ANALYSIS OF SPECIFIED AND HYPOTHETICAL GPS IIIC INTEGRITY FOR LPV 200 OPERATIONS*

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BIOGRAPHY

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ABSTRACT

Modernized GPS IIIC will broadcast an integrity assured user range accuracy (URA) parameter describing a level of satellite ranging error which will only be exceeded with a small probability (10^{-8} per hour) without timely notification. Consequently, there is great interest in the potential of GPS IIIC to provide Localizer Performance with Vertical Guidance to a decision altitude of 200 ft (LPV 200), without need for additional augmentation. This paper addresses the potential integrity performance of GPS IIIC to meet the fault-based integrity requirement for LPV 200 operations. This requirement states that the magnitude of the user's vertical position error in the presence of an undetected satellite range fault cannot exceed 15 m with a probability larger than 10^{-5}. The analysis of achieved integrity performance is based on two alternative concepts: 1) a "specified" concept in which the satellite range error probability is bounded by points (discrete or a continuum) related to a Gaussian probability density function with standard deviation equal to URA and 2) a "hypothetical" monitor concept in which GPS IIIC integrity is modeled to meet a monitor missed detection probability consistent with an assumed prior probability of fault and integrity assured performance. For both concepts, the resulting risk depends on the vertical alert limit (VAL) which the user applies to restrict the satellite geometry and therefore also the translation of any single satellite fault into vertical position error. The analysis finds the largest VAL that will permit the faulted requirement to be met assuming a URA of 0.7 m. To maximize availability it would be desirable to permit the same maximum VAL of 35 m as for LPV 200 operations using the Wide Area Augmentation System (WAAS). For the "specified" concept, if only two discrete points are defined (4.42 and 5.73×URA) as in the current GPS IIIC specifications, the VAL must be limited to about 10.6 m. If, however, a continuous Gaussian bound could be assumed by users, the VAL could be increased to about 19.2 m. For the "hypothetical" integrity performance based on a monitor with Gaussian noise statistics and threshold set for a reasonable false alarm probability, the VAL could be increased to about 24.4 m. Based on these observations it is recommended that: 1) the requirement be revised to describe a continuous Gaussian bound, 2) planned operational tests include data collection at many more points than just the two currently specified and 3) the same analysis concept used for the "hypothetical" monitor be applied to the actual monitor after the detailed monitoring system design of GPS IIIC becomes more extensively defined.

INTRODUCTION

Background

All GPS satellites broadcast a user range accuracy (URA) parameter that may be used to statistically characterize the instantaneous user range error (IURE) due to the satellite signal in space (SIS). As the performance of GPS has evolved and improved, the URA has taken on a very specific role for defining integrity guaranteed for the GPS SIS. The integrity performance guarantee may be stated as "The probability per hour per signal-in-space that the IURE exceeds the NTE Tolerance without a Timely Alert shall be ≤ P" [1]. For the current Block II GPS satellites, the NTE Tolerance is 4.42×URA, a Timely Alert is within 8 to 10 s and P = 10^{-5} [1]. Modernized GPS IIIC will

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provide enhanced integrity of satellite ranging signals in the following ways [2]. First the notification time will be shortened to 5.2 s. Second an additional integrity status flag (ISF) will be added to the broadcast. When the integrity status flag is “off”, the integrity guarantee will be the same as legacy Block II, i.e, 1 - 10^{-5} per hour per SV at a level of 4.42×URA. However, when the integrity status flag is “on”, the integrity performance guarantee will be further extended to a level of 5.73×URA with a probability of 1 - 10^{-8} per hour per SV.

The improved performance of integrity assured GPS IIIC (when integrity status flag is “on”) is consistent with that required by more demanding aviation approach applications. Of particular interest is the potential of GPS IIIC alone to provide Localizer Performance with Vertical Guidance to a decision altitude of 200 ft (LPV 200), without need for additional augmentation. Navigation Sensor Error (NSE) performance requirements have been established for LPV 200 for both fault-free (accuracy) and faulted (integrity) circumstances [3, 4]. However, meeting the integrity requirements on NSE is likely to be more difficult.

Purpose and Organization of Paper

This paper evaluates the potential of integrity assured GPS IIIC integrity to meet LPV 200 integrity requirements. The paper begins with a summary of LPV 200 requirements for both fault-free (accuracy) and faulted circumstances (integrity). Next, the general methodology for the calculation of integrity performance is presented. The first set of results are presented for GPS IIIC integrity meeting an overall performance bound using the general concept as currently specified. The second set of results are presented for GPS IIIC integrity meeting a hypothetical fault monitoring performance requirement. The paper concludes with a summary.

NSE_V PERFORMANCE REQUIREMENTS FOR LPV 200

Navigation Sensor Error (NSE) performance for LPV 200 applications must meet requirements in both the lateral and vertical dimensions. However, requirements on vertical navigation sensor error (NSE_V) are more demanding and they will be the requirements considered in this paper. The requirements on NSE_V assumed for this analysis are based on [3, 4].

In the fault-free condition the following two requirements must be met:

$$\mathrm{Prob}\left\{\left|NSE_V\right| > 4.0\ \mathrm{m}\right\} \le 0.05 \tag{1}$$

$$\mathrm{Prob}\left\{\left|NSE_V\right| > 10.0\ \mathrm{m}\right\} \le 10^{-7} \tag{2}$$

The first requirement is often stated alternatively as demanding that the 95% vertical NSE be no worse than 4.0 m. As stated in [4] these requirements are equivalent to the vertical accuracy of an instrument landing system (ILS) capable of providing Category I (CAT I) precision approach service.

For the faulted condition the following requirement must be met

$$\mathrm{Prob}\left\{\begin{matrix}\left|NSE_V\right| > 15.0\ \mathrm{m} \\ \text{due to undetected fault}\end{matrix}\right\} \le 10^{-5} \tag{3}$$

As discussed in [4], this requirement addresses the effect of a significant undetected NSE_V on the ability of the flight crew to visually acquire the runway and safely land the aircraft after reaching the 200 ft decision altitude indicated by GPS. During this transition from guidance based on GPS to manual flight based on visual cues, it is highly undesirable for the flight crew to perceive that they must take significant action to compensate for a navigation system error. The result may be increased workload and decreased margin of safety. According to [4] “Limited operational trials, in conjunction with operational expertise, have indicated that navigation errors less than 15 meters consistently result in acceptable touchdown performance. For errors larger than 15 meters, there can be a significant increase in the flight crew workload and potentially a significant reduction in the safety margin, particularly for errors that shift the point where the aircraft reaches the decision altitude closer to the runway threshold where the flight crew may attempt to land with an unusually high rate of descent.” Since the hazard severity of such an event is major, the probability that $|NSE_V|$ exceeds 15.0 m in the presence of an undetected fault must be no greater than 10^{-5}.

RELATING LPV 200 REQUIREMENT TO GPS IIIC PERFORMANCE

The integrity requirement for LPV 200 is stated as in the position domain and per approach. However, the requirement/performance for GPS IIIC is expressed in the range domain on a per satellite per hour basis. For convenience in the rest of this paper, the LPV 200 requirement will be converted to the GPS IIIC form. The following conversion of the allowed probability reflects the approach duration of 150 s

$$\begin{aligned}&\text{Prob}\left\{\left|NSE_V\right| \geq 15\,\text{m / hr}\right\}\\&= \frac{3600\,\text{s / hr}}{150\,\text{s / app}} \times 10^{-5}\ /\ \text{app}\\&= 2.4 \times 10^{-4}\ /\ \text{hr}\end{aligned} \tag{4}$$

In order to relate a requirement in the position domain to a requirement in the range domain per SV the same philosophy will be used herein as applied to the integrity analysis of LAAS for CAT I precision approaches. CAT I LAAS integrity has only been specified at a high level [5], with the details of integrity allocations left to the ground station manufacturer. Although such details are typically proprietary, some work in the public domain, such as [6] and [7], has revealed the integrity allocation philosophy that has been deemed acceptable for LAAS. In particular, the total integrity risk in the position domain has been conservatively determined as the risk over all N of the individual satellites in the position solution. Applying this philosophy to LPV 200 gives the acceptable risk on a per SV basis as

$$\begin{aligned}&\text{Prob}\left\{\left|NSE_V\right| \geq 15\,\text{m / hr / SV}\right\}\\&= \frac{\text{Prob}\left\{\left|NSE_V\right| \geq 15\,\text{m / hr}\right\}}{\text{N}}\\&\leq \frac{2.4 \times 10^{-4}\ /\ \text{hr}}{\text{N}}\end{aligned} \tag{5}$$

The work in [6] and [7] indicates that the LAAS ground station must assume integrity responsibility for up to a maximum of 18 visible satellites that could be used in the position solution. However, for this analysis a less conservative and more representative value of N = 10 will be assumed, giving

$$\begin{aligned}&\text{Prob}\left\{\left|NSE_V\right| > 15\,\text{m / hr / SV}\right\}\\&\leq \frac{2.4 \times 10^{-4}\ /\ \text{hr}}{10} = 2.4 \times 10^{-5}\ /\ \text{hr / SV}\end{aligned} \tag{6}$$

INTEGRITY PERFORMANCE EVALUATION METHODOLOGY

In the faulted circumstance NSE_V is assumed to consist of a fault-free component due to the usual fault-free errors on all of the satellites ($NSE_{V,ff}$) and a vertical bias value due to the fault on a single satellite (B_V)

$$NSE_V = NSE_{V,ff} + B_V \tag{7}$$

The value of B_V is given in terms of the satellite range error bias B_R by

$$B_V = S_{vert} \times B_R \tag{8}$$

where S_{vert} is the vertical coefficient of the faulted satellite in the position solution.

Probability $|NSE_V| > 15$ m

In the discussion that follows, the overall probability that $|NSE_V|$ exceeds 15 m in the presence of an undetected fault per hr per SV will be designated simply as $P_{|NSEV|>15}$. Since NSE_V is assumed to consist of both a random component due to the usual fault-free errors and a bias component due to the fault B_R, $P_{|NSEV|>15}$ may in principle be computed from

$$\begin{aligned}&P_{\left|NSE_V\right|>15}\\&= \int_{-\infty}^{\infty}\left[\begin{array}{l}P_{fault} \times p_{UDF}(x)\\ \times P_{\left|NSE_V\right|>15givenUDF}(x)\end{array}\right]dx\end{aligned} \tag{9}$$

Where P_{fault} is the prior probability of fault per hr per SV, where $p_{UDF}(x)$ is the conditional probability density function for undetected faults (i.e., post-detection), satisfying the property that

$$\int_{-\infty}^{\infty} p_{UDF}(x)dx = 1.0 \tag{10}$$

And $P_{|NSEV|>15givenUDF}$ is the conditional probability that $|NSE_V|$ exceeds 15 m due to the usual fault-free errors, given the particular undetected fault bias B_R.

Observe from equation (9) that $P_{|NSEV|>15}$ may also be written as

$$\begin{aligned}&P_{\left|NSE_V\right|>15}\\&= \int_{-\infty}^{\infty}\left[\begin{array}{l}P_{fault} \times p_{UDF}(x)\\ \times P_{NSE_V>15givenUDF}(x)\end{array}\right]dx\\&+ \int_{-\infty}^{\infty}\left[\begin{array}{l}P_{fault} \times p_{UDF}(x)\\ \times P_{NSE_V<-15givenUDF}(x)\end{array}\right]dx\end{aligned} \tag{11}$$

Further realizing that $P_{NSEV>15givenUDF}$ is negligible when $B_R < 0$ and $P_{NSEV<-15givenUDF}$ is negligible when $B_R > 0$, gives

$$P_{|NSE_V|>15} = \int_0^{\infty} \begin{bmatrix} P_{fault} \times p_{UDF}(x) \\ \times P_{NSE_V>15 givenUDF}(x) \end{bmatrix} dx + \int_{-\infty}^{0} \begin{bmatrix} P_{fault} \times p_{UDF}(x) \\ \times P_{NSE_V<-15 givenUDF}(x) \end{bmatrix} dx \quad (12)$$

Negating the variable of integration in the second integral and recognizing that

$$P_{NSE_V<-15 givenUDF}(-x) = P_{NSE_V>15 givenUDF}(x) \quad (13)$$

gives

$$P_{|NSE_V|>15} = \int_0^{\infty} \begin{bmatrix} P_{fault} \times [p_{UDF}(x) + p_{UDF}(-x)] \\ \times P_{NSE_V>15 givenUDF}(x) \end{bmatrix} dx \quad (14)$$

Furthermore, recognizing that the sum of the two density terms is just the density of the magnitude of B_R

$$p_{|UDF|}(x) = p_{UDF}(x) + p_{UDF}(-x) \quad (15)$$

gives

$$P_{|NSE_V|>15} = \int_0^{\infty} \begin{bmatrix} P_{fault} \times p_{|UDF|}(x) \\ \times P_{NSE_V>15 given|UDF|}(x) \end{bmatrix} dx \quad (16)$$

In the analyses that follow two different assumptions will be made for $P_{fault} \times p_{|UDF|}$. In the first analysis denoted "specified" GPS IIIC integrity, a bound is assumed on $P_{fault} \times p_{|UDF|}$ for all values of B_R. For that case $P_{|NSEV|>15}$ can be determined merely by evaluating equation (16). In the second analysis denoted "hypothetical" GPS IIIC integrity, in lieu of any specific knowledge of $p_{|UDF|}$, performance is based on determining the worst-case value of $|B_R|$. That worst-case $|B_R|$ produces the largest product of a hypothetical monitor missed detection probability, $P_{md}(|B_R|)$ and $P_{NSEV>15given|UDF|}(|B_R|)$. The distinction between these two integrity methodologies will become clearer as they are discussed below. However, first it is convenient to present some other aspects of the analysis that are common to both methods.

Probability NSE_V > 15 m given $|B_R|$

The presence of a fault bias $|B_R|$ on a single satellite is assumed to merely bias the fault-free vertical error distribution by an amount equal to $|S_{vert}| \times |B_R|$. Therefore, the probability that NSE_V exceeds 15 m given the fault is undetected may be expressed as

$$P_{NSE_V>15 given|UDF|}(|B_R|) = \int_{15}^{\infty} dnorm\left(x, |S_{vert}| \times |B_R|, \sigma_{NSE_V,ff}\right) dx \quad (17)$$

where

$$dnorm(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (18)$$

Recall from the performance requirements for LPV 200 that the $(1-10^{-7})^{th}$ percentile fault-free error cannot exceed 10 m (refer back to equation (2)). Therefore, for the analysis it will be assumed that $\sigma_{NSEV,ff}$ is given by

$$\sigma_{NSE_V,ff} = \frac{10}{5.33} = 1.88 \text{ m} \quad (19)$$

Note that this assumption is not inconsistent with the constraint VPL ≤ VAL for VAL > 10 m (refer to equations (20) and (21) below for the expressions for VPL). In particular it is anticipated that URA may be somewhat larger than the standard deviation of a Normal distribution characterizing the fault-free SIS ranging error. This is likely true because 5.73×URA must also bound the error in the case of a fault to the same probability $(1-10^{-8})$ as for 5.73 standard deviations of a Normal distribution. Consequently, it is readily possible to have VPL restricted by a VAL > 10 m, but still have $5.33 \times \sigma_{NSEV,ff}$ effectively restricted by a value of 10 m.

Determination of $|S_{vert}|$ for Analysis

The largest magnitude of S_{vert} for any satellite in the position solution depends on what restrictions are placed on the satellite geometry by other limits. For the purpose of this analysis it will be assumed that the satellite geometry is limited by the requirement that the vertical protection level (VPL) not exceed the vertical alert limit (VAL). The VPL for LPV 200 using GPS IIIC is assumed to be of the following form

$$VPL = 5.33 \times \sqrt{\sum_{i=1}^{N} S_{vert,i}^2 \times D_i^2} \le VAL \qquad (20)$$

where D_i characterizes both the GPS III satellite SIS errors and other range errors due to the troposphere and user equipment

$$D_i = \sqrt{URA_i^2 + \sigma_{tropo,i}^2 + \sigma_{user,i}^2} \qquad (21)$$

The analysis assumes that URA takes on what is believed to be a conservatively large value of 0.7 m.
It is further assumed that $\sigma_{tropo,i}$ is given by

$$\sigma_{tropo,i} = 0.12\,\mathrm{m} \times \left(\frac{1.001}{\sqrt{0.002001 + \sin^2(\theta_i)}} \right) \qquad (22)$$

where θ_i is the elevation angle of the i^{th} satellite. The aircraft is assumed to have dual-frequency L1/L5 avionics. Therefore, the expression for $\sigma_{user,i}$ is given as in [8]

$$\sigma_{user,i} = 2.59 \sqrt{RMS_{pr\_air}^2 + \sigma_{multipath}^2} \qquad (23)$$

$$RMS_{pr\_air} = 0.11 + 0.13 e^{-\theta_i/4} \qquad (24)$$

$$\sigma_{multipath} = 0.13 + 0.53 e^{-\theta_i/10} \qquad (25)$$

The largest $|S_{vert,i}|$ allowable by equation (20) will be determined by observing that

$$\max_i \left(|S_{vert,i}| \times D_i \right) \le \sqrt{\sum_{i=1}^{N} S_{vert,i}^2 \times D_i^2} \qquad (26)$$

$$\max_i \left(|S_{vert,i}| \right) \le \frac{\sqrt{\sum_{i=1}^{N} S_{vert,i}^2 D_i^2}}{D_{\min}} \le \frac{VAL}{5.33 \times D_{\min}} \qquad (27)$$

$$D_{\min} = \sqrt{URA^2 + \sigma_{tropo,90\deg}^2 + \sigma_{user,90\deg}^2} = \sqrt{.7^2 + .12^2 + .44^2} = 0.84\,\mathrm{m} \qquad (28)$$

$$\max_i \left(|S_{vert,i}| \right) = \frac{VAL}{5.33 \times 0.84} = 0.224 \times VAL \qquad (29)$$

PERFORMANCE ANALYSIS – "SPECIFIED" GPS IIIC INTEGRITY

As mentioned above and discussed in [2], the ranging error integrity for GPS IIIC is essentially "specified" as

$$\mathrm{Prob}\{|IURE| > 4.42 \times URA\} \le 10^{-5}\,\mathrm{/hr/SV;} \quad \text{integrity status flag "off" (or "on")} \qquad (30)$$

$$\mathrm{Prob}\{|IURE| > 5.73 \times URA\} \le 10^{-8}\,\mathrm{/hr/SV;} \quad \text{integrity status flag "on"} \qquad (31)$$

Thus, when the integrity status flag is "on" the integrity is guaranteed at two values of ranging error, i.e., 4.42×URA and 5.73×URA.

Probability of Undetected Fault

For the purpose of the LPV 200 performance analysis the above GPS IIIC integrity guarantees will be related to the "two-sided tail probability" of $P_{fault} \times p_{UDF}$ in the following manner

$$\int_L^{\infty} P_{fault} \times p_{|UDF|}(x)\,dx = \begin{cases} 1.0\ \mathrm{/hr/SV};\ 0 \le L < 4.42 \times URA \\ 10^{-5}\ \mathrm{/hr/SV};\ 4.42 \times \mathrm{URA} \le \mathrm{L} < 5.73 \times \mathrm{URA} \\ 10^{-8}\ \mathrm{/hr/SV};\ \mathrm{L} \ge 5.73 \times \mathrm{URA} \end{cases} \qquad (32)$$

where "two-sided tail" probability refers to the probability that the magnitude of the random variable exceeds the given value L. This strict interpretation of the specified performance assumes the "tail probability" does not decrease any from the previous value until the next specified point is reached. This interpretation is particularly onerous for assuming that the "tail probability" does not decrease below 1.0 until $|B_R|$ = 4.42×URA. A graph of this worst possible integrity characteristic compliant with the two specified points is shown in Figure 1 assuming URA = 0.7 m.

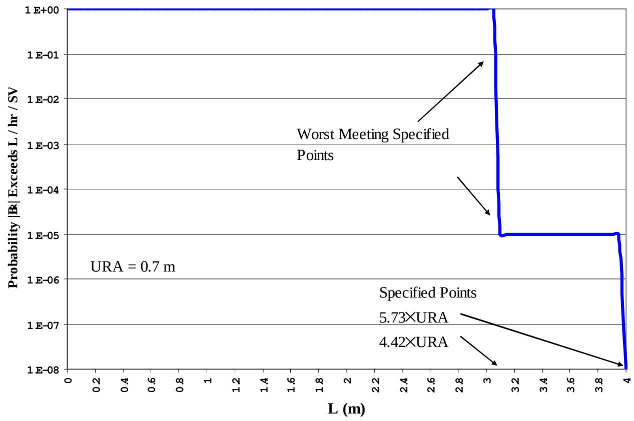

Figure 1. Worst GPS IIIC Integrity Characteristic Compliant with Two Specified Points

Recall that to evaluate $P_{|NSEV|>15}$ using the integral in equation (16) it is necessary to characterize the density of $|B_R|$ rather than the "tail probability" of $|B_R|$. The probability density of $|B_R|$ corresponding to the "tail probabilities" in equation (32) is

$$
\begin{aligned}
&P_{fault} \times p_{|UDF|}(x) \\
&= (1-10^{-5}) \times u_0(x - 4.42 \times URA) \\
&+ \left(10^{-5} - 10^{-8}\right) \times u_0(x - 5.73 \times URA)
\end{aligned}
\qquad (33)
$$

where $u_0(x)$ is the well-known unit impulse function defined by

$$
u_0(x) = \begin{cases} \infty \text{ for x} = 0 \\ 0 \text{ for x} \neq 0 \end{cases} \text{ and } \int_{-\infty}^{\infty} u_0(x) dx = 1 \qquad (34)
$$

Results for Discrete Point Gaussian Bound on $P_{fault} \times p_{|UDF|}$

At this point in the discussion it is informative to show the relationship between $P_{fault} \times p_{|UDF|}$ and $P_{NSEV>15given|UDF|}$. Figure 2 gives a plot of these two functions and also their product.

Recall that the $P_{NSEV>15given|UDF|}$ depends on the magnitude of S_{vert} that converts the range bias to a vertical error bias. The curve for $P_{NSEV>15given|UDF|}$ in Figure 2 is computed assuming VAL = 35 m, with a corresponding $|S_{vert}|$ = 7.84 (refer back to equation (29)). For best availability it is desirable to have VAL as large as possible. Furthermore, a maximum VAL of 35 m has been suggested as providing

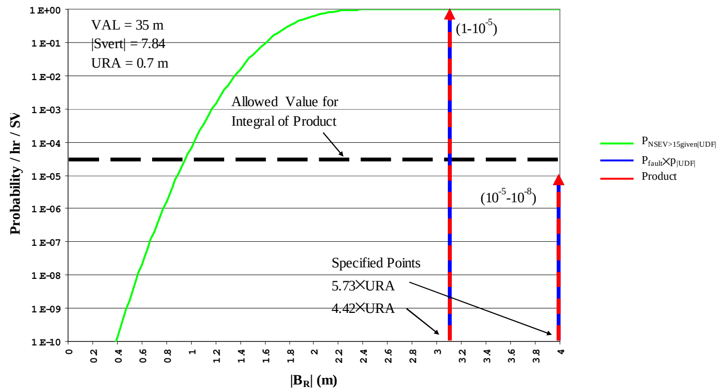

Figure 2. Probability Functions for $P_{fault} \times p_{|UDF|}$ Gaussian at 4.42 and 5.73×URA and $P_{NSE>15given|UDF|}$ for VAL = 35 m

adequate integrity for LPV 200 approaches using the Wide Area Augmentation System (WAAS) [9]. Note that the curve for $P_{NSEV>15given|UDF|}$ in Figure 2 rises from small probability values to exceed the required probability of 2.4×10^{-5} for values of $|B_R|$ greater than about 0.9 m and approaches 1.0 for values of $|B_R|$ greater than about 2.3 m.

Note that the $P_{fault} \times p_{|UDF|}$ function consists of two impulses (as in equation (33)), one at $|B_R|$ = 4.42×URA = 3.1 m and the other at $|B_R|$ = 5.73×URA = 4.0 m. The "area" of each impulse is also shown in the figure. The product function is also a pair of impulses at exactly the same values of $|B_R|$. The area of each product impulse is the product of the original area and the corresponding value of $P_{NSEV>15given|UDF|}$. In this particular example, the value of $P_{NSEV>15given|UDF|}$ is always 1.0 at the impulse location, so the area of each product impulse is the same as the original value. Recall from equation (16) that the value of $P_{|NSEV|>15m}$ is the total integral of the product function. For this example the resulting probability is just the total area of the product impulses, i.e., $1 - 10^{-5} + 10^{-5} - 10^{-8} = 1 - 10^{-8}$. Obviously, this result is much larger than the allowed value of 2.4×10^{-5}.

The astute reader may predict from this example that the requirement would be met without changing the $P_{fault} \times p_{|UDF|}$ function if the $P_{NSEV>15given|UDF|}$ curve could be moved to the right so that it would cross 2.4×10^{-5} at about $|B_R|$ = 3.1 m. Then the area of the product impulse at 4.42×URA would be approximately 2.4×10^{-5} and the area of the product impulse at 5.73×URA would likely be negligible. That situation is illustrated in Figure 3. The necessary value of $|S_{vert}|$ is found to be 2.37 with corresponding VAL of 10.6 m. In summary, the LPV 200 integrity requirement could be met with the worst possible GPS IIIC integrity characteristic compliant with the two currently "specified" points, if VAL were no larger than about 10.6 m. However, requiring such a small VAL may be undesirable due to the accompanying decrease in availability of adequate satellite geometry.

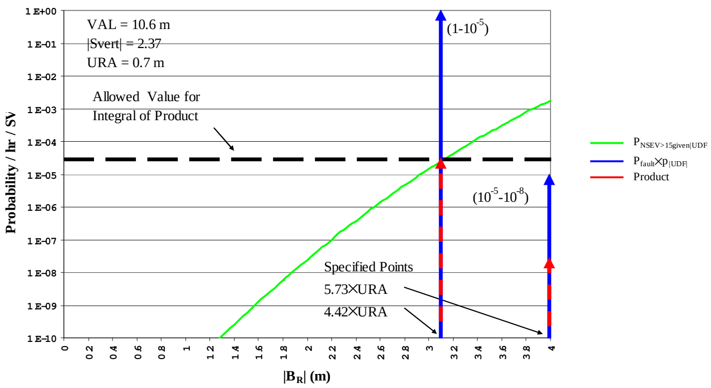

Figure 3. Probability Functions for $P_{fault}\times p_{|UDF|}$ Gaussian at 4.42 and 5.73×URA and $P_{NSEV>15given|UDF|}$ for VAL = 10.6 m

It is very unlikely that the GPS IIIC integrity characteristic would be no better than the "worst compliant" function just illustrated, particularly for values of $|B_R|$ just slightly smaller than the 4.42×URA level. However, it should be clear from the above example, that more knowledge of the GPS IIIC integrity characteristics than just the two points currently specified is needed in order to judge feasibility of meeting the integrity requirement for LPV 200 applications. This is particularly the case for values of $|B_R|$ smaller than 4.42×URA.

Many readers will recognize that the probability values of 10^{-5} and 10^{-8} associated with 4.42×URA and 5.73×URA, respectively, are simply the values of the "two-sided tail" probability associated with a Normal distribution with zero mean and standard deviation equal to URA

$$P_{Gaussian\_two\_tail}[L] = 2\times\int_L^{\infty} dnorm[x,0,1]dx \tag{35}$$

Information currently available in the public domain does not confirm that URA is intended to be the standard deviation of a Normal distribution bounding the IURE. However, previously developed GPS augmentation systems such as WAAS [10] and LAAS (Local Area Augmentation System) [11] use integrity based on Gaussian bounds. The validity of those bounds hinges critically on the assumption that Gaussian statistical properties allow the combination of different error sources in the range domain for each single satellite and further combination of the resulting ranging error from multiple satellites to obtain the resulting error bound in the position domain. Furthermore recently published work by other researchers has dealt extensively with the issue of formulating and verifying Gaussian error bounds for integrity assured GPS IIIC [12]. The work in [12] emphasizes gathering data from operational satellites to verify that the Gaussian tail probability is met at several values smaller than 4.42×URA. Concepts for dealing with non-zero means of the satellite ranging error distributions are also discussed in [12]. However, in this paper, the mean of the range error distribution is assumed to be zero for simplicity of the discussion and for placing the emphasis on compliance of performance with the particular integrity requirement of the LPV 200 application.

As motivated by the above discussion the remainder of the performance analysis for "specified" GPS IIIC integrity in this paper will focus on further examples of a single Gaussian bounding curve varying from discrete points (in addition to the 4.42×σ and 5.73×σ levels) to a continuous curve.

Figure 4 shows the probability functions for assuming the integrity performance characteristic of GPS IIIC is bounded by the Gaussian tail probability at the three additional points suggested in [12], i.e., 1.0, 1.96 and 3.29×URA. Those familiar with the Normal distribution will recognize immediately that 1.96 is the 0.05 "two-sided tail" probability point and 3.29 is the 10^{-3} "two-sided tail" probability point. As was previously the case for just the 4.42 and 5.73 points, the GPS IIIC integrity probability is assumed to decrease only abruptly when each successively larger $|B_R|$ point is reached. Therefore, $P_{fault}\times p_{|UDF|}$ consists of 5 impulses, each at on of the "specified" integrity performance points. The curve for $P_{NSEV>15given|UDF|}$ shown in Figure 4 assumes the largest value of $|S_{vert}|$ that just allows the total area of the product impulses to meet the 2.4×10^{-5} integrity requirement. Thus, specifying the three additional points for GPS IIIC

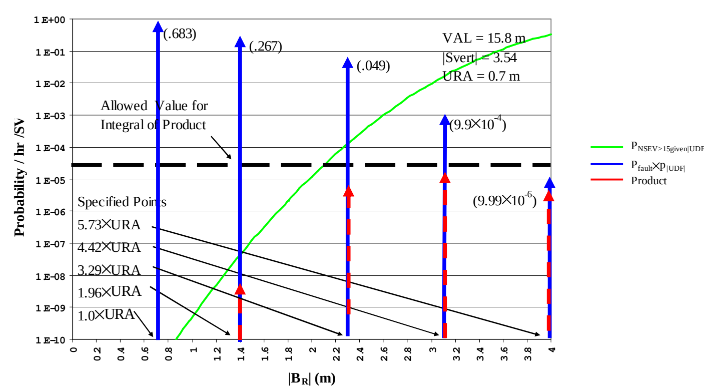

Figure 4. Probability Functions for $P_{fault}\times p_{|UDF|}$ Gaussian at 1.0, 1.96, 3.29, 4.42 and 5.73×URA and $P_{NSEV>15given|UDF|}$ for VAL = 15.8 m

integrity allows $|S_{vert}|$ to be increased from 2.37 to 3.54 with the corresponding VAL increased from 10.6 m to 15.8 m.

It is evident by comparing Figures 3 and 4 that adding these three points reduces the product of peak value of $P_{fault}\times p_{|UDF|}$ and $P_{NSEV>15given|UDF|}$ a considerable amount. It also seems that adding a few more points for intermediate values of $|B_R|$ would provide further benefit. Figure 5 shows the results for adding two more "specified" Gaussian bounding points at 2.58 and 3.89×URA. These additional points would allow $|S_{vert}|$ equal to 3.88 and corresponding VAL of 17.3 m.

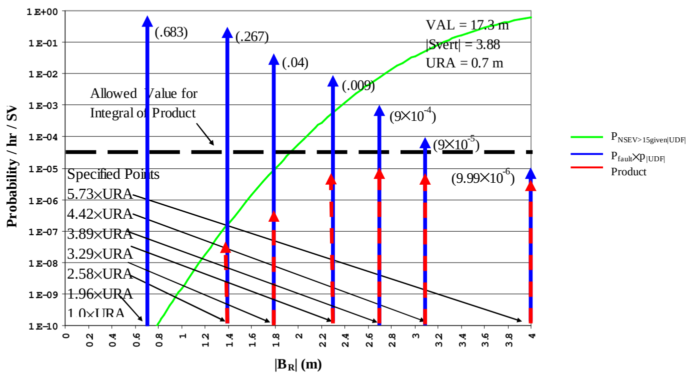

Figure 5. Probability Functions for $P_{fault}\times p_{|UDF|}$ Gaussian at 1.0, 1.96, 2.58, 3.29, 3.89, 4.42 and 5.73×URA and $P_{NSEV>15given|UDF|}$ for VAL = 17.3 m

Results for Continuous Gaussian Bound on $P_{fault}\times p_{|UDF|}$

At this point in the discussion the reader may be wondering whether just assuming a continuous Gaussian bound for $P_{fault}\times p_{|UDF|}$ would allow increasing VAL all of the way up to 35 m. The answer to that question is provided by the plot in Figure 6. For a continuous Gaussian bound the $P_{fault}\times p_{|UDF|}$ function and its product with $P_{NSEV>15given|UDF|}$ are continuous curves, rather than a series of impulses. The resulting value of $P_{|NSEV|>15m}$ is calculated by numerical integration of the product function. As can be seen from Figure 6, with a VAL of 35 m the peak value of the product function is nearly two orders of magnitude larger than the required value of the total integral (2.4×10^{-5}). However, if the VAL were lowered to about 19.2 m ($|S_{vert}|$ = 4.3), the LPV 200 faulted integrity requirement would be met for a continuous Gaussian bound on GPS IIIC integrity based on URA = 0.7 m as shown in Figure 7.

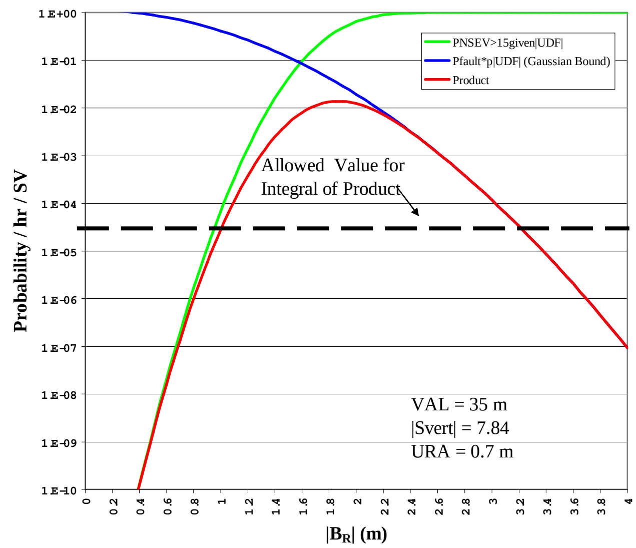

Figure 6. Probability Functions for $P_{fault}\times p_{|UDF|}$ Continuous Gaussian with URA = 0.7 m and $P_{NSEV>15given|UDF|}$ for VAL = 35 m

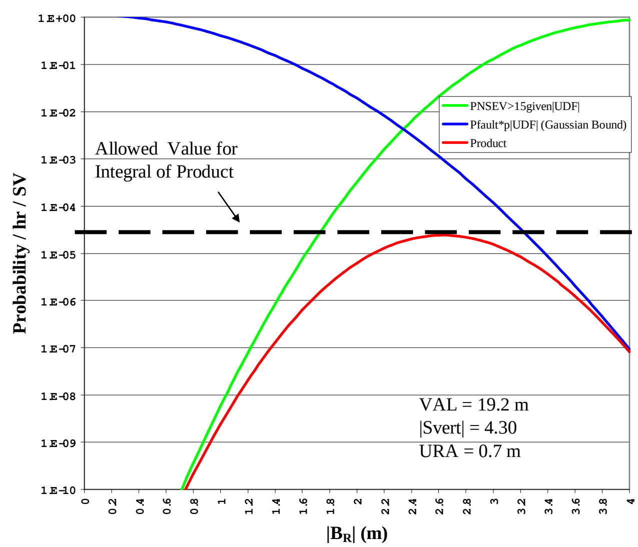

Figure 7. Probability Functions for $P_{fault}\times p_{|UDF|}$ Continuous Gaussian with URA = 0.7 m and $P_{NSEV>15given|UDF|}$ for VAL = 19.2 m

PERFORMANCE ANALYSIS "HYPOTHETICAL" GPS IIIC INTEGRITY

An alternative formulation of the performance analysis will treat GPS IIIC integrity as if specifically related to a fault monitoring process. This alternative concept is likely to be much more appropriate for describing the integrity actually provided by GPS IIIC than the simple Gaussian bounding function just discussed above.

Probability of Undetected Fault

For analysis of this concept it will be assumed that the conditional density of undetected fault magnitudes, $p_{|UDF|}$, is given by

$$p_{|UDF|}(|B_R|) = P_{md}(|B_R|) \times p_{|F|}(|B_R|) \qquad (36)$$

where $P_{md}(|B_R|)$ is the monitor missed detection probability function and $p_{|F|}(|B_R|)$ is the conditional density of range fault magnitudes prior to applying the detection process. The expression for computing $P_{|NSEV|>15}$ then becomes (substituting equation (36) into equation (16))

$$P_{|NSE_V|>15} = \int_0^\infty \begin{bmatrix} P_{fault} \times p_{|F|}(x) \times P_{md}(x) \\ \times P_{NSE_V > 15 given|UDF|}(x) \end{bmatrix} dx \qquad (37)$$

It will be further assumed (as is often the situation) that $p_{|F|}(|B_R|)$ cannot be explicitly characterized as a function of $|B_R|$. However the following observation can be made from equation (37)

$$P_{|NSE_V|>15} \le \max_{|B_R|} \begin{bmatrix} P_{md}(|B_R|) \\ \times P_{NSE_V > 15 given|UDF|}(|B_R|) \end{bmatrix} \times \int_0^\infty P_{fault} \times p_{|F|}(x) dx \qquad (38)$$

Further observing that

$$\int_0^\infty P_{fault} \times p_{|F|}(x) dx = P_{fault} \qquad (39)$$

gives

$$P_{|NSE_V|>15} \le \max_{|B_R|} \begin{bmatrix} P_{md}(|B_R|) \\ \times P_{NSE_V > 15 given|UDF|}(|B_R|) \end{bmatrix} \times P_{fault} \qquad (40)$$

In other words, $P_{|NSEV|>15}$ is no larger than P_{fault} times the largest product of $P_{md}(|B_R|)$ and $P_{NSEV>15given|UDF|}(|B_R|)$ for any value of $|B_R|$.

Hypothetical GPS IIIC P_{md} Function

Since no details of a fault monitoring process for GPS IIIC are yet available in the public domain, a hypothetical monitor behavior will be proposed. In order to form a hypothetical P_{md} function it is first necessary to know the value of P_{fault}. Recall that P_{fault} is the probability of fault per hour per SV. Detailed discussion of fault modes and probabilities for GPS IIIC satellites is not available in the public domain. However, requirements have been publicly stated [13] for prior probabilities of SV fault that need to be met by GPS IIIC satellites for backward compatibility with existing aviation integrity methods or systems, such as Receiver Autonomous Integrity Monitoring (RAIM), WAAS and LAAS. These requirements will be used as the basis for determining P_{fault} for this analysis. The backward compatibility requirements include five single satellite fault modes: 1) excessive acceleration (i.e., steps or ramps in pseudorange), 2) code-carrier divergence, 3) signal deformation, 4) ephemeris errors and 5) low satellite signal power. The first three modes each have a prior probability of 10^{-4} per hour per SV and will all be included in P_{fault}. The probability of ephemeris fault in backward compatibility requirements is stated as 1×10^{-7} per hr for errors larger than 200 m [13]. Ephemeris errors are normally considered in integrity analysis, often with a different P_{md} than typically associated with the other fault modes. However, for simplicity of presentation, ephemeris errors will be ignored in this analysis. The low power fault will also be ignored because it only adds to tracking noise rather than producing an error bias. Consequently, only the first three fault modes will be included giving $P_{fault} = 3\times10^{-4}$ per hour per SV.

Given a value for P_{fault} it is possible to determine one point on the hypothetical GPS IIIC "monitor" characteristic curve. GPS IIIC integrity guarantees that the probability of undetected range fault larger than 5.73×URA does not exceed 10^{-8} (per hour). For the hypothetical monitor concept, the probability of undetected fault larger than 5.73×URA is

$$P_{|UDF|>5.73\times URA} = \int_{5.73\times URA}^{\infty} P_{fault} \times p_{|F|}(x) \times P_{md}(x) dx \qquad (41)$$

Assuming that P_{md} is a monotonically decreasing function of $|B_R|$

$$P_{UDF>5.73\times URA} \le P_{fault}$$
$$\times P_{md}(5.73\times URA)\times \int_{5.73\times URA}^{\infty} p_{|F|}(x)dx \qquad (42)$$

Since even if all of the faults are concentrated in magnitude at 5.73×URA and beyond

$$\int_{5.73\times URA}^{\infty} P_{|F|}(x)dx \le 1.0 \qquad (43)$$

Then it follows that

$$P_{|UDF|>5.73\times URA} \le P_{fault}\times P_{md}(5.73\times URA) \qquad (44)$$

This leads to the choice of one value of $P_{md}(|B_R|)$

$$P_{md}(5.73\times URA) = \frac{P_{|UDF|>5.73\times URA}}{P_{fault}}$$
$$= \frac{10^{-8}\text{ per hr}}{3\times 10^{-4}\text{ per hr}} = 3.33\times 10^{-5} \qquad (45)$$

To determine the remainder of the hypothetical $P_{md}(|B_R|)$ curve several simplifying assumptions will be made. First it will be assumed that the monitor behaves as if it were a single monitor with Gaussian noise characterized by standard deviation σ_{mon}. Second it will be assumed that the monitor threshold T_{mon} is set as low as possible to achieve a false alarm rate on the order of 10^{-7} per decision. This requires $T_{mon} = 5.33\times\sigma_{mon}$. A missed detection probability of 3.33×10^{-5} for a monitor with Gaussian noise corresponds to a fault that is $3.99\times\sigma_{mon}$ above the monitor threshold. Therefore,

$$T_{mon} + 3.99\times\sigma_{mon}$$
$$= 9.32\times\sigma_{mon} = 5.73\times URA \qquad (46)$$

Solving equation (46) for σ_{mon} assuming URA = 0.7 m gives σ_{mon} = 0.43 m and T_{mon} = 2.29 m. The monitor P_{md} function is then given by

$$P_{md}(|B_R|) = \int_{-T_{mon}}^{T_{mon}} dnorm(x,|B_R|,\sigma_{mon})dx$$
$$= \int_{-2.29}^{2.29} dnorm(x,|B_R|,0.43)dx \qquad (47)$$

Results

A plot of the resulting hypothetical GPS IIIC P_{md} function multiplied by P_{fault} is shown in Figure 8. Note that as $|B_R|$ increase from zero, the curve remains equal to $P_{fault} = 3\times 10^{-4}$ until $|B_R|$ nears T_{mon} at about 2.3 m. Then the curve decreases and passes through 10^{-8} at 5.73×URA = 4.0 m, as designed. It should be pointed out that this curve also meets the 10^{-5} GPS IIIC integrity guarantee at 4.42×URA = 3.1 m. The figure also shows $P_{NSEV>15given|UDF|}$ for VAL = 35 m (same curve as in Figure 2). Figure 8 shows the product of P_{md} and $P_{NSEV>15given|UDF|}$. Recall that in the "specified" performance concept presented first in this paper, the LPV 200 faulted integrity requirement is assumed to be met if the total integral under the product curve in Figures 2 thru 7 is less than 2.4×10^{-5}. However, in this second "hypothetical" monitor concept, the requirement is assumed to be met if the maximum value of the product curve does not exceed 2.4×10^{-5}. As is evident from Figure 8, the hypothetical GPS IIIC P_{md} characteristic does not meet the LPV 200 faulted integrity requirement for VAL = 35 m. More insight into why this occurs is shown by two additional points in the Figure. For VAL = 35 m, $|S_{vert}|$ = 7.84 and the monitor threshold in the range domain (2.3 m) corresponds to an effective monitor threshold in the vertical position domain of 7.84×2.3 = 18.0 m. However, the mean of the vertical error distribution is 15 m ($P_{NSEV>15given|UDF|}$ = 0.5) when $|B_R|$ is only 15 / 7.84 = 1.9 m. Therefore, the effective monitor threshold is not small enough to provide sufficient fault mitigation.

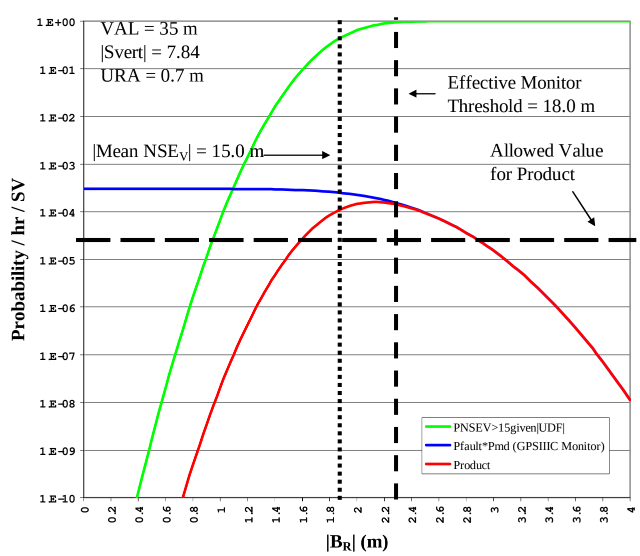

Figure 8. Probability Functions for Hypothetical GPS IIIC Monitor with URA = 0.7 m and $P_{NSEV>15given|UDF|}$ for VAL = 35 m

Figure 9 shows results with the curve for $P_{NSEV>15given|UDF|}$ based on VAL = 24.4 m. In this case $|S_{vert}|$ = 5.47 and the effective monitor threshold in the vertical position domain (12.6 m) is somewhat smaller than 15 m. Furthermore, the value of $|B_R|$ corresponding to a 15 m mean of the vertical error distribution (about 2.75 m) is now somewhat larger than the monitor threshold of 2.3 m. As can be seen from the figure, the peak of the product curve does not exceed 2.4×10^{-5} and thus the LPV 200 faulted integrity requirement could be met by the hypothetical GPS IIIC monitor and a user VAL of 24.4 m.

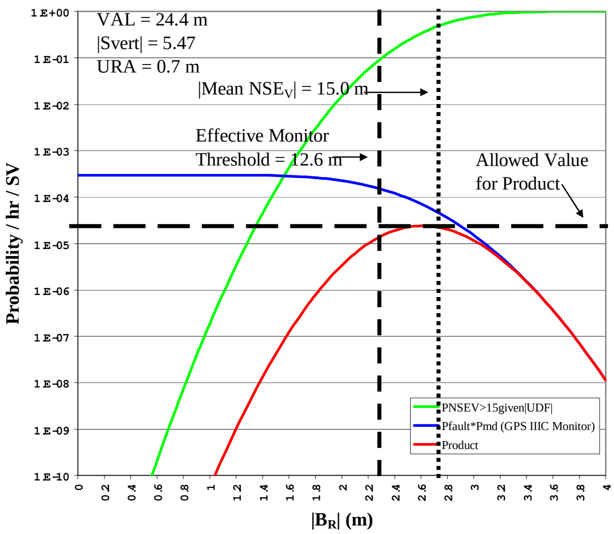

Figure 9. Probability Functions for Hypothetical GPS IIIC Monitor with URA = 0.7 m and $P_{NSEV>15given|UDF|}$ for VAL = 24.4 m

SUMMARY

- GPS IIIC satellites will provide enhanced integrity by broadcasting an integrity status flag in addition to the currently broadcast range accuracy parameter (URA). When the integrity status flag is "on" the probability that the magnitude of the satellite ranging error exceeds 5.73×URA will be assured to be less than 10^{-8} per hour per SV.

- This paper has examined the potential of integrity assured GPS IIIC to meet integrity requirements for LPV 200 applications. Those requirements establish a limit of 10^{-5} per approach on the probability that the magnitude of the vertical error exceeds 15 m in the presence of an undetected satellite ranging fault.

- The aircraft flying an LPV 200 approach applies a vertical alert limit (VAL) to restrict the geometry of satellites in the position solution and thereby to limit the vertical error resulting from an undetected satellite range fault. A reasonable goal is a VAL of 35 m (same as WAAS for LPV 200).

- Two GPS IIIC integrity analysis concepts have been presented: 1) a "specified" concept in which the satellite range error probability is bounded by points (discrete or a continuum) related to a Gaussian probability density function with standard deviation equal to URA and 2) a "hypothetical" monitor concept in which the GPS IIIC integrity monitoring process is modeled to meet a missed detection probability consistent with integrity assured performance at 5.73×URA. Results are presented for each concept assuming the URA is 0.7 m.

- For the "specified" concept, if only two discrete points are defined (4.42 and 5.73×URA) as in the current GPS IIIC specifications, the VAL must be limited to about 10.6 m. If, however, a continuous Gaussian bound could be assumed by users (particularly for range errors less than 4.42×URA in magnitude), the VAL could be increased to about 19.2 m.

- Therefore, it is recommended that: 1) the requirement be revised to describe a continuous Gaussian bound and 2) planned operational tests include data collection at many more points than just the two currently specified, even though the data will not likely include many faults.

- For the "hypothetical" integrity performance based on a monitor with Gaussian noise statistics and threshold set for a reasonable false alarm probability, the VAL could be increased to about 24.4 m.

- Therefore, it is recommended that the same analysis concept used for the "hypothetical" monitor be applied to the actual monitor after the detailed monitoring system design of GPS IIIC becomes more extensively defined.

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