) + \ddot{x}_0 \end{aligned} \tag{5}$$

C. Spacing with Non-Constant Distances in the Inertial Frame

If a rigid-body-like formation is desired wherein the trajectories do not cross for a circular reference trajectory, control laws can still be designed in the inertial frame. However, the desired distances between vehicles are not constant in the inertial frame, and the first and second derivatives of the distances must be included in the control laws. For example, given a C^2 reference trajectory $\{x_0(t), y_0(t)\}$, a rigid-body orientation, ψ, is defined as the orientation of the reference trajectory in the $\hat{\boldsymbol{b}}$ frame relative to the inertial frame, $\hat{\boldsymbol{n}}$.

$$\psi = \tan^{-1} \left(\frac{\dot{y}_0}{\dot{x}_0} \right) \tag{6}$$

Constant distances between vehicles can be defined in a virtual body-fixed reference frame, where the $\hat{\boldsymbol{b}}_1$ direction is aligned with the orientation of the reference trajectory: $\boldsymbol{d} = l\hat{\boldsymbol{b}}_1 + m\hat{\boldsymbol{b}}_2$ (l and m are constant distances aligned with and perpendicular to the reference trajectory). These distances and their derivatives can be coordinatized in the inertial frame.

$$\begin{aligned} \boldsymbol{d} &= \left(l \cos\psi - m \sin\psi \right) \hat{\boldsymbol{n}}_1 + \left(l \sin\psi + m \cos\psi \right) \hat{\boldsymbol{n}}_2 \\ \dot{\boldsymbol{d}} &= \left(l\dot{\psi} \sin\psi - m\dot{\psi} \cos\psi \right) \hat{\boldsymbol{n}}_1 + \left(l\dot{\psi} \cos\psi - m\dot{\psi} \sin\psi \right) \hat{\boldsymbol{n}}_2 \\ \ddot{\boldsymbol{d}} &= \left(-l\ddot{\psi} \sin\psi - l\dot{\psi}^2 \cos\psi - m\ddot{\psi} \cos\psi + m\dot{\psi}^2 \sin\psi \right) \hat{\boldsymbol{n}}_1 + \\ &\quad + \left(l\ddot{\psi} \cos\psi - l\dot{\psi}^2 \sin\psi - m\ddot{\psi} \sin\psi - m\dot{\psi}^2 \cos\psi \right) \hat{\boldsymbol{n}}_2 \end{aligned} \tag{7}$$

The control inputs in equation (5) are modified to include the first and second derivatives of the distances between vehicles. To yield homogeneous error dynamics, the control input to the ith vehicle also includes the second derivatives of the desired distances for the first through ith vehicles. Note that the $\ddot{d}_i$ terms shown in equation (8) are the $\hat{\boldsymbol{n}}_1$ components from equation (7); control inputs in the y direction use the distances in the $\hat{\boldsymbol{n}}_2$ direction.

$$u_1 = k_1 (x_0 - x_1 - d_1) + c_1 \left(\dot{x}_0 - \dot{x}_1 - \dot{d}_1\right) + \ldots$$
$$\ldots + \bar{k}_1 (x_0 - x_1 - d_1) + \bar{c}_1 \left(\dot{x}_0 - \dot{x}_1 - \dot{d}_1\right) + \ddot{x}_0 - \ddot{d}_1$$

$$u_2 = k_2 (x_1 - x_2 - d_2) + c_2 \left(\dot{x}_1 - \dot{x}_2 - \dot{d}_2\right) + \ldots$$
$$\ldots + \bar{k}_2 \left(x_0 - x_2 - \sum_{j=1}^{2} d_j\right) + \bar{c}_2 \left(\dot{x}_0 - \dot{x}_2 - \sum_{j=1}^{2} \dot{d}_j\right) + \ddot{x}_0 - \left(\ddot{d}_1 + \ddot{d}_2\right) \tag{8}$$
$$\vdots$$
$$u_i = k_i (x_{i-1} - x_i - d_i) + c_i \left(\dot{x}_{i-1} - \dot{x}_i - \dot{d}_i\right) + \ldots$$
$$\ldots + \bar{k}_i \left(x_0 - x_i - \sum_{j=1}^{i} d_j\right) + \bar{c}_i \left(\dot{x}_0 - \dot{x}_i - \sum_{j=1}^{i} \dot{d}_j\right) + \ddot{x}_0 - \left(\ddot{d}_1 + \ddot{d}_2 + \ldots + \ddot{d}_i\right)$$

Therefore, to implement a spacing-control law with non-zero $\dot{d}$ and $\ddot{d}$ terms, each vehicle must have knowledge of the reference trajectory and the desired spacing between all of the preceding vehicles in the formation.

Figure 1 compares vehicle trajectories for a circular reference trajectory: (a) shows the overlapping trajectories when vehicles are spaced with constant distances in the inertial frame as described by equation (5), and (b) shows the rigid-body-like motion that results when the desired distances between vehicles are constant in a non-inertial frame as described by equation (8). The vehicles start with initial condition errors (starting positions are denoted by the asterisks), and converge to the desired formation.

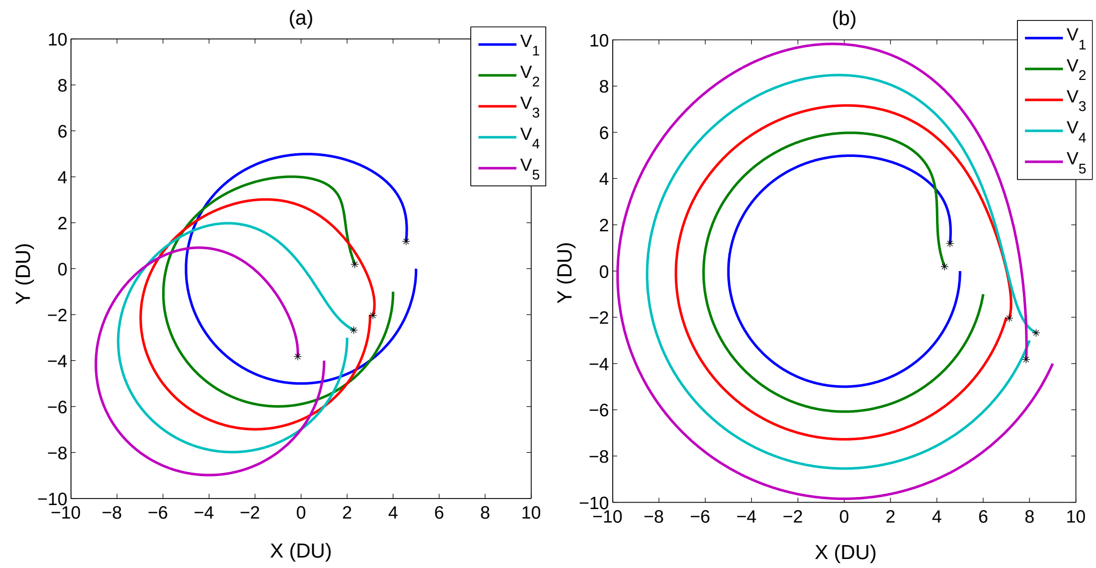

Figure 1. Vehicle positions for a circular reference trajectory: (a) constant distances in the inertial frame, and (b) constant distances in a non-inertial frame; x and y are shown for some generic distance units (DU).

III. Main Results

In this section, control laws to space vehicles along an arbitrary path are investigated. The arbitrary path is parameterized in terms of arclength, or distance along the path, and the arclength parameter is the spacing variable that motivates the control-law design. This approach still requires that all vehicles have knowledge of the reference trajectory; however, the control law design does not require that vehicles know the $\ddot{d}$ terms for all preceding vehicles in the formation.

A. Definition of the Path Reference Frame

Paths in n-dimensional space can be described using geometric properties, such as arclength and curvature. Given a C^1, or smooth, path in n-dimensions and parameterized in time, the arclength and curvature can be determined. The arclength of a path is an intrinsic parameter, which depends only on how the path bends and not on how quickly the path is traced in time.[20]

Given a vector of inertial positions, $\boldsymbol{r}(t)$, and velocities, $\dot{\boldsymbol{r}}(t)$, the length of the path can be determined by integrating the speed along the path for a given time interval.

$$L\left(\boldsymbol{r}(t_0, t_f)\right) = \int_{t_0}^{t_f} ||\dot{\boldsymbol{r}}(t)||\, dt \tag{9}$$

Similarly, the arclength $s(t)$ is an intermediate point on the path. This integral can be used to parameterize the path as a function of arclength rather than time.

$$s(t) = \int_{t_0}^{t} ||\dot{\boldsymbol{r}}(\tau)||\, d\tau \tag{10}$$

To parameterize the path as a function of arclength, equation (10) must be an invertible function. Equation (10) will yield a theoretically invertible expression if the speed along the path is never equal to zero. Given $\dot{\boldsymbol{r}}(t) \neq 0$, the speed along the path is always positive, and the arclength $s(t)$ is strictly increasing. Thus, time is written as a function of arclength, and the path can be described using the arclength parameter. Challenges arise in determining time as a function of arclength when equation (10) does not yield a closed-form solution.

To define the path reference frame used in this approach, consider a two-dimensional path. The path reference frame is defined along the path with the $\hat{\boldsymbol{b}}_1$ vector tangent to the path, or aligned with the velocity vector, and the $\hat{\boldsymbol{b}}_2$ vector is perpendicular to the path in the plane. The velocity and acceleration of a vehicle along the path are coordinatized in the path reference frame.

$$\dot{\boldsymbol{r}}(t) = v\hat{\boldsymbol{b}}_1 \tag{11}$$

$$\ddot{\boldsymbol{r}}(t) = \dot{v}\hat{\boldsymbol{b}}_1 + v\dot{\psi}\hat{\boldsymbol{b}}_2 \tag{12}$$

Here, v and $\dot{v}$ are the velocity and acceleration along the path, respectively; $\dot{\psi}$ is the angular turn rate of the path frame relative to the inertial axis; and, ψ is the angle of the path reference frame with respect to the horizontal inertial axis. Acceleration components in the $\hat{\boldsymbol{b}}_1$ and $\hat{\boldsymbol{b}}_2$ directions indicate that a multivehicle system can be spaced along the path and perpendicular to the path, which motivates the definition of along-path and perpendicular-to-path spacing errors.

B. Arclength Spacing

Given a reference trajectory parameterized by arclength $\{X_0(s), Y_0(s)\}$, vehicles can be spaced along and perpendicular to the reference trajectory. Figure 2 illustrates the spacing of a vehicle string along an arbitrary reference trajectory. Each vehicle's (x, y) position is projected onto the reference trajectory to determine the along-path distance, s_i, and the perpendicular distance, p_i, from the trajectory.

Along-path errors are written as a function of arclength between two adjacent vehicles.

$$e_i^s = s_{i-1} - s_i - d_i^s; \quad i = 1, ..., n \tag{13}$$

Here, d_i^s is a constant arclength to the preceding vehicle, and s_0 is the arclength of the reference trajectory (at a given time). Perpendicular-to-path errors are similarly defined, where d_i^p is a constant perpendicular distance to the preceding vehicle.

$$e_i^p = p_{i-1} - p_i - d_i^p; \quad i = 1, ..., n \tag{14}$$

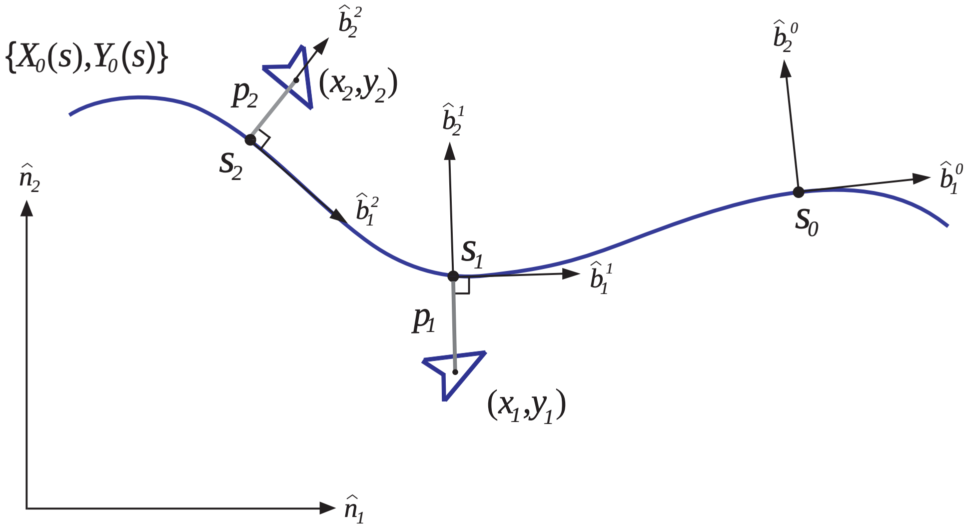

Figure 2. Projection of vehicle positions onto the reference trajectory. Path reference frames are shown at the different arclength positions ($s_2 < s_1 < s_0$).

The perpendicular distance of the reference trajectory, p_0, is zero for all s. The desired arclength and perpendicular distance for the ith vehicle relative to its lead vehicle, denoted as s_i^* and p_i^*, respectively, are determined from Equations (13) and (14) when the spacing errors are equal to zero.

$$s_i^* = s_{i-1} - d_i^s; \qquad p_i^* = p_{i-1} - d_i^p \tag{15}$$

The along-path position at the desired arclength can be determined from the reference trajectory: $\{X_0\left(s_i^*\right), Y_0\left(s_i^*\right)\}$. Velocity and acceleration at the desired along-path position are also easily determined from the reference trajectory. From the velocity at the along-path position, the orientation of the path reference frame at the desired arclength, ψ_i, is determined.

$$\psi_i(s_i^*) = \tan^{-1}\left(\frac{\dot{Y}_0\left(s_i^*\right)}{\dot{X}_0\left(s_i^*\right)}\right) \tag{16}$$

The desired perpendicular distance, p_i^*, is used to generate the ideal trajectory for the ith vehicle, relative to the (i-1)th vehicle through s_i^*, in the inertial reference frame.

$$\begin{array}{l|l} X_i = X_0\left(s_i^*\right) - p_i^* \sin\psi_i & Y_i = Y_0\left(s_i^*\right) + p_i^* \cos\psi_i \\ \dot{X}_i = \dot{X}_0\left(s_i^*\right) - \dot{\psi}_i p_i^* \cos\psi_i & \dot{Y}_i = \dot{Y}_0\left(s_i^*\right) - \dot{\psi}_i p_i^* \sin\psi_i \\ \ddot{X}_i = \ddot{X}_0\left(s_i^*\right) - \ddot{\psi}_i p_i^* \cos\psi_i + \dot{\psi}_i^2 p_i^* \sin\psi_i & \ddot{Y}_i = \ddot{Y}_0\left(s_i^*\right) - \ddot{\psi}_i p_i^* \sin\psi_i - \dot{\psi}_i^2 p_i^* \cos\psi_i \end{array} \tag{17}$$

C. Spacing Control Law

A proportional-derivative (PD) control law can be designed for the ith vehicle to track the ideal trajectory designed relative to the (i-1)th vehicle: $\{X_i, Y_i\}$. However, the inertial velocities derived from an arbitrary reference trajectory can lead to string instabilities. Tracking velocities that include high-frequency content will cause spacing errors to grow through the formation when a PD control law is implemented. Swaroop and Hedrick proposed a sliding-mode controller that uses information from the preceding vehicle and a reference trajectory to yield a string-stable system; that sliding-mode control law is used here.[5] Simulation results in Section IV compare the results for the PD and sliding-mode control laws to illustrate the performance of different control methodologies.

Along-path and perpendicular-to-path spacing errors for the ith vehicle relative to the reference trajectory, $\{X_0(s), Y_0(s)\}$, are defined similarly to those in Equations (13) and (14).

$$\hat{e}_i^s = s_0 - s_i - d_{0_i}^s; \qquad \hat{e}_i^p = p_0 - p_i - d_{0_i}^p; \qquad i = 1, ..., n \tag{18}$$

The distances $d_{0_i}^s$ and $d_{0_i}^p$ are the desired distances of the ith vehicle from the reference arclength and perpendicular offset, respectively. The ideal arclength and perpendicular offset of the ith vehicle relative to the reference trajectory are shown below.

$$\hat{s}_i^* = s_0 - d_{0_i}^s; \qquad \hat{p}_i^* = p_0 - d_{0_i}^p \tag{19}$$

Ideal parameters relative to the reference trajectory are denoted by the overhat.

The ideal trajectory of the ith vehicle relative to the reference trajectory is determined in a similar manner to the trajectory in equation (17), and is shown below for clarity; $\hat{\psi}_i$ is the orientation of the reference frame at $\hat{s}_i^*$.

$$\begin{array}{l|l} \hat{X}_i = X_0\left(\hat{s}_i^*\right) - \hat{p}_i^* \sin\hat{\psi}_i & \hat{Y}_i = Y_0\left(\hat{s}_i^*\right) + \hat{p}_i^* \cos\hat{\psi}_i \\ \dot{\hat{X}}_i = \dot{X}_0\left(\hat{s}_i^*\right) - \dot{\hat{\psi}}_i\hat{p}_i^* \cos\hat{\psi}_i & \dot{\hat{Y}}_i = \dot{Y}_0\left(\hat{s}_i^*\right) - \dot{\hat{\psi}}_i\hat{p}_i^* \sin\hat{\psi}_i \\ \ddot{\hat{X}}_i = \ddot{X}_0\left(\hat{s}_i^*\right) - \ddot{\hat{\psi}}_i\hat{p}_i^* \cos\hat{\psi}_i + \dot{\hat{\psi}}_i^2\hat{p}_i^* \sin\hat{\psi}_i & \ddot{\hat{Y}}_i = \ddot{Y}_0\left(\hat{s}_i^*\right) - \ddot{\hat{\psi}}_i\hat{p}_i^* \sin\hat{\psi}_i - \dot{\hat{\psi}}_i^2\hat{p}_i^* \cos\hat{\psi}_i \end{array} \tag{20}$$

Spacing errors of the ith vehicle relative to the ideal trajectories with respect to the (i-1)th vehicle and reference trajectory are now defined. The expressions below are in the x direction only; the development in the y direction is identical.

$$\begin{array}{r|l} e_i = X_i - x_i & \hat{e}_i = \hat{X}_i - x_i \\ \dot{e}_i = \dot{X}_i - \dot{x}_i & \dot{\hat{e}}_i = \dot{\hat{X}}_i - \dot{x}_i \end{array} \tag{21}$$

The control input to the ith vehicle in the x direction, u_i, is shown below for the sliding-mode controller.

$$u_i = \frac{1}{1+q_3}\left(\ddot{X}_i + q_3\ddot{\hat{X}}_i + (q_1+\lambda)\,\dot{e}_i + (q_4+\lambda q_3)\,\dot{\hat{e}}_i + q_1\lambda e_i + \lambda q_4 \hat{e}_i\right) \tag{22}$$

The constants q_1, q_3, q_4, and λ in equation (22) are related to the sliding surface defined in reference [5] and should be chosen to achieve a desired rate of convergence. Note that the control input u_i does not require spacing information from any vehicles in the formation other than its lead, which is a significant advantage over designing the control laws in an inertial frame. However, each vehicle must have knowledge of the common reference trajectory in order to implement this spacing scheme.

It should be noted that the string stable, sliding-mode control law provides system robustness to disturbances. Disturbances to the system are mitigated through the formation in the same manner that the effects of the arbitrary trajectory are mitigated. The effect of disturbances on each individual vehicle can be minimized through the selection of control gains, q_1, q_3, q_4, and λ. This aspect of the control design is not explored here as it is dependent upon how disturbances affect the vehicle model.

IV. Simulation Results

A. Smoothing Algorithm for Trajectory Generation

A smoothing algorithm developed by Junkins and Jancaitis is used to generate trajectories through a sequence of arbitrary points.[21] The algorithm, as described in the reference, generates a C^1 trajectory $\{X(s), Y(s)\}$. For the spacing concept presented here, the smoothing algorithm was modified to generate trajectories that are continuous through the third derivative. C^3 continuity is required in order to find reference trajectories with continuous positions, velocities, and accelerations. Third derivatives $\dddot{X}(s)$ and $\dddot{Y}(s)$ are also needed to determine $\ddot{\psi}$, which is used to find the ideal accelerations in the inertial frame.

The modified smoothing algorithm generates $\{X_0(s), Y_0(s)\}$, where s is defined as a function of time. Time derivatives of the reference trajectory use the chain rule, and the reference velocities in the x and y directions must be normalized to achieve the desired along-path speed, $\dot{s}$. In further development below, the subscript zero is dropped from the reference trajectory notation for simplicity.

$$\dot{X}(s) = \frac{1}{N}\left(\frac{dX}{ds}\right)\dot{s}; \qquad \dot{Y}(s) = \frac{1}{N}\left(\frac{dY}{ds}\right)\dot{s}; \qquad N = \sqrt{\left(\frac{dX}{ds}\right)^2 + \left(\frac{dY}{ds}\right)^2} \tag{23}$$

Higher derivatives of the reference trajectory are made easier by defining $f(s)$ and $g(s)$ as shown.

$$f(s) = \frac{1}{N}\left(\frac{dX}{ds}\right); \qquad g(s) = \frac{1}{N}\left(\frac{dY}{ds}\right) \tag{24}$$

Second and third time derivatives of the reference trajectory use spatial derivatives of $f(s)$ and $g(s)$ and time derivatives of $s(t)$.

$$
\begin{array}{l|l}
\ddot{X}(s) = \frac{df}{ds}\dot{s}^2 + f\ddot{s} & \ddot{Y}(s) = \frac{dg}{ds}\dot{s}^2 + g\ddot{s} \\
\dddot{X}(s) = \frac{d^2 f}{ds^2}\dot{s}^3 + 3\frac{df}{ds}\dot{s}\cdot\ddot{s} + f\dddot{s} & \dddot{Y}(s) = \frac{d^2 g}{ds^2}\dot{s}^3 + 3\frac{dg}{ds}\dot{s}\cdot\ddot{s} + g\dddot{s}
\end{array}
\tag{25}
$$

The reference trajectory used for the simulation results in Section IV.B is shown in Figure 3. The red circles denote a sequence of points that the reference trajectory passes through. Clustered points at the start and end of the trajectory are used to generate the starting and ending legs as described in reference [21]. Time derivatives are generated for a constant along-path speed $\dot{s} = 1$ distance unit/time unit (DU/TU). Additionally, equation (3) is confirmed to equal zero along the path; i.e., $\dot{v} = \ddot{s} = 0$.

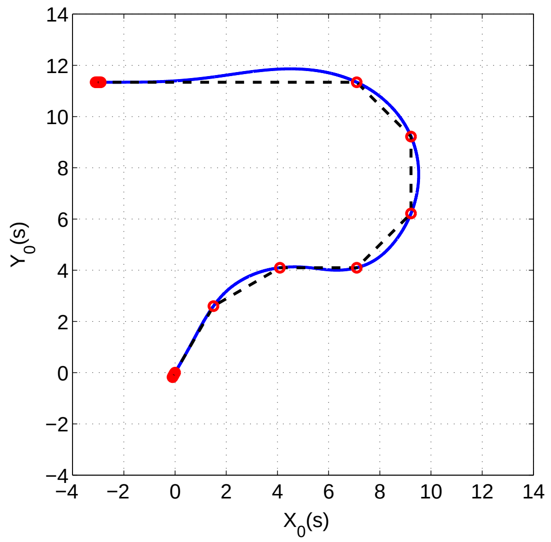

Figure 3. Reference trajectory generated using the modified smoothing algorithm.

B. Spacing Results

In the simulation, vehicle positions are projected onto the reference trajectory using nonlinear least squares to determine the arclength and perpendicular offset for each (x,y) position.[22] A look-up table containing the reference trajectory positions as a function of arclength is used to generate an initial guess for the nonlinear-least-squares algorithm.

1. Proportional-Derivative Control Law

Figure 4 shows simulation results for a five-vehicle formation using a PD control law to track the ideal trajectory relative to each vehicle's lead. The x, y positions of each vehicle are shown with starting positions of the vehicles denoted by the asterisks; the desired spacing between vehicles is $d_i^s = 0.1$ DU and $d_i^p = 0.1$ DU. String instabilities are revealed in the tracking behavior, where the fifth vehicle in particular, incurs larger spacing errors around the turn. The string-unstable behavior is more evident by the along-path and perpendicular-to-path spacing errors, also shown in Figure 4, which show the growth of spacing errors through the formation; e.g., the maximum spacing errors of the fifth vehicle are greater than the fourth vehicle's maximum errors.

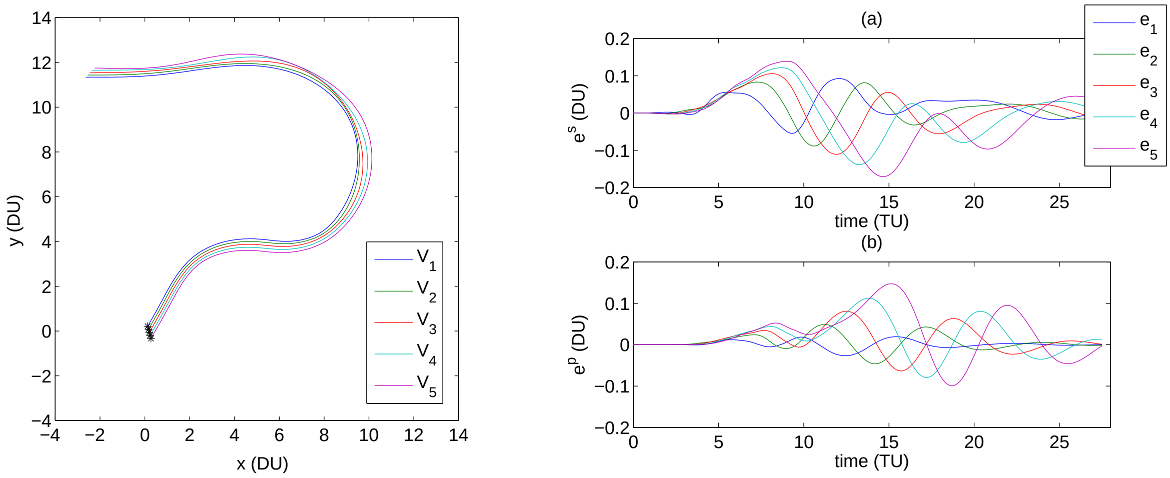

Figure 4. The five-vehicle formation spacing along an arbitrary path using the proportional-derivative control law is shown on the left. The resulting along-path (a) and perpendicular-to-path (b) spacing errors are shown on the right.

2. Sliding-Mode Control Law

Figure 5 shows the simulation results using the sliding-mode controller in equation 22. The tracking performance of the sliding-mode control law appears significantly improved over the tracking performance achieved with the PD control law. The resulting along-path and perpendicular-to-path spacing errors confirm that the sliding-mode controller yields string-stable behavior, where spacing errors are decreased through the formation. Although the sliding-mode control law uses spacing errors relative to both the preceding vehicle and the reference trajectory, only spacing errors relative to the preceding vehicle are shown here.

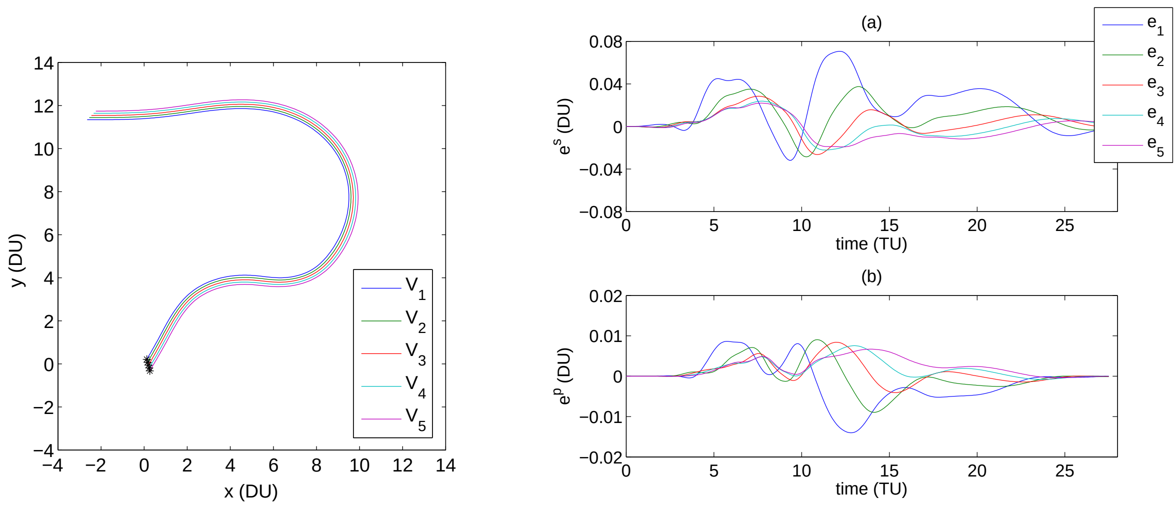

Figure 5. The five-vehicle formation spacing along an arbitrary path using the sliding-mode control law is shown on the left. The resulting along-path (a) and perpendicular-to-path (b) spacing errors are shown on the right.

V. Conclusions

An approach to spacing vehicles along an arbitrary path was presented. Previous research was used to motivate a new approach for spacing vehicles along and perpendicular to an arbitrary reference trajectory. Along-path and perpendicular-to-path spacing errors were defined, and these errors were shown to generate an ideal trajectory for a vehicle to follow relative to its lead vehicle. Tracking an arbitrary path can lead to string instabilities, or the growth of spacing errors along the formation, which necessitated the application of a string-stable control law. A sliding-mode controller was implemented, which uses information for the reference trajectory and the ideal trajectory to achieve the desired formation. To demonstrate the feasibility of the approach for a five-vehicle formation, a reference trajectory was designed using a smoothing algorithm. Simulation results compared formation performance for both a string-unstable, proportional-derivative control law and the string-stable, sliding-mode control law.

Future research will continue to investigate the feasibility of this approach for spacing aircraft in future air traffic systems, including further definition of the spacing application (e.g., en-route spacing or approach spacing) and the communication of trajectory information to the aircraft involved in the spacing application. This research approach will also be extended to a time-based spacing application, where aircraft are spaced in time across a common point.

Acknowledgments

The authors would like to thank John Koelling from the FAA's Surveillance and Broadcast Services (SBS) office and Jonathan Hammer from MITRE for their thorough reviews and helpful comments in the preparation of this paper.

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