Overview of Space-Time Wireless Communications in Quasi-static Rayleigh Flat Fading Channels

Robert M. Taylor Jr.

***Abstract*— This paper presents a brief overview of space-time algorithms for communication systems operating in quasi-static Rayleigh flat fading environments. The space-time algorithms exploit a multi-input multi-output (MIMO) channel and are designed to meet criteria for optimality under three different channel state information (CSI) conditions: 1) CSI at receiver and transmitter, 2) CSI at receiver but not transmitter, and 3) CSI at neither receiver or transmitter. We analyze the theoretical capacities of the MIMO channels and give the design criteria for space-time codes (STC) under each condition. We briefly describe Grassmannian beamforming (quantized beamforming), space-time block coding (STBC), space-time trellis coding (STTC), and unitary space-time modulation (USTM) for a 2×2 MIMO system operating at a spectral efficiency of 2 bps/Hz. We show through simulation the comparative performance of each of the space-time transmission schemes presented.**

***Index Terms*— space-time coding, performance comparison, space-time block codes, space-time trellis codes, unitary space-time modulation**

I. Introduction

Increasing demand for greater use of the radio frequency spectrum and the complexity of multipath fading channels has prompted the use of multiple antenna communications systems to overcome the deficiencies inherent in single antenna communications. With proper usage of multiple antennas we can achieve higher data rates and better quality of service (QoS). In fact, multiple-input multiple-output (MIMO) antenna systems have been shown theoretically and experimentally to break Shannon's bound established for single-input single-output (SISO) by a factor proportional to the minimum of the number of transmit elements and receive elements.

Given an arbitrary wireless communication system, we consider a link for which the transmitting end as well as the receiving end is equipped with multiple antenna elements as in Fig. 1. In MIMO communications the signals on the transmit and receive antennas are combined in such a way that the quality (bit-error rate or BER) and/or the data rate (bits/sec) of the communication for each MIMO user will be improved. A core idea in MIMO systems is space–time signal processing in which time is complemented with the spatial dimension inherent in the use of multiple spatially distributed antennas. A key feature of MIMO systems is the ability to turn multipath propagation, traditionally a pitfall of wireless transmission, into a benefit for the user. MIMO systems take advantage of random fading and multipath delay spread to create parallel information channels. Multiple antennas at both the transmitter and the receiver create a matrix channel (of size the number of receive antennas times the number of transmit antennas). The advantage lies in the possibility of transmitting over several spatial modes of the matrix channel within the same time-frequency slot at no additional power expenditure [6]. A MIMO system differs from conventional coherent antenna arrays in that the antennas expect to see decorrelated fading.

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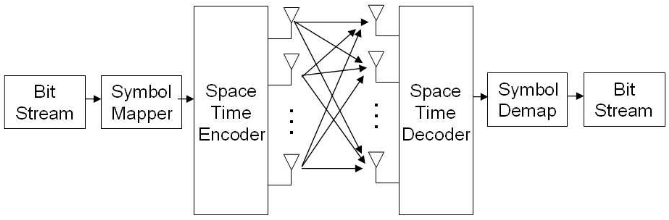


Fig. 1. High level diagram of a multi-input multi-output (MIMO) system employing space–time coding.

This paper gives an overview of the theoretical capacities and the simulated performance of space–time coding (STC) schemes developed to exploit the MIMO channel. Specifically, we look at STC algorithms that are optimal under differing assumptions on the availability of channel state knowledge. If a transmitter or receiver possesses channel state information (CSI), it is said to be *informed* borrowing from the terminology of [2]. We consider the following three cases:

- Case A: CSI at Receiver and Transmitter (Informed Receiver, Informed Transmitter)

CSI case	*Informed Transmitter*	*Informed Receiver*	*Space–Time Coding Algorithm*
A	Yes	Yes	Grassmannian Beamformer
B	No	Yes	Space-time block/trellis code
C	No	No	Unitary space–time modulation

TABLE I

CHANNEL STATE INFORMATION CONDITIONS FOR MIMO CHANNELS AND ASSOCIATED SPACE–TIME CODING SCHEMES

- Case B: CSI at Receiver but not Transmitter (Informed Receiver, Uninformed Transmitter)
- Case C: CSI not at Receiver or Transmitter (Uninformed Receiver, Uninformed Transmitter)

The fourth case, which is impossible, would be an informed transmitter and uninformed receiver. Table I summarizes the three types of channel state information and shows the associated space–time coding algorithms to be discussed.

The remainder of the paper is organized as follows. Section II gives an overview of the expressions for the MIMO system capacity as a function of CSI knowledge. Performance bounds and criteria for optimality in designing STC are given in Section III. Here we also describe transmitter precoding for Case A. We briefly describe space–time block coding (STBC) in Section V and space–time trellis coding (STTC) in Section VI which are designed for Case B. Case C is handled by unitary space–time modulation (USTM) described in Section VII. We compare simulation results of a 2×2 space–time block code to a single-input single-output (SIS0) system in Section VIII.

II. CAPACITY OF THE MIMO CHANNEL

Before we introduce space–time coding in Section III, it is important to understand what the theoretical limits are on the capacity of a MIMO channel that a space–time code would be designed for. The capacity is the maximum mutual information between the transmitted codeword and the received signal maximized over all possible pdfs of the space–time codeword. A space–time codeword for n_t transmit antennas and n_c symbols (sampling rate is symbol rate here) is an $n_t \times n_c$ matrix of complex signals written as:

$$\mathbf{S} = [\mathbf{s}[1], \mathbf{s}[2], \cdots, \mathbf{s}[n_c]] = \begin{bmatrix} s_1[1] & \cdots & s_1[n_c] \\ \vdots & & \vdots \\ s_{n_t}[1] & \cdots & s_{n_t}[n_c] \end{bmatrix} \quad (1)$$

where each row of $\mathbf{S}$ is a *space–time symbol* and the received signal model over that single time epoch is

$$\mathbf{Y} = \sqrt{\frac{E_s}{n_t}} \mathbf{H}\mathbf{S} + \mathbf{W} \quad (2)$$

where $\mathbf{Y} = [\mathbf{y}[1], \cdots, \mathbf{y}[n_c]] \in \mathcal{C}^{n_r \times n_c}$ is the received signal matrix, $\mathbf{H} \in \mathcal{C}^{n_r \times n_t}$ is the channel matrix (or channel state information, CSI), E_s is the average transmit symbol energy per antenna, and $\mathbf{W} = [\mathbf{w}[1], \cdots, \mathbf{w}[n_c]] \in \mathcal{C}^{n_r \times n_c}$ is the zero-mean additive white complex Gaussian noise with covariance matrix $E[\mathbf{w}[n]\mathbf{w}[n]^H] = N_0 \mathbf{I}_{n_r}$. We will define the covariance of the transmitted symbol matrix $\mathbf{S}$ as $\mathbf{R}_{ss} = n_c E[\mathbf{s}[n]\mathbf{s}[n]^H]$ such that $tr[\mathbf{R}_{ss}] = n_t$. The MIMO capacity is then:

$$C = \max_{p(\mathbf{S})} I(\mathbf{S}, \mathbf{Y}) \quad (3)$$

where I denotes mutual information.

A. Case I: Informed Receiver and Transmitter

An informed transmitter is created by feeding back the CSI at the receiver. The capacity of the MIMO channel for the case of perfect CSI knowledge at both transmitter and receiver was shown in [1] to be:

$$C = \max_{tr[\mathbf{R}_{ss}] = n_t} \log_2 \det(\mathbf{I}_{n_r} + \frac{E_s}{n_t N_0} \mathbf{H}\mathbf{R}_{ss}\mathbf{H}^H) \quad (4)$$

where $\mathbf{I}_{n_r}$ is the $n_r \times n_r$ identity matrix representing noise uncorrelated across the receiver antennas. We see from (4) that we must compute $\mathbf{R}_{ss}$ in order to solve for the final capacity expression. To this end we singular value decompose the channel matrix as $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$. Premultiplying both sides of the received signal model (2) by the unitary matrix $\mathbf{U}^H$ gives:

$$\begin{aligned} \mathbf{U}^H\mathbf{Y} &= \sqrt{\tfrac{E_s}{n_t}} \mathbf{\Sigma}\mathbf{V}^H\mathbf{S} + \mathbf{U}^H\mathbf{W} \\ \tilde{\mathbf{Y}} &= \sqrt{\tfrac{E_s}{n_t}} \mathbf{\Sigma}\tilde{\mathbf{S}} + \tilde{\mathbf{W}} \end{aligned} \quad (5)$$

where $\tilde{\mathbf{Y}} = \mathbf{U}^H\mathbf{Y}$, $\tilde{\mathbf{S}} = \mathbf{V}^H\mathbf{S}$, and $\tilde{\mathbf{W}} = \mathbf{U}^H\mathbf{W}$. From (5) we can explicitly decompose the received signal vector into $r = \mathrm{rank}(\mathbf{H})$ parallel SISO channels as:

$$\tilde{y}_i[n] = \sqrt{\frac{E_s}{n_t}} \sqrt{\lambda_i} \tilde{s}_i[n] + \tilde{w}_i[n], \quad i = 1, 2, ..., r \quad (6)$$

where λ_i are the eigenvalues of $\mathbf{H}\mathbf{H}^H$. Defining $\gamma_i = E[|s_i|^2]/n_c \quad \forall i = 1, ..., r$, the capacity in (4) can be rewritten as

$$C = \max_{\sum_{i=1}^{r} \gamma_i = n_t} \sum_{i=1}^{r} \log_2 \left(1 + \frac{E_s \gamma_i \lambda_i}{n_t N_0}\right) \quad . \quad (7)$$

The optimal energy allocation policy γ_i^* that maximizes (7) is found through the waterfilling algorithm [17] and yields

$$\gamma_i^* = \left(\mu - \frac{n_t N_0}{E_s \lambda_i}\right)_+ \tag{8}$$

where μ is a constant and $(x)_+$ denotes the non-negative portion of x. Once the optimal power allocation is determined in (8), we can compute the optimal transmit signal covariance matrix $\mathbf{R}_{ss}^*$ in (4) using

$$\mathbf{R}_{ss}^* = \mathbf{V}\mathrm{diag}\{\gamma_1^*, \gamma_2^*, \ldots, \gamma_r^*\}\mathbf{V}^H \quad . \tag{9}$$

B. Case II: Informed Receiver, Uninformed Transmitter

In this case, the $\mathbf{R}_{ss}$ from (4) is completely non-preferential and can be assigned $\mathbf{R}_{ss} = \mathbf{I}_{n_t}$ yielding the capacity expression

$$C = \log_2 \det\left(\mathbf{I}_{n_r} + \frac{E_s}{n_t N_0}\mathbf{H}\mathbf{H}^H\right) \tag{10}$$

When we describe the channel matrix $\mathbf{H}$ stochastically it assumes a zero-mean complex Gaussian matrix form in the case of quasi-static Rayleigh flat fading. The capacity in (10) is then a function of a Wishart random matrix $\mathbf{H}\mathbf{H}^H$ and is thus a random variable itself. In Fig. 2 (borrowed from [6]) we show cumulative distribution plots for the capacity expression in (10) using 3×3 and 10×10 MIMO channel matrices at $\frac{E_s}{n_t N_o} = 1$. For comparison, two different single-input multi-output (SIMO) channels and two different multi-input single-output (MISO) channels are also shown.

C. Case III: Uninformed Receiver, Uninformed Transmitter

The capacity for the uninformed transmitter/receiver pair in quasi-static Rayleigh fading remains an open research issue. However, Marzetta and Hochwald [3] and Hughes [9] showed that the form of the transmitted codeword must be

$$\mathbf{S} = \mathbf{D}\mathbf{G} \tag{11}$$

where $\mathbf{G}$ is a member of a group code whose members are unitary matrices and $\mathbf{D} \in \mathcal{C}^{n_t \times n_c}$ is a transmission matrix. Marzetta and Hochwald further provided a lower bound on the capacity for block sizes of n_c symbols. Defining $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ..., \lambda_{n_r}]$ where λ_i is the i^{th} eigenvalue of $\mathbf{H}\mathbf{H}^H$ and assuming that $n_c > n_r$ they show:

$$\begin{aligned} &C \geq -n_c n_r \log_2 e - n_r n_t \log_2(1 + \tfrac{E_s n_c}{N_0 n_t}) \\ &- \int p(\boldsymbol{\lambda}) f(\boldsymbol{\lambda}) \left[\log_2 f(\boldsymbol{\lambda}) - (\log_2 e) \sum_{i=1}^{n_r} \lambda_i\right] d\boldsymbol{\lambda} \end{aligned} \tag{12}$$

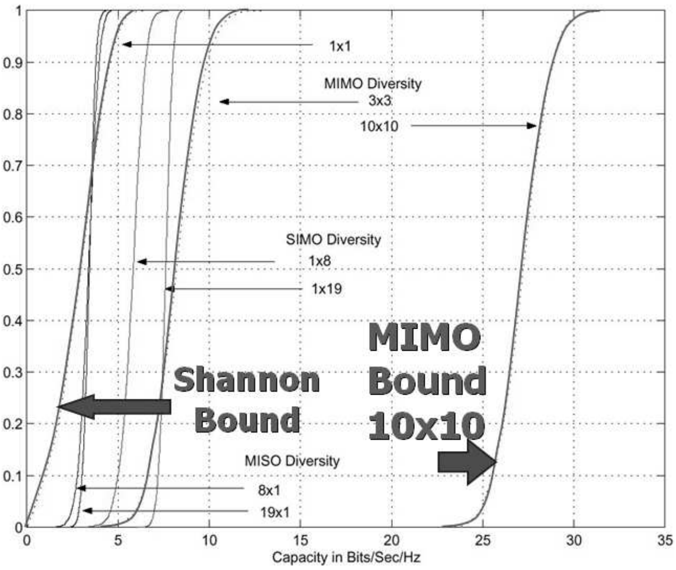


Fig. 2. Cumulative distribution function of capacity for varying channel matrix dimensions. Single-input multi-output (SIM0), multi-input single-output (MISO), and single-input single-output (SIS0) curves are shown for comparison. Figure adapted from [6].

where

$$\begin{aligned} &f(\boldsymbol{\lambda}) = \left(1 + \tfrac{E_s n_c}{N_0 n_t}\right)^{-n_c n_t} \int p(\mathbf{G}) \times \\ &\exp\left\{\sum_{i=1}^{n_r}\sum_{m=1}^{n_t} \lambda_i \left(\tfrac{E_s n_c}{N_0 n_t + E_s n_c}\right) |G_{im}|^2\right\} d\mathbf{G} \end{aligned} \tag{13}$$

and

$$p(\mathbf{Y}) = \frac{\mathrm{etr}(-\mathbf{Y}\mathbf{Y}^H)}{\pi^{n_c n_r}} f(\boldsymbol{\lambda}) \tag{14}$$

III. Design Criteria for Diversity Maximization

Space–time coding (or STC), which is often more accurately described as space–time modulation, refers to the design of “symbol-to-multiantenna” assignments at the transmit side and symbol recovery from multiple antennas at the receiver side. A spatial multiplexer simply does a serial-to-parallel conversion of the input bit stream and sends the parallel bit streams out separate antennas. However, a space–time encoder keeps the information rate constant (or lower in case of non-full-rate) and “assigns” symbols and functions of the symbols to the different transmit elements according to the channel conditions. The receiver then exploits the knowledge of how the transmitted ST codewords were formed to optimally detect the symbols and consequently the bits. The first usage of the phrase *space–time code* in 1998 [13] was really an extension of trellis coded modulation (TCM) to the case of multiple antennas to create what was referred to as space–time trellis codes (STTC). Since then space–time block codes (STBC), space–time turbo trellis codes

(ST turbo TC), and unitary (or differential) space–time modulation (USTM) have been developed among others. Quantized beamforming for diversity maximization was introduced in 2003 [7], [8] as an alternative to space-time coding for systems that employed feedback.

Up until now we have considered only a single codeword $\mathbf{S}$ in our analysis. We now expand upon the notation by defining $\mathbf{S}_k$ to be the codeword at block k, such that a string of M codewords transmitted in succession would be $[\mathbf{S}_1 \quad \mathbf{S}_2 \quad \cdots \quad \mathbf{S}_M]$. Similarly, the observation matrix and channel matrix at block k are $\mathbf{Y}_k$ and $\mathbf{H}_k$ respectively. In keeping with the pattern of analysis for varying conditions of CSI, we first consider the design of MIMO algorithms for an informed transmitter and receiver in Section III-A. In Section III-B we state two criteria for design of space-time coding algorithms that assume an informed receiver only. Finally, Section III-C describes two different design criteria for an uninformed transmitter and receiver.

A. Case I: Informed Receiver and Transmitter

Channel knowledge at the transmitter may be exploited to improve data link reliability since the capacity expression (4) admits a capacity higher than the cases of the uninformed transmitter. An encoded signal $\mathbf{X}$ to be transmitted can be prefiltered by pre-multiplication with a beamformer matrix $\mathbf{B}$ to make the codeword matrix (1) have the form $\mathbf{S} = \mathbf{B}\mathbf{X}$. The covariance matrix $\mathbf{R}_{ss}$ of the transmitted signal then has the form (assuming $E[\mathbf{X}\mathbf{X}^H] = \mathbf{I}_{n_t}$):

$$\mathbf{R}_{ss} = \mathbf{B}\mathbf{B}^H \quad . \tag{15}$$

The beamformer matrix $\mathbf{B}$ must satisfy the power constraint

$$\| \mathbf{B} \|_F^2 = n_t \tag{16}$$

since $tr[\mathbf{R}_{ss}] = n_t$. Recall that the optimum $\mathbf{R}_{ss}^*$ was found in (9) in Section II-A through the waterfilling algorithm. It follows from (15) that the optimal beamforming matrix $\mathbf{B}^*$ is:

$$\mathbf{B}^* = \mathbf{V}\mathbf{D}^{1/2} \tag{17}$$

where $\mathbf{V}\mathbf{D}\mathbf{V}^H$ is the eigendecomposition of $\mathbf{R}_{ss}^*$.

The optimal beamforming matrix is impossible to implement in practice. The constraint of a low-rate feedback channel and a dynamic environment means that only a few bits can be sent back to the transmitter during a coherent time interval. The best we can do is feedback the label of the best beamformer from a predefined codebook. If we employ maximum ratio combining at the receiver, then the design of a quantized maximum ratio transmission codebook involves packing lines in a complex Grassmannian manifold. This is described in more detail in Section IV and the reference therein.

B. Case II: Informed Receiver, Uninformed Transmitter

The Chernoff bound for the pairwise error probability (PEP) was shown in [13] to have the final form:

$$\begin{aligned} P(\mathbf{S}_l \to \mathbf{S}_m) &\leq \left(\frac{1}{\prod_{i=1}^{n_t} \left(1 + \frac{E_s \lambda_i}{4N_0 n_t}\right)} \right)^{n_r} \\ &\leq \left(\prod_{i=1}^{n_t} \lambda_i \right)^{-n_r} \left(\frac{E_s}{4N_0 n_t} \right)^{n_r n_t} \quad \forall l \neq m \end{aligned} \tag{18}$$

where λ_i are the eigenvalues of $\mathbf{H}\mathbf{H}^H$. The PEP bound derived in (18) suggests the following commonly accepted design criteria for space–time codes possessing CSI at the receiver.

- The Rank Criterion: To achieve maximum *diversity gain* the kernel matrix $\mathbf{K}_{l,m} = (\mathbf{S}_l - \mathbf{S}_m)(\mathbf{S}_l - \mathbf{S}_m)^H$ has to have full rank $\forall l, m$.
- The Determinant Criterion: The minimum product $\prod_{i=1}^{n_t} \lambda_i = det(\mathbf{K}_{l,m}) \ \forall l, m$ needs to be maximized to give maximum *coding gain*.

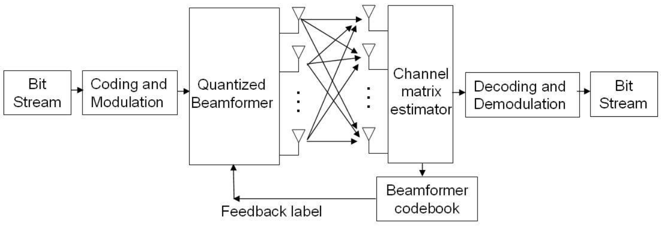
The space–time block codes described in Section V and space–time trellis codes described in Section VI implicitly make use of these design criteria.

C. Case III: Uninformed Receiver, Uninformed Transmitter

Here we consider a class of space–time coding methods called unitary space–time modulation (USTM) that do not require CSI at either end of the system. They can be thought of as an extension of differential phase-shift keying (DPSK) to the case of multiple antennas. A differential receiver operates non-coherently in that it does not require estimates of the channel state to recover the symbols. We show in Section VII that this non-coherent USTM receiver is quadratic and requires no channel estimates. This is an extremely powerful feature of USTM since accurate channel estimates may not be easy to obtain. For example, if we're restricted in how much training data we can use, or if the channel is highly nonstationary, channel estimation is problematic.

The Chernoff bound for the PEP in this case was shown in [9] to be:

$$P(\mathbf{S}_l \to \mathbf{S}_m) \leq \frac{1}{\left| \mathbf{I} + \frac{E_s^2 n_c^2}{4N_0^2 n_t^2 + E_s N_0 n_t n_c} [\mathbf{I} - (\frac{1}{n_c^2}) \mathbf{S}_l \mathbf{S}_m^H \mathbf{S}_l \mathbf{S}_m^H] \right|^{n_r}} \tag{19}$$



Hochwald and Marzetta [10] observed that the maximum diversity is obtained when the value “1” is not a singular value of $\frac{1}{n_c}\mathbf{S}_l\mathbf{S}_l^H$. The coding advantage is given by the *angular distance* [9]:

$$\begin{aligned}\Lambda(\mathbf{S}_l,\mathbf{S}_m) &= \left|n_c\mathbf{I}-(1/n_c)\mathbf{S}_m\mathbf{S}_l^H\mathbf{S}_l\mathbf{S}_m^H\right|^{1/n_t} \\ &= (1/n_c)\left|\begin{bmatrix}\mathbf{S}_l\\ \mathbf{S}_m\end{bmatrix}[\mathbf{S}_l^H \quad \mathbf{S}_m^H]\right|^{1/n_t}\end{aligned} \tag{20}$$

This leads to the following design criteria:

- Independence Criteria: The rows of $\mathbf{S}_l$ and $\mathbf{S}_m$ must be linearly independent $\forall\ l,m$.
- The Determinant Criterion: For

$$\mathbf{K}_{l,m}=\begin{bmatrix}\mathbf{S}_l\\ \mathbf{S}_m\end{bmatrix}[\mathbf{S}_l^H \quad \mathbf{S}_m^H] \tag{21}$$

  $det(\mathbf{K})\ \ \forall l,m$ needs to be maximized to give maximum *coding gain*. The USTM algorithm described in Section VII implicitly makes use of this design criterion.

IV. MIMO Grassmannian Beamforming

This section briefly describes Grassmannian beamforming for MIMO systems as taken from Love et al.[7]. The focus is on transmission of a single bit stream out multiple transmitters employing quantized maximum ratio transmission (QMRT) beamforming. Maximum ratio combining (MRC) is assumed at the receiver. Fig. IV shows the high-level components of a feedback communication system utilizing quantized beamforming at the transmitter.

Rewriting the signal model from (2) for a single time instance gives the vector form:

$$\mathbf{y}[n]=\sqrt{\frac{E_s}{n_t}}\mathbf{H}\mathbf{s}[n]+\mathbf{w}[n]\ \ . \tag{22}$$

We define the vectors $\mathbf{b}$ and $\mathbf{z}$ to be the beamforming and combining vectors respectively. Letting $\mathbf{s}[n]=\mathbf{b}x[n]$, where $x[n]$ is a single channel bandlimited symbol sequence, we can write (22) after MRC as

$$\mathbf{z}^H\mathbf{y}[n]=\sqrt{\frac{E_s}{n_t}}\mathbf{z}^H\mathbf{H}\mathbf{b}x[n]+\mathbf{z^H}\mathbf{w}[n]\ \ . \tag{23}$$

.8393-j.2939	-.1677+j.4256
-.3427*j.9161	.0498+j.2019
-.2065+j.3371	.9166+j.0600
.3478-j.3351	.2584+j.8366
.1049+j.6820	.6537+j.3106
.0347-j.2716	.0935-j.9572
-.7457+j.1181	-.4533-j.4719
-.7983+j.3232	.5000+j.0906

TABLE II
CODEBOOK (TAKEN FROM [7]) FOR QUANTIZED BEAMFORMING FOR $n_t=2$ TRANSMITTERS AND $n_b=3$ FEEDBACK BITS

The product $\mathbf{z}^H\mathbf{H}\mathbf{b}$ is the quantity of interest. For the beamforming/combining system under consideration we will design $\mathbf{b}$ and $\mathbf{z}$ to maximize SNR–which will in turn minimize average probability of error. The MRC vector can be shown to be

$$\mathbf{z}=\frac{\mathbf{H}\mathbf{b}}{\parallel \mathbf{H}\mathbf{b}\parallel_2} \tag{24}$$

and, thus, we are left with the design of a unit vector $\mathbf{b}$. Since we are assuming the existence of a low-rate, error-free, zero-delay, n_b-bit feedback link, $\mathbf{b}$ must correspond to a quantized beamforming vector chosen from a codebook of $N=2^{n_b}$ beamforming vectors and can be written as:

$$\hat{\mathbf{b}}=\arg\max_{\mathbf{b}}\parallel \mathbf{H}\mathbf{b}\parallel_2 \tag{25}$$

The question remaining is how to create the optimal beamformer codebook. For the case of a Rayleigh-fading channel matrix, Love et al. [7] and Mukkavilli et al. [8] independently showed that the problem is equivalent to real Grassmannian subspace packing (or line packing for complex Grassmannian). The interested reader is referred to the above citations for details concerning bounds on codebook size for full diversity and distance metrics among other issues. Here, we simply state the results in Table II given in [7] for the 2×2 MIMO quantized beamformer codebook using $n_b=3$ bits of feedback. This is the codebook used in Section VIII where we simulate the QMRT system.

V. Space–Time Block Coding

We focus attention here on the class of STBC that make use of orthogonal designs–namely space–time orthogonal block (STOB) codes. In this regime, the kth transmitted ST codeword is such that

$$\mathbf{S}_k\mathbf{S}_k^H=\sum_{i=1}^{n_c}|s_{i,k}|^2\mathbf{I}_{n_t} \tag{26}$$

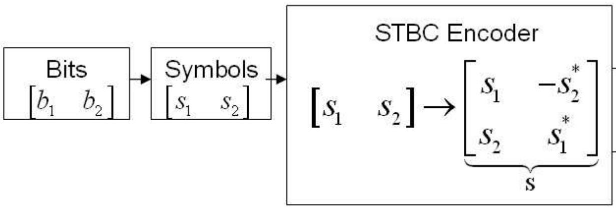


Fig. 3. Alamouti [11] space–time block encoder for a 2×2 system

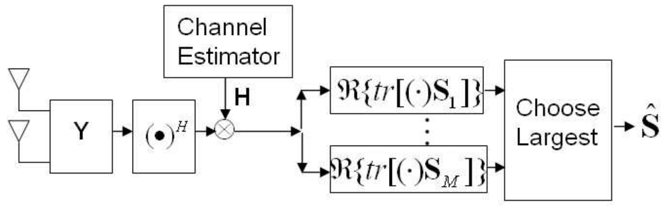


Fig. 4. Space–time block decoder (31) corresponding to an encoder of the form of Fig. 3.

A. Space–Time Block Encoding

The Alamouti scheme [11] is the simplest of this type. The 2×2 code word for complex symbols is the only full-rate orthogonal complex block code. It is given as:

$$\mathbf{S}_k = \begin{bmatrix} s_{1,k} & -s_{2,k}^* \\ s_{2,k} & s_{1,k}^* \end{bmatrix} \tag{27}$$

Real code matrices and non-full-rate complex codes for STOB are discussed in [12]. A branch of number theory known as Radon-Hurwitz theory dictates that real full-rate orthogonal blocks exist only for the 2×2, 4×4, and 8×8 cases. As an example, the 4×4 real orthogonal code matrix is:

$$\mathbf{S}_k = \begin{bmatrix} s_{1,k} & s_{2,k} & s_{3,k} & s_{4,k} \\ -s_{2,k} & s_{1,k} & -s_{4,k} & s_{3,k} \\ -s_{3,k} & s_{4,k} & s_{1,k} & -s_{2,k} \\ -s_{4,k} & -s_{3,k} & s_{2,k} & s_{1,k} \end{bmatrix} \tag{28}$$

In Fig. 3 we show the block diagram for the Alamouti [11] space–time block encoder for a 2×2 system.

B. Space–Time Block Decoding

From (2) and noting that the receiver has knowledge of the channel matrix, we can write the conditional pdf of the received signal conditioned on the channel matrix and space–time codeword as:

$$p(\mathbf{Y}_k|\mathbf{H}_k, \mathbf{S}_k) = \frac{\mathrm{etr}(-\frac{1}{2N_0}\mathbf{Z}_k\mathbf{Z}_k^H)}{\pi^{n_r n_c}} \tag{29}$$

where, for notational compactness, we have defined $\mathbf{Z}_k = \mathbf{Y}_k - \sqrt{\frac{E_s}{n_t}}\mathbf{H}_k\mathbf{S}_k$ and $\mathrm{etr}(x) = \exp(tr[x])$. The maximum-likelihood receiver obtained from (29) requires an informed receiver and leads to the following form for the space–time block decoder:

$$\begin{aligned} \hat{\mathbf{S}}_k &= \arg\max_{\mathbf{S}_k} p(\mathbf{Y}_k|\mathbf{H}_k, \mathbf{S}_k) \\ &= \arg\min_{\mathbf{S}_k} tr[(\mathbf{Y}_k - \sqrt{\tfrac{E_s}{n_t}}\mathbf{H}_k\mathbf{S}_k)(\mathbf{Y}_k - \sqrt{\tfrac{E_s}{n_t}}\mathbf{H}_k\mathbf{S}_k)^H] \\ &= \arg\min_{\mathbf{S}_k} tr[\tfrac{E_s}{n_t}\mathbf{H}_k\mathbf{S}_k\mathbf{S}_k^H\mathbf{H}_k^H] - 2\Re\{tr[\sqrt{\tfrac{E_s}{n_t}}\mathbf{H}_k\mathbf{S}_k\mathbf{Y}_k^H]\} \\ &= \arg\min_{\mathbf{S}_k} \Re\{tr[\mathbf{H}_k\mathbf{S}_k\mathbf{Y}_k^H]\} \end{aligned} \tag{30}$$

Using the identity $tr[\mathbf{AB}] = tr[\mathbf{BA}]$, the last line of (30) reduces to

$$\hat{\mathbf{S}}_k = \arg\min_{\mathbf{S}_k} \Re\{tr[\mathbf{Y}_k^H\mathbf{H}_k\mathbf{S}_k]\} \tag{31}$$

which admits the receiver design shown in Fig. 4.

VI. Space–Time Trellis Coding

Space-time trellis codes (STTC), like space-time block codes, achieve maximum diversity gain. However, STTCs have the additional advantage of providing coding gain as well as diversity gain. This coding gain comes at the expense of a more computationally complex receiver usually implemented with Viterbi decoding. In this section we restrict our attention to 4-state M-PSK signaling.

A. Space–Time Trellis Encoding

The space-time trellis encoder maps binary data streams into modulation symbols for which the mapping is described by a trellis diagram. For an M-PSK modulation we assign $m = \log_2 M$ and the memory order of the shift register is defined to be ν. For a given bit at time t denoted b_t, we group m bits at a time into the data structure $b_t^l, \quad \forall l = 1, ..., m$. The convolutionally encoded output at time t for transmit antenna i, denoted by $s_{i,t}$, is computed as:

$$s_{i,t} = \sum_{l=1}^{m}\sum_{j=0}^{\nu} g_{j,i}^l b_{t-j}^l \mod M, \quad i = 1, 2, ..., n_t \tag{32}$$

For $\nu = 2$ we can cast our bit stream as coefficients of a polynomial as:

$$\begin{array}{l} \mathbf{b}^1(z^{-1}) = b_0^1 + b_1^1 z^{-1} \\ \mathbf{b}^2(z^{-1}) = b_0^2 + b_1^2 z^{-1} \end{array} \quad . \tag{33}$$

This allows us to rewrite the convolutional encoder of (32) as a polynomial matrix multiplied by a polynomial vector:

$$\mathbf{S} = \begin{bmatrix} \mathbf{g}_1^1(z^{-1}) & \mathbf{g}_1^2(z^{-1}) \\ \mathbf{g}_2^1(z^{-1}) & \mathbf{g}_2^2(z^{-1}) \end{bmatrix} \begin{bmatrix} \mathbf{b}^1(z^{-1}) \\ \mathbf{b}^2(z^{-1}) \end{bmatrix} \tag{34}$$

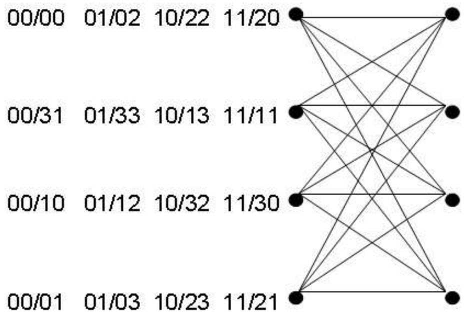


Fig. 5. Space–time trellis diagram for the generator polynomials given in (35) from [14].

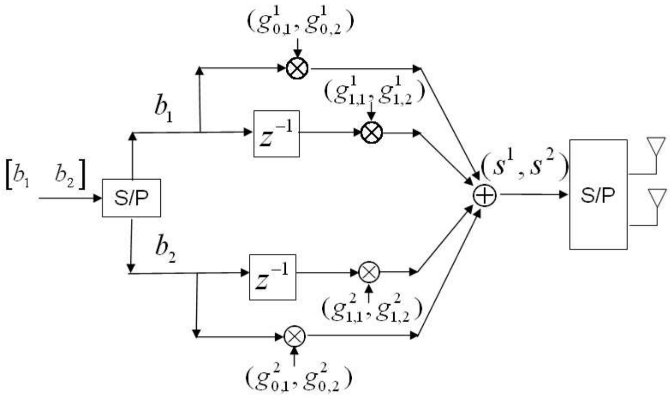


Fig. 6. Space–time trellis encoder for a QPSK constellation with memory order 2 and 2 transmitters.

These outputs are from the M-PSK signal constellation and form the elements of the transmitted ST codeword $\mathbf{S} = \{s_{i,t}\}$.

We consider here the space–time trellis codes developed by Baro, Bauch, and Hansmann [14] since they are optimal in the sense of meeting the rank and determinant criteria established previously in this section. They are an improvement over the original space–time trellis codes proposed in [13]. The generator polynomials from [14] are given as:

$$\begin{aligned}
\mathbf{g}_1^1(z^{-1}) &= g_{0,1}^1 + g_{1,1}^1 z^{-1} = 2 + z^{-1} \\
\mathbf{g}_2^1(z^{-1}) &= g_{0,2}^1 + g_{1,2}^1 z^{-1} = 2 \\
\mathbf{g}_1^2(z^{-1}) &= g_{0,1}^2 + g_{1,1}^2 z^{-1} = 3z^{-1} \\
\mathbf{g}_2^2(z^{-1}) &= g_{0,2}^2 + g_{1,2}^2 z^{-1} = 2 + z^{-1}
\end{aligned} \tag{35}$$

Fig. 5 shows the trellis for these generator polynomials following the format similar to Tarokh et al. [13]. For the generator polynomials given in (35) and for a 2^{nd} order shift register, QPSK modulation, and 2 transmit elements, $\nu = 2$, $M = 4$, $m = 2$, and $n_t = 2$ in (32). The STTC encoder for this case is shown in Fig. 6.

B. Space–Time Trellis Decoding

Decoding follows the trellis using a sequence metric calculator. The Viterbi algorithm provides the maximum likelihood decoder for space–time trellis codes. Assuming perfect CSI available at the receiver, one useful branch metric $m(t)$ used by the Viterbi algorithm is the squared Euclidean distance between the received signal and the hypothesis:

$$m(t) = \| \mathbf{y}(t) - \mathbf{H}\mathbf{s}(t) \|^2 \quad . \tag{36}$$

The Viterbi algorithm chooses the path with minimum path metric as the decoded sequence.

VII. Unitary Space–Time Modulation

Unitary space–time modulation methods send orthogonal signals over the n_t transmit antennas. Orthogonal designs can be decoded non-coherently (without CSI). In USTM the current transmitted orthogonal signal matrix is constructed from the previous transmitted codeword through multiplication by a unitary matrix. It was shown in [3] that, for the block-wise constant channel (quasi-static assumption), USTM can achieve the capacity for non-coherent detection.

For the uninformed transmitter/receiver pair we write the conditional pdf of the received signal as a function of the code matrix only:

$$p(\mathbf{Y}_k|\mathbf{S}_k) = \frac{\mathrm{etr}(-\mathbf{Y}_k \Sigma^{-1} \mathbf{Y}_k^H)}{|\pi\Sigma|_r^n} \tag{37}$$

where $\Sigma = \mathbf{I} + \frac{E_s}{N_0 n_t} \mathbf{S}_k^H \mathbf{S}_k$. Through use of the matrix inversion lemma, we can show that

$$\Sigma^{-1} = \mathbf{I} - \frac{E_s}{n_c E_s + N_0 n_t} \mathbf{S}_k^H \mathbf{S}_k \tag{38}$$

Using (37) and (38) we can write the maximum likelihood estimator for the general non-coherent space–time detector as:

$$\begin{aligned}
\hat{\mathbf{S}}_k &= \arg\max_{\mathbf{S}_k} p(\mathbf{Y}_k|\mathbf{S}_k) \\
&= \arg\max_{\mathbf{S}_k} tr[\mathbf{Y}_k \mathbf{S}_k^H \mathbf{S}_k \mathbf{Y}_k^H] \quad .
\end{aligned} \tag{39}$$

The codeword in (39) applies to the general form for a codeword $\mathbf{S}_k$ of arbitrary temporal size n_c. However, to reduce computational complexity, we wish to restrict attention to a detector form that uses a ST codeword with a small n_c so that the search space is manageable. We can do this by using two consecutive "small" space–time codewords and grouping them into a composite codeword.

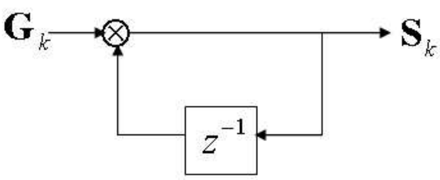


Fig. 7. Block diagram of unitary space-time modulator.

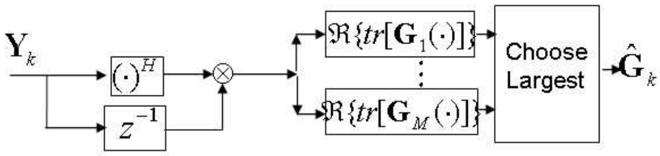


Fig. 8. Block diagram of unitary space–time demodulator for the modulator of Fig. 7.

A. Unitary Space–Time Encoding

We will need the current and previous codewords to be able to perform the differential demodulation. To this end we construct a composite STC codeword matrix consisting of two unitary STC words $\bar{\mathbf{S}}_k = [\mathbf{S}_{k-1} \quad \mathbf{S}_k]$ which produce the pair of observation matrices $\bar{\mathbf{Y}}_k = [\mathbf{Y}_{k-1} \quad \mathbf{Y}_k]$. In Fig. 7 we show the differential encoder for USTM that creates the composite codeword. The z^{-1} in the figure represents a delay of an entire codeword. Here $\mathbf{S}_k$ and $\mathbf{S}_{k-1}$ are temporally consecutive $n_t \times n_c$ complex matrices that are formed according to the rule:

$$\mathbf{S}_k = \mathbf{S}_{k-1}\mathbf{G}_k, \quad \mathbf{S}_0 = \mathbf{D} \tag{40}$$

where $\mathbf{G}_k \in \mathcal{G}$ is the unitary matrix at time slot k described in Section II-C in (11) and $\mathcal{G}$ is the set of unitary matrices composing the multichannel group code [9]. Consequently, the composite ST codeword is:

$$\bar{\mathbf{S}}_k = [\mathbf{S}_{k-1} \quad \mathbf{S}_{k-1}\mathbf{G}_k] \ . \tag{41}$$

As an example, for a QPSK signal constellation $\{1, j, -1, -j\}$ and a $n_t = 2$ system the set $\mathcal{G}$ is given as:

$$\mathcal{G}_k = \left\{ \begin{array}{c} \pm \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \pm \begin{bmatrix} j & 0 \\ 0 & -j \end{bmatrix}, \\ \pm \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \pm \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} \end{array} \right\} \tag{42}$$

and

$$\mathbf{D} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \tag{43}$$

B. Unitary Space–Time Decoding

In keeping with the ML detector form in (39) the detector for the 2-block code $\bar{\mathbf{Y}}_k = [\mathbf{Y}_{k-1} \quad \mathbf{Y}_k]$ selects

$$\hat{\mathbf{S}}_k = \arg\max_{\mathbf{S}_k} tr[\bar{\mathbf{Y}}_k \bar{\mathbf{S}}_k^H \bar{\mathbf{S}}_k \bar{\mathbf{Y}}_k^H] \tag{44}$$

Since, $\mathbf{S}_k = \mathbf{S}_{k-1}\mathbf{G}_k$, knowledge of $\mathbf{S}_k$ is captured by $\mathbf{G}_k$ allowing us to carry out the derivation for the detector as:

$$\begin{aligned} \hat{\mathbf{G}}_k &= \arg\max_{\mathbf{G}_k \in \mathcal{G}} tr\,[\mathbf{Y}_{k-1} \quad \mathbf{Y}_k] \begin{bmatrix} n_t\mathbf{I} & n_t\mathbf{G}_k \\ n_t\mathbf{G}_k^H & n_t\mathbf{I} \end{bmatrix}^H \times \\ &\qquad [\mathbf{Y}_{k-1} \quad \mathbf{Y}_t]^H \\ &= \arg\max_{\mathbf{G}_k \in \mathcal{G}} \Re\left\{ tr\left[\mathbf{Y}_{k-1}\mathbf{G}_k\mathbf{Y}_k^H\right]\right\} \end{aligned} \tag{45}$$

Again, using $tr[\mathbf{AB}] = tr[\mathbf{BA}]$, this receiver reduces to

$$\hat{\mathbf{G}}_k = \arg\max_{\mathbf{G}_k \in \mathcal{G}} \Re\left\{ tr\left[\mathbf{G}_k\mathbf{Y}_k^H\mathbf{Y}_{k-1}\right]\right\} \tag{46}$$

which is shown in Fig. 8.

As Hughes [9] states, the receiver has an estimator-correlator interpretation. If $\mathbf{H}_k$ and $\mathbf{S}_k$ were known at the receiver, then the optimal detector would be the minimum Euclidean distance rule as in (30). Hughes [9] shows that for unitary codes, this reduces to a correlation receiver:

$$\hat{\mathbf{G}}_k = \Re\left\{ tr[\mathbf{H}_k\mathbf{S}_{k-1}\mathbf{G}_k\mathbf{Y}_k^H]\right\} \tag{47}$$

which we note is the same as (30) in which $\mathbf{S}_k$ is replaced by $\mathbf{S}_{k-1}\mathbf{G}_k$.

VIII. Simulation Results

In this section we compare the simulated performance of the Grassmannian beamformer (GB) from Section IV, the Alamouti [11] space–time block code (STBC) from Section V, the 4-state space-time trellis code (STTC) from Section VI, and the unitary space-time modulator (USTM) from Section VII to the single-input single-output (SISO) baseline modem. All the MIMO algorithms operate with a 2×2 antenna array system in a quasi-static Rayleigh fading channel. The channel $\mathbf{H}$ is simulated by taking samples from the complex normal distribution. All modems operate with a spectral efficiency of 2 bps/Hz with the exception of the USTM algorithm which is designed here to operate at 1.5 bps/Hz. The USTM is the only algorithm to not require training data and, hence, we lower its required spectral efficiency with an implicit assumption that the other MIMO algorithms require a factor of $\frac{4}{3}$ training overhead. In all cases, we estimate the channel matrix and

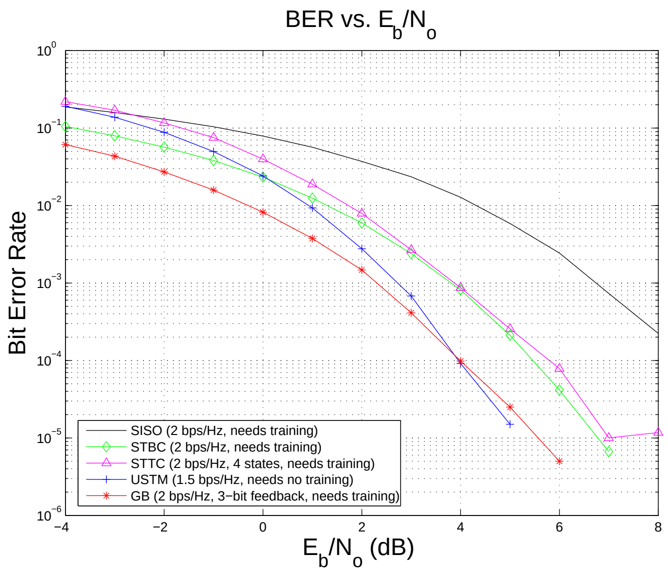


Fig. 9. Performance comparison of the 2×2 MIMO space-time algorithms discussed in this paper.

insert the estimated channel into the space-time decoding modules. In Fig. 9 we show performance in terms of bit error rate (BER) as a function of the received bit energy per noise ratio E_b/N_0. The Grassmannian beamformer significantly outperforms the space-time coding algorithms for the same spectral efficiency. The space-time trellis code performs slightly worse than the space-time block code most likely because the number of states is only 4. If we were to go to a more typical 16-, 32-, or 64-state trellis we would see the curve significantly improve (see e.g. [13]). All the algorithms achieve full diversity of $n_t n_r = 4$.

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