The Spreading and Overlay Codes for the L1C Signal

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BIOGRAPHY

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ABSTRACT

The creation of the new L1C GPS signal presented the opportunity to choose both a family of spreading codes and an associated family of overlay codes. This paper describes the rationale and construction of these families. The families were created from extensive searches with each search requiring its own fine-tuned techniques and search criteria.

The L1C spreading codes comprise 210 pilot/data pairs of length-10230 sequences. The data code modulates the data message bits while the pilot code modulates the overlay code symbols, which represents a fixed repeating pattern. The codes are perfectly balanced and exhibit good auto- and crosscorrelation (both in the "odd" case, when there is a bit transition across the code boundary, and the "even" case when there is no such transition). The length 10230 precluded the immediate adaptation of well-known spreading code families, such as Gold codes. Instead, the relatively new Weil sequence construction was adapted. Weil codes are prime length sequences constructed via shift-and-add from the well-known Legendre sequence and one of its shifts. Weil code correlation sidelobes are bounded by twice the square root of the length, which is no worse than 3 dB from commonly used Gold codes.

The L1C codes were created by using Weil-codes of prime length 10223. Selected Weil-codes were padded with a fixed 7-bit pad to yield the L1C spreading code. The correlation properties of Weil-codes with pad are highly dependent on both the Weil code and the pad insertion point. Thus a search over all Weil codes and insertion points was required. The search criteria were derived from threshold bounds on the sidelobes for both auto- and cross-correlation and for both the even and odd cases. The search criteria need to be adjusted occasionally to yield more candidates codes. The overall search yielded a large set of codes from which the final set of 420 codes could be selected. Because L1C is currently considering two separate modulation schemes (BOC(1,1) and TMBOC), two separate families of codes were constructed that are optimized to the modulation

The 210 L1C overlay codes are length 1800, which corresponds to a frame length of 1800 symbols. The search to construct the overlay codes used two types of criteria. First, full period even auto- and cross-correlation sidelobe bounds were specified. Second, criteria were given for the correlation sidelobes when a small subsequence of the code is correlated against the full code. For this case, lengths of 100 (1 second) and 200 (2 seconds) were used. The overlay codes are based on truncated linear feedback shift register sequences of length 2047, either m-sequences or Gold sequences. The choices for truncation points ensured good auto and cross-correlation sidelobes while also allowing flexibility to bound the short window correlations.

INTRODUCTION

One part of the ongoing Global Positioning System modernization is the creation of a new civil signal on the next generation of satellites (GPS III). This new signal, called L1C, will be transmitted on the L1 carrier frequency. The L1C signal design has been designed with many innovative and effective features that will provide improved performance to all users. These features include separate pilot and data components, new spreading and overlay codes, flexible data messaging with separation of clock and ephemeris, state-of-the-art forward error correction (FEC). The L1C signal has been designed to be compatible with other international GNSS signals. A complete description of the L1C signal is at [1], while an overview of the final design is [2].

We summarize the L1C design features pertinent to the spreading and overlay codes. The L1C signal nominally uses a BOC(1,1) modulation, and thus fits into other GPS signals on L1 as shown in Figure 1. BOC(1,1) is a binary offset carrier modulation with 1.023 MHz spreading code chipping rate and 1.023 MHz square wave subcarrier frequency, sine phased [3]. An alternative modulation scheme, called TMBOC, has been recommended and may be adapted in the future; the effects of the modulation scheme on the spreading code families is dealt with below.

The L1C signal consists of a pilot and a data component. Power is divided unequally between the two components, with 75% of the power in the pilot component and the remaining 25% in the data component. The phase relationship between the two components is not specified ahead of time, and thus is not assumed in this paper. The data symbol period is 10 ms, which is also the spreading code period. Since the chipping rate is 1.023 MHz, the spreading code period is 10230.

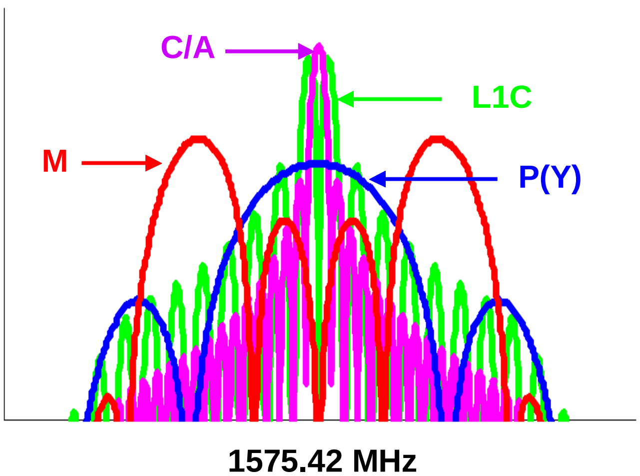

Figure 1. Future Collection of Signals on L1

The families of spreading and overlay codes have their own constraints and requirements, and thus each required their own tailored searches. The bulk of the paper is devoted to spreading codes and their novel use of Weil sequences. The overlay codes are based on conventional linear feedback shift register sequences.

FAMILIES OF SPREADING CODES

In general, a spreading sequence family consists of a set of (often binary) sequences that exhibit good auto- and cross correlation properties. For a periodic sequence **a** of length N with elements a_i for i = 0 to N – 1, the autocorrelation is

$$\mathrm{auto}(\mathbf{a};\tau) = \sum_{i=0}^{N-1} a_i a_{i+\tau},$$

where throughout all subscript arithmetic is modulo N. As such, we only consider periodic correlations. When τ is not congruent to 0 modulo N, the values of the autocorrelation are called *sidelobes*. Similarly, the crosscorrelation is defined for distinct **a** and **b** as

$$\mathrm{cross}(\mathbf{a},\mathbf{b};\tau) = \sum_{i=0}^{N-1} a_i b_{i+\tau}.$$

All values of the crosscorrelation are sidelobes. The *balance* of a sequence is just the sum of its elements.

Finding large families of sequences with good correlation properties is difficult. The Welch bound [4] provides a lower bound for the maximum sidelobe for a family of M sequences of length N:

$$\sqrt{N}\sqrt{\frac{MN-N}{MN-1}}.$$

Table 1 indicates most of the well-known sequence family constructions, along with the respective length, family size, and maximum sidelobe magnitude. Notice that some families essentially meet the Welch bound. The last line of Table 1 has the relatively new Weil sequences, which are described in the next section. The data for Table 1 is derived from [5-8].

Table 1. Good Sequence Families

Name	Length N	Family Size	Max Sidelobe
Gold (odd)	2^n-1, n odd	$N+2$	$1+\sqrt{2}\sqrt{N+1}$
Gold (even)	2^n-1, $n=4k+2$	$N+2$	$1+2\sqrt{N+1}$
Kasami (small)	2^n-1, n even	$\sqrt{N+1}$	$1+\sqrt{N+1}$
Kasami (large)	2^n-1, $n=4k+2$	$(N+2)\sqrt{N+1}$	$1+2\sqrt{N+1}$
Bent	2^n-1, $n=4k$	$\sqrt{N+1}$	$1+\sqrt{N+1}$
No	2^n-1, $n=2k$	$\sqrt{N+1}$	$1+\sqrt{N+1}$
Gong	$(2^n-1)2$	$\sqrt{N}$	$3+2\sqrt{N+1}$
Paterson, Gong	p^2, p prime 3 mod 4	$\sqrt{N}+1$	$3+2\sqrt{N+1}$
Paterson	p^2, p prime 3 mod 4	N	$5+4\sqrt{N+1}$
Z_4-linear, family I	$2(2^n-1)$, n odd	$N/2+1$	$2+\sqrt{N+2}$
Z_4-linear, family II	$2(2^n-1)$, n odd	$(N+2)^2/4$	$2+2\sqrt{N+2}$
Weil	p, prime	$(N-1)/2$	$5+2\sqrt{N}$

An important lesson from Table 1 is the lack of variety for the lengths of the various families. The fact that Weil codes exist for any prime length gives them a flexibility that may suggest other applications besides the L1C spreading code construction.

Table 1 reports maximum sidelobe in the so-called *even* case, when there is no sign transition across a sequence period. *Odd* correlation is defined when such a sign transition occurs; for example, odd crosscorrelation is defined as

$$\mathrm{oddcross}(\mathbf{a},\mathbf{b};\tau) = \sum_{i=0}^{N-\tau-1} a_i b_{i+\tau} - \sum_{i=N-\tau}^{N-1} a_i b_{i+\tau}$$

(odd autocorrelation defined similarly). It is an unfortunate reality that sequence families designed for good even correlation often fail to have good odd correlation (note that the Welch bound still serves as a lower bound in the odd case).

L1C SPREADING CODE SEARCH OVERVIEW

The design for the L1C signal specified several requirements for the length-10230 L1C spreading codes:

- A total of 210 pilot code/ data code pairs are needed.
- BOC(1,1) modulation should be used initially.
- Because of the greater power on the pilot component, greater emphasis should be placed on the correlation properties of the pilot codes.
- The codes should be optimized for both even and odd correlation.
- The sequences should have near-zero balance.
- Within a given pilot code/data code pair, the codes should be near orthogonal at zero lag.
- The new family should improve on other length 10230 GNSS spreading code families, including the GPS L5 codes.

Finally, the codes were desired to be easy to implement. This last criteria meant that memory codes were not considered.

Given these goals, several initial constructions were considered with little success. Examples included looking at concatenations and truncations of Gold codes [9] and attempts at modifying the interleave constructions of Gong and Paterson [6-7]. Eventually, the Weil sequence constructions was re-discovered and adapted to the L1C task.

WEIL SEQUENCES

Weil sequences exist for any prime length p. They are constructed via a "shift-and-add" procedure from the length-p Legendre sequence. Legendre sequences in turn are based on which integer values are squares modulo p. An integer x is a square modulo p if there is some integer y such that $x \equiv y^2 (\mathrm{mod}\ p)$ (so, for example, 2 is a square modulo 7). We will use the term "square" only for values of x that are not divisible by p. Define the Legendre symbol as

$$\left(\frac{x}{p}\right) = \begin{cases} 0 & p \text{ divides } x \\ 1 & x \text{ is a square} (\mathrm{mod}\ p) \\ -1 & x \text{ is not a square} (\mathrm{mod}\ p) \end{cases}$$

Then the Legendre sequence Leg_p is defined as $\mathrm{Leg}_p(0) = -1$ and $\mathrm{Leg}_p(i) = \left(\frac{i}{p}\right)$ otherwise. It is well-known (see [10]) that for k not divisible by p,

$$\sum_{i=0}^{p-1} \left(\frac{i}{p}\right)\left(\frac{i+k}{p}\right) = -1.$$

From this equation it follows that the sidelobes for τ not divisible by p

$$\mathrm{auto}(\mathrm{Leg}_p;\tau) = \begin{cases} +1 \text{ or } -3, & p \equiv 1 (\mathrm{mod}\ 4), \\ -1, & p \equiv 3 (\mathrm{mod}\ 4). \end{cases}$$

In particular, Leg_p has the same 2-valued autocorrelation function as an m-sequence in the second case.

Weil sequences are defined from a Legendre sequence using an index k by

$$\mathrm{Weil}_p^k(i) = \mathrm{Leg}_p(i)\mathrm{Leg}_p(i+k),$$

where k = 0 to $(p-1)/2$. This construction is a "shift-and-add" construction set in a multiplicative form since the elements are ±1. We will omit the value p from the notation if it is understood from context.

The Weil sequence construction was originally proposed for primes congruent to 3 modulo 4 in [11]. We re-discovered the construction, extended it to all primes, and proved the important correlation properties in [8]. Some of the properties of Weil sequences are listed in Table 1. In addition, Weil sequences have balance -1 or +1 and -3, depending on the value of p modulo 4; this fact is equivalent to the autocorrelation of the Legendre sequence.

The proof of the bound of $5+2\sqrt{p}$ in Table 1 for the Weil sequence sidelobes is beyond the scope of this paper and can be found in [8]. It is established by bounding sums of the form (see [10])

$$\sum_{i=0}^{p-1} \left(\frac{i}{p}\right)\left(\frac{i+k}{p}\right)\left(\frac{i+\tau}{p}\right)\left(\frac{i+\ell+\tau}{p}\right).$$

A general bound on such sums was proved by Weil in 1948 [12], hence the naming of these sequences.

Figure 2 shows the maximum sidelobe between Weil^1 and Weil^2 for all primes up to 10000 compared to the theorem bound in Table 1. We see that the bound is tight for many values of p. The prevalence of the primes is also evident.

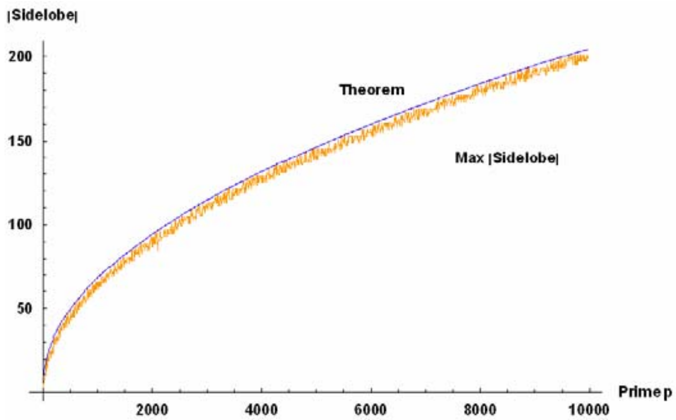

Figure 2. Bound versus Actual Maximum Sidelobe Magnitude

One intriguing aspect of Weil sequences, which is still not completely understood, is the behavior of the odd correlation sidelobes. Consider Figure 3, which shows a plot of the maximum autocorrelation sidelobes for all 5111 Weil sequences when $p = 10223$. The vertical axis is in dB, which is measured here and throughout as

$$10\log_{10}\left(\frac{\text{corr}^2}{\text{length}^2}\right).$$

The bottom curve shows the maximum even autocorrelation sidelobe. This line is flat at about -34 dB, which reflects the bound magnitude bound of $5+2\sqrt{10223}$. The top curve shows the maximum odd autocorrelation. We see a 2 to 4 dB variation at any given point on the curve, and an almost 8 dB variation between the smallest values to the largest. Other experiments (not shown) inidcate that this behavior between the odd and even curves, including the shape of the odd curve, holds in general for all primes. Interestingly, although Weil sequences are 3 dB worse than Gold sequences, experiments have seen comparable performance between Gold and Weil sequences when odd correlation is considered.

The importance of Figure 3 is that Weil sequences with indices in the upper range are more likely to have both good even and odd correlation. Such facts influence the actual searches conducted below.

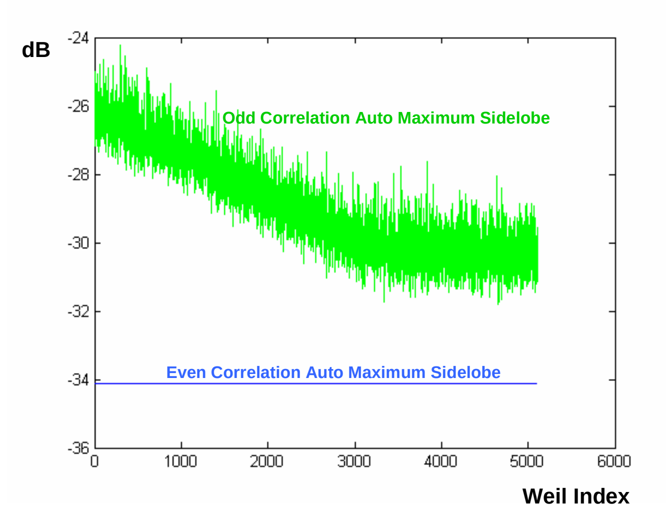

Figure 3. Maximum Even and Odd Autocorrelation Sidelobe (dB)

THE WEIL-BASED L1C CONSTRUCTION

The prevalence of primes allowed some flexibility in initial attempts to adapt Weil codes to L1C task. For example, 10223 and 10243 are the two primes that are nearest to 10230. In the first case, we would need to pad Weil sequences by 7 values to obtain the desired length; in the second case, we would need to truncate Weil sequences by 13 values. Some preliminary experiments determined that padding yielded slightly better performance (measured in correlation sidelobes), and hence that was the approach ultimately taken. In particular, we fix $p = 10223$ for now on.

The construction is shown in Figure 4; the figure is adapted from [1], but uses values that are ±1 consistent with the mathematical formulation in this paper. The figure indicates how Weil sequences are a shift-and-add of the core Legendre sequence, and how the Weil sequence is then augmented with a 7-bit pad to obtain the final length-10230 code (the choice of pad is elaborated on below).

Notice that the resulting sequences are easily implemented in logic. Indeed, even the Legendre sequence could be constructed on the fly using shift-register logic to generate the location of the square values, but in practice the single Legendre sequence would probably be stored and used to derive all subsequent codes.

This construction suggests the search strategy given in Figure 5. There are 5111 Weil sequences of length 10223. There are 10223 positions where a 7 bit pad could be inserted into the sequence, and there are 128 possible 7-bit pads. The family of codes is constructed by progressing through this parameter space, creating a given code, and testing it against the previously found members of the family to determine whether or not it should be added to the family. The test criteria are in terms of auto- and crosscorrelation for both the even and odd cases.

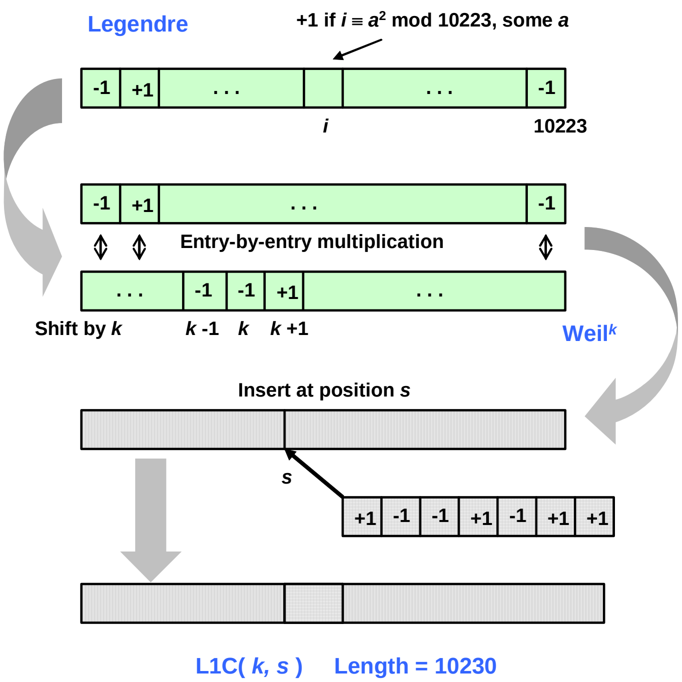

Figure 4. Construction of Length-10230 Weil-Based Spreading Codes

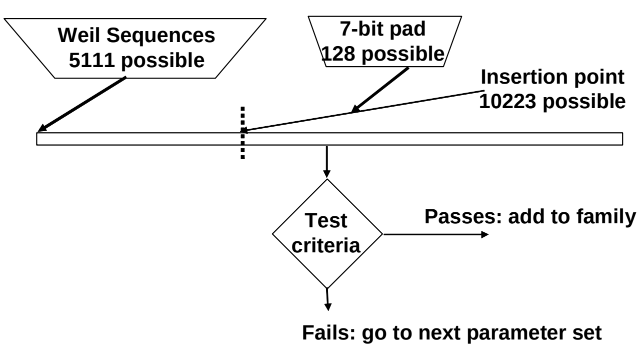

Figure 5. Weil-Based Search Strategy

Motivated by Figure 3, some experiments were run to establish the dependency on the pad and insertion point in this construction. The results are shown in Figures 6 and 7. In each figure, we show the maximum even and odd autocorrelation sidelobes in dB (two separate plots) for each insertion point. The two curves in each plot indicate the minimum (green) and maximum (blue) of this maximum sidelobe over all 128 possible 7-bit pads. Figure 6 yields results for the length-10223 Weil Index 1 sequence, while Figure 7 is for Weil index 5111.

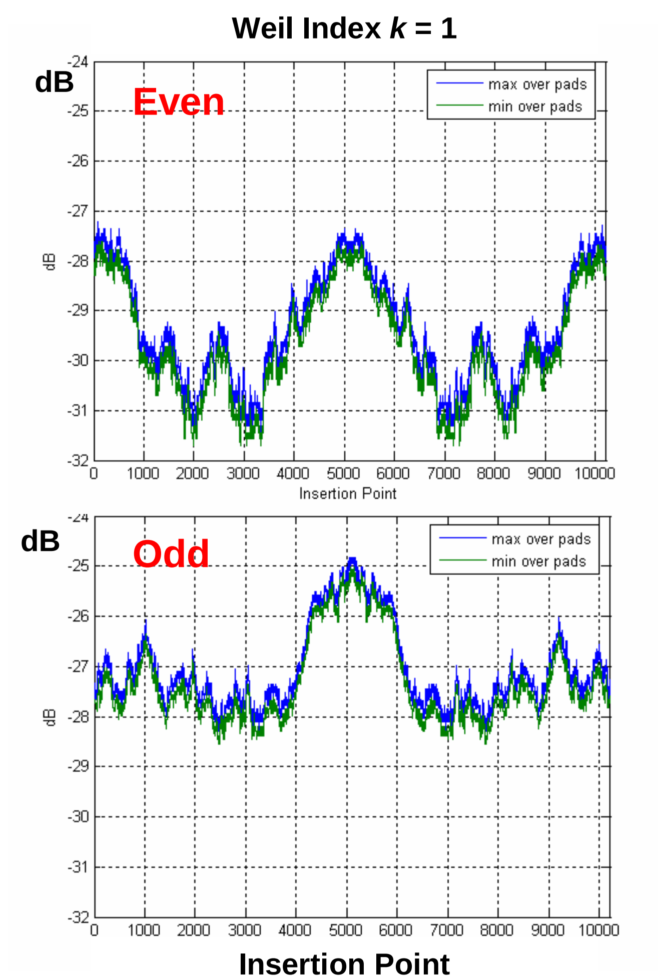

Figure 6. Variations Due to Pad and Insertion Point for Weil Index 1

Several observations are immediate from the figures. First, since we are looking at autocorrelation properties of the Weil sequence plus a pad, we lose the uniform performance evident in the even case in Figure 3. For Weil Index 1 in Figure 6, we see how bad the odd correlation compared to the even correlation; again consistent with Figure 3. Similarly, Figure 7 shows how good the odd correlation is for Weil Index 5111. Indeed, it is better than the even correlation result for most insertion points. Finally, both Figures 6 and 7 indicate that there is little to gain by varying the 7-bit pad. As such, the final strategy fixed a pad to be the values +1 -1 -1 +1 -1 +1 +1 as indicated in Figure 4. One benefit to using this pad is that all resulting Weil-based codes will have balance equal to 0.

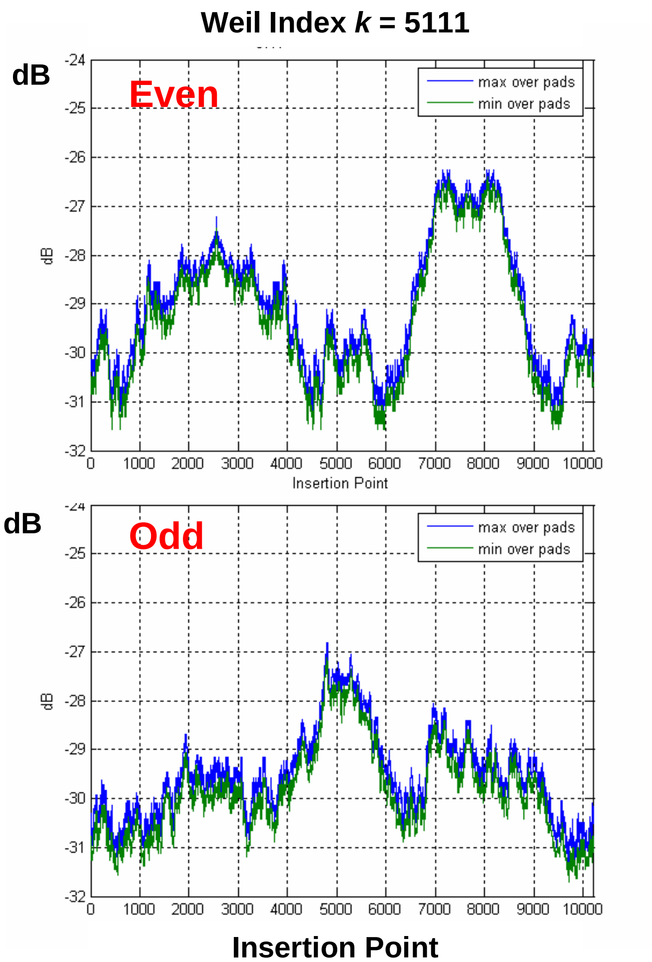

Figure 7. Variations Due to Pad and Insertion Point for Weil Index 5111

SEARCH RESULTS

An exhaustive search was conducted over all Weil indices *k* and insertion point values *s*. The search began with k = 5111 and progressed downward. The search first used several days to gather all potential codes that satisfied an autocorrelation threshold of -31 dB for even correlation and -28 dB for odd correlation. The rationale was that it was relatively quick to gather this set, and that autocorrelation is an easier criterion to meet and thus would not unduly degrade the larger follow on search. The actual threshold values were based on preliminary test searches.

Once that set of potential codes had been gathered, the longer exhaustive search was conducted. The resulting set of codes divides naturally into three subsets based on criteria for crosscorrelation:

1. An even crosscorrelation threshold at -28 dB and an odd crosscorrelation threshold at – 26.5 dB yielded 109 codes.
2. An even threshold of -27.5 dB and an odd threshold of -26.5 yielded an additional 150 codes.
3. A final setting of an even threshold of -27.2 dB and an odd threshold of -26.2 dB yielded 480 codes.

In the second and third cases, the full set of potential codes was re-examined; thus in essence three exhaustive searches were conducted. The end result is a family of 739 Weil-based length-10230 codes that meet the above criteria. Figure 8 captures the full family of codes by graphing the Weil index versus insertion point. Note that the indices toward 5111 are more heavily used, and the values of the insertion point are consistent with the phenomena observed in Figures 6 and 7.

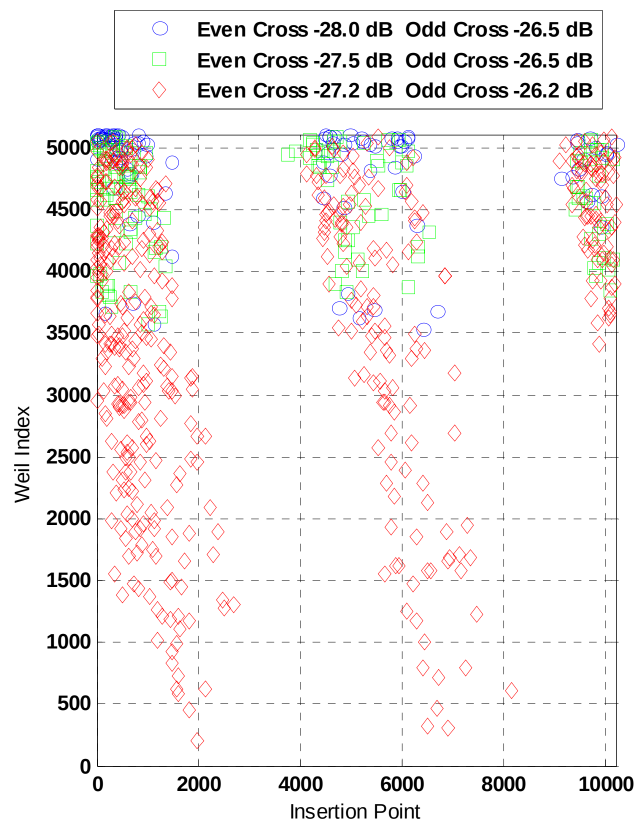

Figure 8. Insertion Points and Weil indices

The final operation required to create the set of L1C spreading codes was to match pilot/data pairs that are near orthogonal at zero lag. Because the sequences have zero balance, the smallest absolute correlation between them is an absolute value of 2. This was obtainable in practice using standard graph matching techniques.

From the set of 739 codes, we created the 210 pilot/data pairs that are found in [1]. The individual codes are specified by the Weil index and insertion point. Because the pilot component will be allocated 75% of the power, the best codes (from the first subset above) were used for the pilot codes when possible. In particular, this is true for the first 63 codes reserved for GPS.

TMBOC

The search and selection of the 739 Weil-based codes was a search on binary sequences, and as such is relevant only when the implicit one sample per chip applies to the modulation scheme, is the case with BOC(1,1) modulation. However, there are currently two different modulation variants that are being considered. In addition to BOC(1,1), the alternative spreading modulation is called multiplexed BOC (MBOC); it has been recommended by the GPS-GALILEO Working Group on Interoperability and Compatibility [13]. MBOC has a spectrum produced by 10/11 of the total signal power in a BOC(1,1) component and 1/11 of the total signal power in a BOC(6,1) component; see [14]. Currently it has not been decided whether BOC(1,1) or MBOC will be used.

The L1C implementation of MBOC is called time-multiplexed BOC (TMBOC). In TMBOC, the data component spreading still uses BOC(1,1), while the pilot component replaces four out of every 33 BOC(1,1) spreading symbols with BOC(6,1) spreading symbols. Note that the fraction of total power devoted to BOC(6,1) symbols is (4/33) × (3/4) = 1/11, since the pilot component has 75% of the power. Figure 9 shows the "4 out if 33" replacement pattern. This pattern was found using search techniques that compared possible patterns versus compatible with the L1C spreading codes.

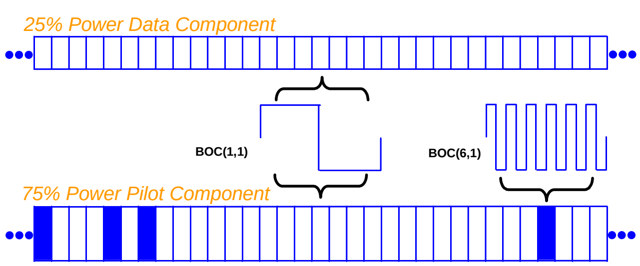

Figure 9. TMBOC Modulation, Including the "4 out of 33" Pattern

A family of spreading codes optimized for one type of modulation scheme will not necessarily be optimized for a different modulation scheme. In particular, the family of codes constructed above for BOC(1,1) performs worse with TMBOC. Fortunately, the set of 739 potential codes resulting from the exhaustive searches permits the creation of a different family of spreading codes optimized for TMBOC. In particular, an initial family of 210 pilot/data pairs of Weil-based codes has been created for TMBOC and is currently being evaluated.

PERFORMANCE SUMMARY

Table 2 summarizes the performance of the two Weil-based spreading code families (BOC(1,1) and TMBOC) along with the other GNSS codes of length 10230. We specify the even auto- and crosscorrelation maximum sidelobe, since this is the prominent metric used in the literature to discuss spreading code families. We also give a measure of the tails of the distribution of the sidelobes for both even and odd combined by giving the 99.9999% cutoff.

The calculations use one sample per spreading symbol for all cases except TMBOC, where 12 samples per spreading symbol are used to capture the structure of the BOC(6,1) spreading symbol. Only 0 Hz Doppler is considered. The low sidelobes for the L1C families are impressive given the large number of codes involved, especially when one considers that only 420 of the possible 739 codes are used in each L1C family. Notice that the tail cutoff points begin to become comparable due to effect of the codes beginning to behave as a collection of random sequences. Even so, the TMBOC family out-performs the other families, because of the cancellation effect of BOC(1,1) and BOC(6,1) symbols being orthogonal.

Table 2. Correlation Sidelobes for Various Length-10230 Spreading Codes

Code Family	Number of Codes	Max. Auto Even Sidelobe (dB)	Max. Cross Even Sidelobe (dB)	99.9999% Auto Even/Odd Sidelobe (dB)	99.9999% Cross Even/Odd Sidelobe (dB)
L1C BOC(1,1)	420	–31.1	–27.3	–28.1	–26.9
L1C TMBOC	420	–31.1	–27.7	–29.4	–28.7
Galileo E5a	200	–28.6	–25.5	–28.6	–26.4
L5 (I5 and Q5)	420	–28.6	–26.0	–26.9	–27.0
L2C CM	37	–27.0	–25.4	–27.0	–25.4

THE OVERLAY CODES

Each pilot component is modulated by an overlay code, which serves several purposes. The overlay code enhances the correlation properties of the pilot spreading code by making the overall period much longer. Narrowband interference suppression is improved through decreasing spectral lines. Finally, the overlay code enables synchronization of the data messaging. For the L1C signal, the overlay code is unique to each pilot code. Each bit of the overlay code is modulated to one period (10 ms) of the spreading code.

The length of the overlay code is 1800 bits, which corresponds to the number of symbols in a data frame for L1C (900 information symbols with half-rate forward error correction). The criteria used to create the family of 210 overlay codes divides into two parts. The first part is the desire for the overlay codes to have good auto- and crosscorrelation as a family of length-1800 spreading codes. This goal was accomplished in the search by setting appropriate thresholds in the same manner as the spreading code search. Notice that only even correlation needs to be considered.

The second part of the criteria relates to the goal of aiding synchronization. In practice, a receiver may wish to synchronize within an overlay code period using a much shorter window. Figure 10 depicts this situation, where a small window of length *L* is correlated periodically against the full overlay code.

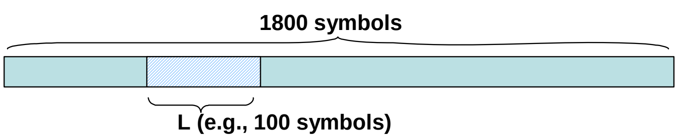

Figure 10. Short Window Synchronization in an Overlay Code

Optimization for every value of *L* is impractical, so two nominal values were used: $L = 100$ and $L = 200$. These values correspond to a 1-second and a 2-second window duration. For each value of *L*, one considers all correlations from each possible length-*L* subsequence taken from the overlay code. The maximum sidelobe of all of these correlations was required to be below a given threshold. For $L = 100$, the threshold was -7 dB; this corresponds to a raw value of about 45, which represents about 4.5 standard deviations if one assumes a normal approximation for the correlations. Similarly, when $L = 200$, the threshold was -10.5 dB, which corresponds to a value of about 60 or 4.2 standard deviations.

Given these two sets of criteria, a search took place to find the best overlay code family. Initially, we considered a Weil-based approach, since 1801 is a prime number, so that there are in fact 900 Weil sequences of length 1801. These sequences can be used to construct a family of codes of length 1800 (by truncation) that meet the first set of correlation goals. However, the resulting codes do not meet the synchronization goals.

Instead, we focused on using linear feed back shift register based sequences of length 2047 to construct the overlay codes. Experimentation found that truncated m-sequences gave comparable crosscorrelation performance to truncated Gold sequences, while having better autocorrelation properties. Thus the first 63 overlay codes, which are reserved for GPS, are truncated m-sequences. The remaining overlay codes are truncated Gold sequences.

The search for the overlay codes was analogous to that of the spreading code listed in Figure 5. First, for each m-sequence, all possible truncation points were checked. If one truncation point was such that the resulting length-1800 sequence passed all of the criteria, it was then added to the family. Testing all of the m-sequences yielded the 63 overlay codes for GPS. Subsequently, each possible Gold sequence was similarly examined to find an appropriate truncation point. Each Gold sequence is derived from a so-called preferred pair, and there are 2047 non-m-sequence Gold sequences for each preferred pair. Thus only a few preferred pairs needed to be examined to complete the desired set of 210 overall codes. The specific parameters are given in [1].

Table 3 shows the auto- and crosscorrelation of the overlay codes. The 2 dB improvement in autocorrelation is due to the better autocorrelation properties of an m-sequence. All of the overlay codes met the above synchronization thresholds. The entire calculation used one sample per code element, and only 0 Hz Doppler was considered.

Table 3. Maximum Sidelobes at 0 Hz Frequency Shift for the Overlay Codes

Overlay Codes	Even Auto-Correlation	Even Cross-Correlation
Index 1 to 63	–24.8 dB	–19.6 dB
Index 1 to 210	–22.7 dB	–19.6 dB

SUMMARY

The new L1C signal presented the opportunity to create better families of spreading and overlay codes. The Weil-based L1C spreading code achieve that goal. They have improved correlation properties, are well-balanced, and easy to construct. The large sets of codes that have been collected permits flexibility with regard to the final modulation scheme. The overlay codes likewise meet the goals enhancing correlation on the pilot component while also permitting synchronization to the data message. Both the spreading and overlay codes are examples of many of the improvements to be found in the new L1C signal.

ACKNOWLEDGMENTS

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