# Coherent Integration With Range Migration Using Keystone Formatting

R. P. Perry, (rpp@mitre.org), R. C. DiPietro, rcd@mitre.org , R. L. Fante,
The MITRE Corp., 202 Burlington Road, Rte. 62,
Bedford, MA 01730-1420, Ph: 781-271-7031 FAX 781-271-7045;

## 1. Introduction

Coherent integration has been used in matched filter radar signal processing to increase target signal to noise ratios. Currently radar designers apply a rule of thumb that a target must remain in the same range resolution cell during the coherent integration time. Thus Doppler processing depends mainly on the change in phase from pulse to pulse. This results in limiting either the range resolution or Doppler resolution of the radar enabling very high range resolution or very high Doppler resolution but not both. We present here an approach to radar matched filtering which does not have these limitations and can produce simultaneous high range and high Doppler resolution matched filter outputs. The new matched filter coherently integrates the radar data even though the target scatterers move through many range resolution cells during the coherent integration time.

The new matched filter uses as its primary component a new transformation which we call Keystone formatting. Keystone formatting provides the ability to match filter all scatterers moving through range cells with linear velocities regardless of their individual velocity magnitudes or directions. Intuitively, there is a direct relationship between a scatterer's Doppler and linear range migration. The Keystone formatting is based on this relationship. The processing also includes the capability to hypothesize range migration acceleration, i.e. quadratic migration, and higher order terms as well as to accommodate under sampled radar PRF's producing folded Doppler spectrums. The processing is in principal transparent to any range migration. Although higher order range migration can be hypothesized, a practical approach would be to limit the hypothesis to acceleration and foldover. The Keystone formatting can be applied prior to these hypothesis to reduce computational redundancies.

## 2. Theoretical Background and the Keystone Transformation

In this section we show how range migration can be compensated, so that coherent integration can be obtained of target scatterers that have migrated through many range cells during a integration interval.

Consider a radar on an aircraft emitting a series of pulses

$$v(\tau) = p(t - kT_1)\exp[- i2\pi \; f_c(t - kT_1)] \quad (1)$$

where $t$ = time, $k$ = pulse number, $T_1$ is the interpulse period, $f_c$ is the carrier frequency and, typically*, $p(t)$ is a chirp pulse, given by

$$p(t) = \exp\left(-i\frac{\pi B t^2}{T_o}\right) rect\left(\frac{t}{T_o}\right) \quad (2)$$

where $T_o$ is the pulse length and $B$ is the bandwidth. Now suppose this pulse train illuminates a moving point scatterer at a range $R(t)$, and the received signals are recorded in a two-dimensional array $s(t', t_k)$ where $t' = t - kT_1$ is known as the "fast time" and $t_k = kT_1$ is known as the "slow time". Then, in terms of these variables the received signal after downconversion, can be written as

$$s(t', t_k) = A\,\mathrm{p}\left[t' - \frac{2R(t_k)}{c}\right]\exp\left[i\frac{4\pi f_c}{c}R(t_k)\right] \quad (3)$$

where $A$ is a constant that depends on the scatterer strength and the scatterer range has been assumed to vary negligibly during the pulse interval $T_o$.

The Fourier transform of Equation (3) over the fast-time variable can be written as

$$S(f, t_k) = A\,P(f)\exp\left[i\frac{4\pi}{c}(f + f_c)R(t_k)\right] \quad (4)$$

* Although $p(t)$ is represented as a chirp pulse, our analysis is valid for arbitrary $p(t)$.

where $P(f)$ is the Fourier transform of $p(t')$. Next, we can generalize Equation (4) from a single point scatterer to an assemblage of N moving point scatterers at ranges $R_n(t_k)$, which can be used as a simplified model. Then Equation (4) becomes

$$S(f, t_k) = P(f)\sum_{n=1}^{N} A_n \exp\left[i\frac{4\pi}{c}(f + f_c)R_n(t_k)\right] . \quad (5)$$

The range $R_n(t_k)$ to scatterer $n$ can be expanded in a Taylor series about $t_k = 0$ as

$$R_n(t_k) = r_n + \dot{r}_n\,t_k + \frac{\ddot{r}_n}{2}t_k^2 + \cdots \quad . \quad (6)$$

where $r_n = R_n(0), \dot{r}_n = \dot{R}_n(0)$, etc. If Equation (6) is used in (5) we obtain

$$S(f, t_k) = P(f)\sum_{n=1}^{N} A_n \exp\left\{i\frac{4\pi}{c}(f + f_c)\left[r_n + \dot{r}_n\,t_k + \frac{\ddot{r}_n}{2}t_k^2\right]\right\} \quad (7)$$

Now recall that the matched filter in range-Doppler space is

$$I(\rho, f_d) = \int_{-\infty}^{\infty} df \int_{-T/2}^{T/2} dt_k S(f, t_k) \exp\left[-i\frac{4\pi f}{c}\rho - i2\pi f_d t_k\right] \quad (8)$$

where $\rho$ is range, $f_d$ is Doppler, $T$ is the coherent integration interval, and we have approximated the summation over pulses by an integral. Then, if (7) is used in (8) it is evident that the term $\exp[i4\pi f\, \dot{r}_n\, t_k / c]$ represents a coupling between range and Doppler due to linear range migration of the scatterers. This range migration is the dominant blurring mechanism that must be compensated in order to obtain a high-resolution matched filter output. This effect can be ignored if $\pi B \Delta R / c << 1$, where $\Delta R$ is the total range migration during the coherent integration interval. We can remove the linear range migration if we rescale the time axis for each frequency by the transformation

$$t_k = \left( \frac{f_c}{f + f_c} \right) \tau \qquad (9)$$

If Equation (9) is used in (7) the rescaled data in the new *(f, τ)* domain, eliminating higher than quadratic terms is:

$$S(f,\tau) = P(f) \sum_{n=1}^{N} A_n \exp\left\{ \begin{aligned} &\left\{ i\frac{4\pi}{c}(f + f_c) r_n \right. \\ &+ i\frac{4\pi}{c}\dot{r}_n\, f_c \tau \\ &+ i\frac{4\pi}{c}(f + f_c) a_n \left( \frac{f_c \tau}{f + f_c} \right)^2 \end{aligned} \right\} \qquad (10)$$

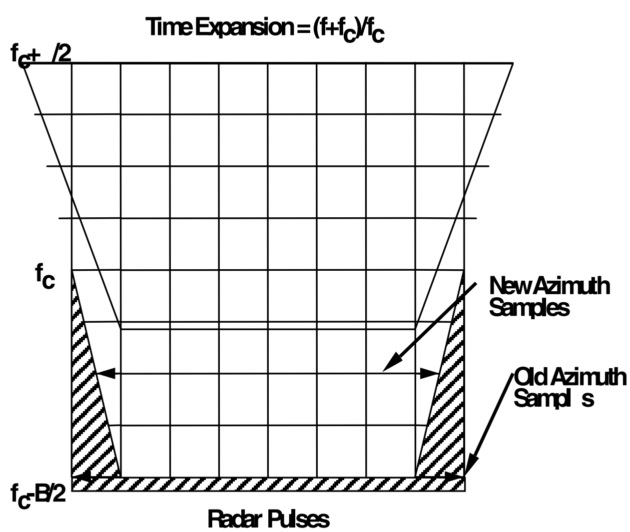


Figure 1. Keystone Transformation of the 2-Dimensional Data

Thus, the coupling effect of the linear range migration has been removed

Figure 1 shows the remapping process. We have called the process “Keystone Formatting” due to its Keystone shape. The target scatterers can now be matched filtered if the acceleration, i.e. $a_n = \ddot{r}_n$ can be estimated, and corrected by hypotheses

## 3. Keystone Formatting of Undersampled Data

Range migration correction using Keystone formatting can still be done even if the data is undersampled. Let us ignore acceleration and then rewrite Equation (10) as

$$S(f,\tau) = \sum_{n=1}^{N} G_n(f) \exp(i2\pi f_{dn}\tau) \qquad (11)$$

where

$$G_n(f) = P(f) A_n \exp\left[ i \frac{4\pi}{c} (f + f_c) r_n \right] \qquad (12)$$

And

$$f_{dn} = \frac{2 f_c}{c} \dot{r}_n \qquad (13)$$

is the true Doppler frequency of scatterer *n*.

Now suppose the data is undersampled and $f_{bn}$ is the folded Doppler frequency of scatterer n. Then Equation (13) can be rewritten as

$$f_{dn} = f_{bn} + F f_1 \qquad (14)$$

where $F$ is the fold factor and $f_1 = 1/T_1$ is the pulse repetition frequency. If Equation (14) is substituted into (11) we obtain

$$S(f,\tau) = e^{i2\pi f_1 F \tau} S_u(f,\tau) \qquad (15)$$

where

$$S_u(f,\tau) = \sum_{n=1}^{N} G_n(f) \exp(i2\pi f_{bn}\tau) \qquad (16)$$

is the undersampled data. Thus, the undersampled data in the $(f, \tau)$ domain can be corrected by multiplying by $\exp(i2\pi f_1 F\tau)$ after Keystone formatting. If the foldover, F, is unknown it must be hypothesized. The foldover correction can also be applied before the Keystone formatting.

## 4. Examples Using Keystone Formatting

### 4.1 Moving Target Matched Filtering

The effectiveness of the Keystone formatting can be shown by synthesizing four simultaneous point targets whose range migration is shown in figure 3. All four targets have different velocities but the same acceleration.

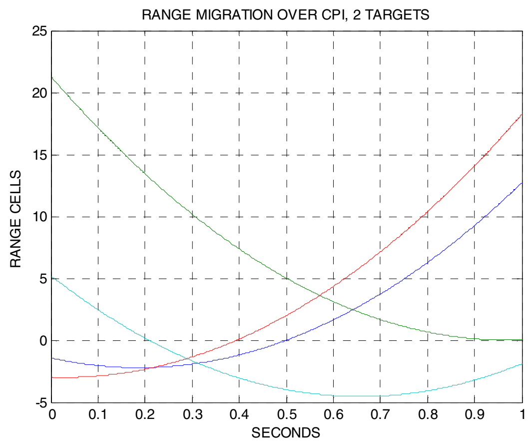


Figure 3

Figure 4 shows the target data matched filtered in range for each pulse with no Keystone formatting or acceleration hypotheses. Figure 5 shows the range matched filter output after Keystone formatting and the correct acceleration hypotheses. The range migration has been eliminated for all four targets simultaneously.

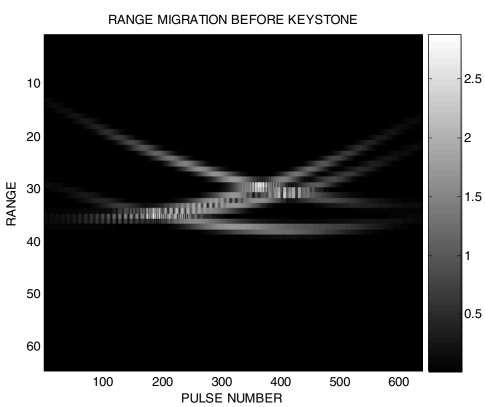


Figure 4- Range Migration With No Keystone Formatting

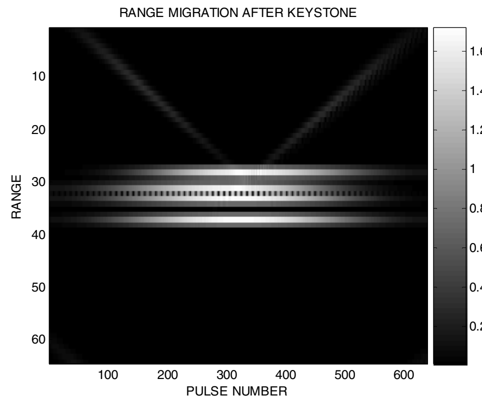


Figure 5 - Range Migration After Keystone Formatting

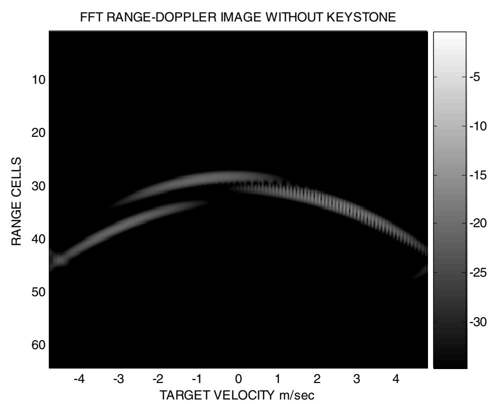


Figure 6

Figure 6 shows the 2D range/Doppler matched filter output with no Keystone Formatting. Figure 7 shows the 2D range/Doppler matched filter output after Keystone formatting and acceleration hypotheses. There is about a 24 dB improvement in coherent integration with Keystone formatting.

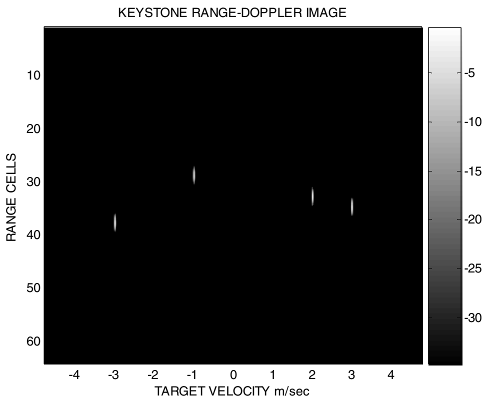


Figure 7

## 4.2 Foldover Example

The target motion shown in figure 3 was increased by 8 m/sec producing a fold factor of one. Figure 8 shows the Keystone matched filter output with a foldover hypothsis of zero.

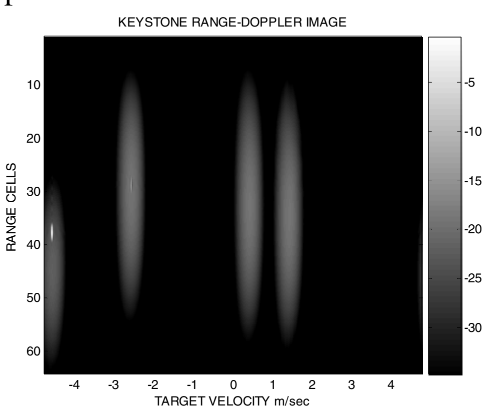


Figure 8

Figure 9 shows the Keystone matched filter with the correct fold factor hypothesis.

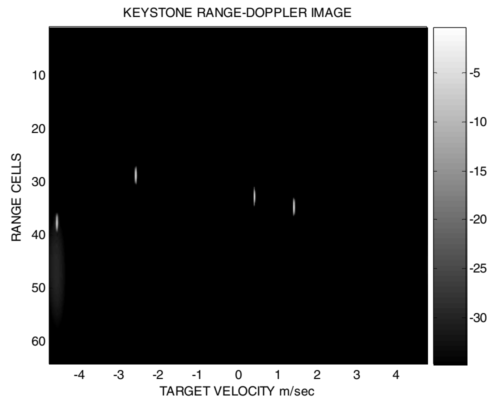


Figure 9

## Summary

We have developed and demonstrated the Keystone format to simultaneously remove linear range migration for all targets regardless of their velocities. Higher order motion and under sampling foldover can be removed by hypotheses.

## References

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