BLINDLY ESTIMATING AND LOCALIZING MULTIPLE SIGNALS FROM THE MATRIX CHANNEL IMPULSE RESPONSE

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ABSTRACT

We consider the problem of estimating and localizing a set of unknown real-valued signals simultaneously arriving at a sensor array composed of elements spaced far enough apart to induce measurable baseband time difference of arrivals. The delayed and mixed line-of-sight signals form a convolutive mixture model. We recover the source signals through multichannel blind deconvolution (MBD) in which the channel impulse response estimate provides the direction of arrival (DOA) information. We consider the two cases in which either the source signal probability density is known or the attractor space for which the source density resides is known. The new MBD algorithm we present works with known source densities to minimize the symmetric Kullback-Liebler distance to the standardized equalized output. Simulation results show successful performance on acoustic data.

1. INTRODUCTION

We pose a solution to the problem of blindly estimating and localizing real-valued multiple simultaneous signals impinging on a sensor array for a line-of-sight (LOS) environment. We consider the case where the sensors are spaced far enough apart (or the signal bandwidth is wide enough) such that all elements in the array observe time delayed versions of all source signals–thereby creating a convolutive mixture model. We derive a new multichannel blind deconvolution (MBD) algorithm to simultaneously estimate the channel impulse response (from which we can localize the signals) and recover the sources up to an unknown amplitude, delay, and permutation.

Research in MBD has shown that the channel impulse response and source signals can be estimated using only knowledge (or partial knowledge) of the probability densities of the source signals (see e.g. [2]). If the source densities lie in the domain of attraction of a stable limit distribution (e.g. Gaussian, Cauchy, Levy) but not on the *attractor* density itself, the source realizations can generally be recovered. Mixture models cause the observed signal densities to "move" towards the attractor while the separation model moves the equalized and unmixed signals away from the attractor and towards the source densities. Most MBD [2] and independent component analysis (ICA) [1] algorithms are designed only to move the unmixed signals away from the attractor (Type-I objective) and do not attempt to minimize the distance to a known source density (Type-II objective). It turns out that the Type-I objective often yields a statistically inconsistent estimator for many cases (of which we show one case by simulation) whereas the Type-II estimator is always consistent by virtue of the distance metric attaining zero only when densities are identical.

The main contributions of this paper are: 1) We form multi-source localization as a MBD problem, 2) We derive ambiguous form of maximum likelihood estimator for source signal realizations, and 3) We derive a consistent DOA vector estimator based on minimum distance between source and equalized signal probability densities. The remainder of the paper is organized as follows. We give the signal model in Section 2. In Sections 3 and 4 we describe source estimation and localization respectively. We show simulation results in Section 5 and conclude in Section 6.

2. SIGNAL MODEL

Consider a number of observable signals arriving in planar wavefronts upon an arbitrary collection of sensors at known positions $\mathbf{P} = \{\mathbf{p}_i = [p_{x_i}, p_{y_i}]^T \;\; \forall i\}$ as in Fig. 1. We have n_t real-valued baseband transmitted signals $\{s_j[n], \forall j, n\}$ sampled every T seconds coming from direction θ_j at speed c impinging on a sensor array composed of $n_r \geq n_t$ receivers. We observe the vector process $\mathbf{Y} = \{y_i[n], \forall i, n\}$ where sensor i captures the sequence

$$y_i[n] = \sum_{j=1}^{n_t} \sum_{l=-\infty}^{\infty} h_{ij}[l] s_j[n-l] + w_i[n], \quad \forall i = 1, ..., n_r \; . \tag{1}$$

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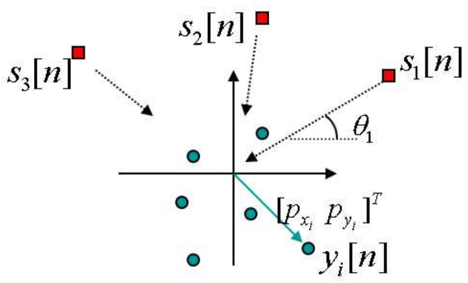

Fig. 1. We illustrate a sensor array with source signals $s_j[n]$ impinging at angles of arrival θ_j to create a received signal $y_i[n]$ at sensor position $[p_{x_i}, p_{y_i}]^T$.

The thermal additive white gaussian noise (AWGN) sequence is given by $w_i[n]$ and $h_{ij}[n]$ are the coefficients of the matrix channel impulse response.

Proposition 1 *For a* line-of-sight *(LOS) channel model, the channel impulse response is given by:*

$$h_{ij}[n] = \alpha_{ij}\text{sinc}\left(n - \frac{\tau_{ij}}{T} - \frac{\Delta_j}{T}\right) \tag{2}$$

where Δ_j is the propagation delay of the j-th source signal to the sensor array origin and

$$\tau_{ij} = \frac{-1}{c}\left[p_{x_i}\cos(\theta_j) + p_{y_i}\sin(\theta_j)\right] \tag{3}$$

is the relative time delay of the j-th source signal between the origin of the sensor array and receiver i.

Proof: Ignoring the additive noise term, the continuous-time received signal model corresponding to (1) is $y_i(t) = \int_{\tau=-\infty}^{\infty} h_{ij}(\tau)s_j(t-\tau)d\tau$. From the sampling theorem, we know that $s_j(t) = \sum_{k=-\infty}^{\infty} s_j[k]\text{sinc}(\frac{1}{T}(t-kT))$. Plugging this in and sampling at $t = nT$ gives:

$$y_i(t = nT) = \sum_k s_j[k] \underbrace{\int_\tau h_{ij}(\tau)\text{sinc}(n - k - \frac{\tau}{T})d\tau}_{h_{ij}[n-k]} \tag{4}$$

Letting $q = n - k$ allows us to write

$$h_{ij}[q] = \int_{\tau=-\infty}^{\infty} h_{ij}(\tau)\text{sinc}(q - \frac{\tau}{T})d\tau \ . \tag{5}$$

Now, we define the 2-D directional unit vector for source j as $[-\cos\theta, -\sin\theta]^T$ and project it onto the position vector $\mathbf{p}_i = [p_{x_i}, p_{y_i}]^T$ for sensor i. This gives the distance between the sensor and the origin along the direction of arrival (DOA) of the j-th source signal. The relative time delay is then simply this distance divided by the speed c of propagating wavefront to give (3). For a real-valued signal in a LOS channel, $h_{ij}(\tau) = \alpha_{ij}\delta(\tau - \tau_{ij} - \Delta_j)$. Plugging this into (5) gives (2). □

The channel amplitude α_{ij} can be approximated $\alpha_{ij} \approx \alpha_j$ since the sources lie in the far field. Furthermore, the channel amplitudes α_j can be wrapped into the source signal amplitudes since we are not concerned with how far away the sources are or what their transmit power is. Since the propagation delay Δ_j is also of no interest, it is sufficient to consider a representation of the channel impulse response in (2) that is parameterized solely by θ_j and $\mathbf{p}_i$ as:

$$\xi_{ij}[n;\theta_j,\mathbf{p}_i] = \text{sinc}\left(n + \frac{1}{cT}(p_{x_i}\cos(\theta_j) + p_{y_i}\sin(\theta_j))\right) \ . \tag{6}$$

3. SOURCE ESTIMATION

Taking the DFT of (1) we write

$$\tilde{y}_i[k] = \sum_{j=1}^{n_t} \tilde{h}_{ij}[k]\tilde{s}_j[k] + \tilde{w}_i[k] \quad k = 0, 1, ..., N-1 \tag{7}$$

which in matrix-vector notation is $\tilde{\mathbf{y}}[k] = \tilde{\mathbf{H}}[k]\tilde{\mathbf{s}}[k] + \tilde{\mathbf{w}}[k]$. Taking the DFT of (6) to get $\tilde{\xi}_{ij}[k;\theta_j,\mathbf{p}_i]$ and packing into $\tilde{\Xi}[k;\Theta,\mathbf{P}]$ we note

$$\tilde{\mathbf{H}}[k] = \tilde{\Xi}[k;\Theta^*]\Pi\Lambda_k \tag{8}$$

where $\Lambda_k = \text{diag}(\alpha_j \exp(-\jmath 2\pi k\Delta_j/TN)), j = 1, ..., n_t$, Θ^* is the true DOA vector, and Π is a (possible) permutation matrix. For notation following, we will use $[\mathbf{A}]_{j,:}$ to denote the jth row a matrix $\mathbf{A}$ and $\mathbf{A}^\dagger$ to denote the Moore-Penrose pseudoinverse.

Proposition 2 *If we define the j-th equalized signal to be*

$$\hat{x}_j[n;\Theta,\mathbf{P},\mathbf{Y}] = \frac{1}{N}\sum_{k=0}^{N-1}\left[\tilde{\Xi}^\dagger[k;\Theta,\mathbf{P}]\right]_{j,:}\tilde{\mathbf{y}}[k]e^{\jmath 2\pi kn/N} \ , \tag{9}$$

then the quantity $\hat{x}_j[n;\Theta^*,\mathbf{P}]$ is the maximum likelihood estimate (MLE) of $s_j[n]$ up to an unknown amplitude, delay, and (possibly) permutation of the source index j.

Proof: If we define $\tilde{\mathbf{x}}[k] = \Pi\Lambda_k\tilde{\mathbf{s}}[k]$, the likelihood function for $\tilde{\mathbf{y}}[k]$ can be written

$$\mathcal{L}(\tilde{\mathbf{x}}[k];\tilde{\mathbf{y}}[k]) \propto \exp\left(-||\tilde{\mathbf{y}}[k] - \tilde{\Xi}[k;\Theta^*,\mathbf{P}]\tilde{\mathbf{x}}[k]||_2^2\right) \tag{10}$$

since the covariance matrix of $\tilde{\mathbf{w}}[k]$ is just a scaled identity matrix if $\mathbf{w}[n]$ possesses a scaled identity covariance matrix (which is the case for AWGN). Therefore, the MLE of $\tilde{\mathbf{x}}[k]$ for k is given as

$$\hat{\tilde{\mathbf{x}}}[k] = \tilde{\Xi}^\dagger[k;\Theta^*,\mathbf{P}]\tilde{\mathbf{y}}[k] \quad k = 0, 1, ..., N-1 \tag{11}$$

Since the linear transformation of a vector of MLEs is also a MLE, it follows that (9) is the time domain MLE of (11) since the inverse DFT is a linear operation. □

4. SOURCE LOCALIZATION

We saw in section 3 that the source signal realizations can be recovered up to an unknown amplitude and delay by finding the DOA vector $\Theta = [\theta_1, ..., \theta_{n_t}]^T$ that corresponds to the true vector. Our goal here is to find an estimate of that parameter (call it $\hat{\Theta}$) which is a function of the observation and possibly the source densities $\{g_j(x), \forall j\}$ when they are available. Depending on how much is known about the source signals, we can estimate the channel using knowledge only of which random variable attractor space the source densities reside (Type-I) or the source density functions themselves (Type-II). In the first case the source densities are unknown, but we know that they lie within a certain domain of attraction to a limiting distribution. In the second case, the source densities are known either theoretically from the physics of the transmitting object or have been empirically derived *a priori*. We describe these different objectives geometrically by considering "distance-like" measures on a Riemannian manifold of probability densities.

4.1. Designing the Objective Function

We define the estimator $\hat{\Theta}^{(\mathrm{I})}$ for Θ that works with *unknown* source densities to maximize distance between an attractor density $a(x)$ and the equalized (estimated) source densities the Type-I estimator and write it as

$$\hat{\Theta}^{(\mathrm{I})} = \arg_{\Theta} \max \sum_{j=1}^{n_t} D\left(f_j(x|\Theta, \mathbf{P}, \mathbf{Y})||a(x)\right) \qquad (12)$$

where $D(f||a)$ is a divergence between two probability densities f and a. We define the estimator $\hat{\Theta}^{(\mathrm{II})}$ for Θ that works with *known* source densities $g_j(x)$ to minimize the distance to the equalized source densities the Type-II estimator and write it as

$$\hat{\Theta}^{(\mathrm{II})} = \arg_{\Theta} \min \sum_{j=1}^{n_t} D\left(f_j(x|\Theta, \mathbf{P}, \mathbf{Y})||g_j(x)\right) \quad . \qquad (13)$$

In Fig. 2 we illustrate a conceptual Riemannian manifold M that contains three probability densities of interest: 1) the attractor density $a(x)$ (which is Gaussian in our problem) 2) the j-th source density $g_j(x)$, and the estimated j-th source density $f_j(x|\Theta, \mathbf{P}, \mathbf{Y})$–the density of the equalized output. When $\Theta = \Theta^*$ and the AWGN $w_i[n]$ in (1) is zero, the estimated source density agrees exactly with $g_j(x)$.

The divergence (or contrast) function we consider in this study is the J-divergence (symmetric Kullback-Liebler divergence) written $D^J(f||g) = D^{KL}(f||g) + D^{KL}(g||f)$ where $D^{KL} = \int_{x \in \mathcal{X}} f(x) \log \frac{f(x)}{g(x)} dx$ is the Kullback-Liebler divergence. The J-divergence is a function of a special class of single-parameter information-type divergences developed by Csiszar of the form $E[k(f/g)]$ for k convex (see e.g. [3] and reference therein). This class also includes the Bhattacharyya distance, Hellinger distance, and Kullback-Liebler divergence among others. Using the Parzen method, we form the probability density function of the equalized sources as

$$f_j(x|\Theta, \mathbf{Y}) = \frac{1}{N} \sum_{n=0}^{N-1} \phi\left(x - \hat{x}_j[n; \Theta, \mathbf{P}, \mathbf{Y}]\right) \qquad (14)$$

where $\phi(\cdot)$ is the normalized kernel function chosen to be Gaussian in our case.

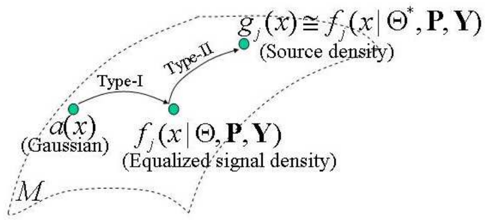

Fig. 2. Conceptual Riemannian manifold of probability density functions containing the attractor density, source densities, and equalized output densities. If we know the source densities, we want to "move" towards them. Otherwise, we want to simply "move" away from the attractor.

4.2. Optimizing the Objective Function

We optimize the objective functions in (12) and (13) by multiscale sampling the n_t-dimensional space of Θ at lattice points correspond to centers of spheres in a n_t-dimensional *sphere packing* ([4]). In the first stage we find the lattice point with minimum value of the function evaluation. In the second stage we consider a local sphere packing with half the radius about that point and evaluate the objective function for lattice points that "kiss" the sphere about the first stage lattice point. We can continue this process until the multi-resolutional lattice point remains unchanged.

Fig. 3 shows contour plots of the objective surface for the Type-I and Type-II estimator as a function of the DOA parameters θ_j for SNR=30dB on the data described in section 5. The multiresolutional sphere packed samples generated from the optimization algorithm are overlayed to show where the parameter estimates converge. For symmetric objective functions such as those produced by the Type-I estimator, we only have to sample the convex polytope containing the unique permutation of parameter elements θ_j (half space for $n_t = 2$ case). Note that the location of the global minima for the Type-I objective function differs from the true vector causing the estimator to incorrectly converge.

This simple example illustrates how objective functions based on maximizing distance from a Gaussian attractor density (e.g. kurtosis maximization, entropy minimization, and minimization of mutual information [1]) can lead to

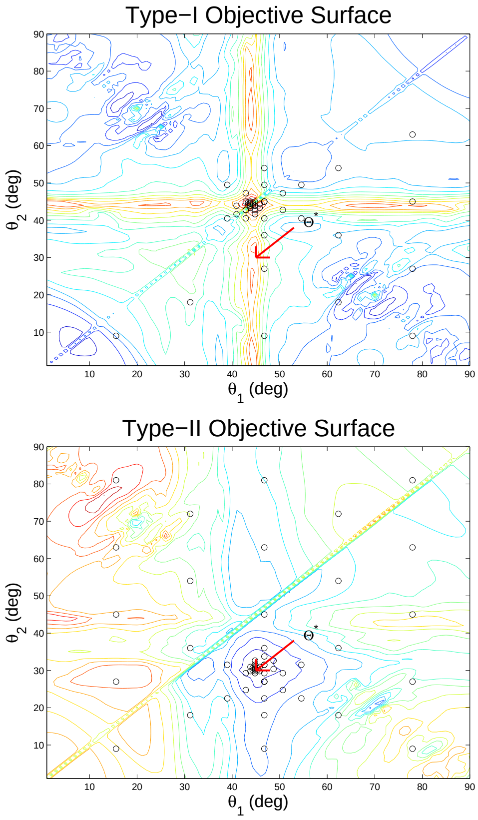

Fig. 3. We show contour plots of the 2-D ($n_t = 2$ case) objective surfaces for the estimators described in (12) and (13). The sphere-packed samples used to perform the multi-resolutional global optimization are overlayed to indicate where the algorithm converges. Note that the Type-I objective surface yields an inconsistent estimator.

a statistically inconsistent estimator. The Type-II estimator, on the other hand, must yield a consistent estimator by construction. The non-negative objective function is minimized when the distance between probability density functions is zero–which can only happen when the two densities are identical.

5. SIMULATION RESULTS

We test our algorithms using 400 samples of two 8kHz sampled acoustic waveforms impinging on a four-element circular array within a random uniformly distributed $90°$ angular space in which the two sources cannot be located at the same angle. The LOS channel is simulated according to (2). We then add artificial white Gaussian noise to drive the received signal SNR to specified values. We estimate average mean squared error ($\overline{MSE}$) as

$$\overline{MSE}(\hat{\Theta}) = \frac{1}{M}\sum_{m=1}^{M} ||\hat{\Theta}_m - \Theta^*||_2^2 \qquad (15)$$

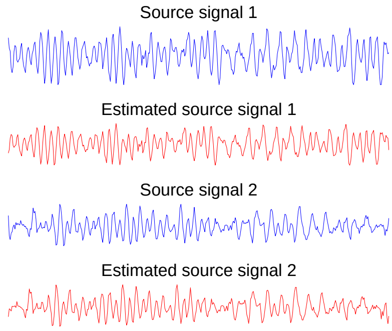

Fig. 4. The original source signals and equalized signals (estimated sources) computed from (9) at the solution to (13).

where $\hat{\Theta}_m$ is the m-th estimate of $M = 50$ Monte Carlo trials and n_t is the number of sources (length of Θ). The average mean square error values of $\hat{\Theta}^{(II)}$ (13) at the SNR values of 10dB, 20dB, and 30dB is 0.5191, 0.0092, and 0.0022 respectively. The estimated sources computed using (9) for the 30dB SNR case is plotted in Fig. 4.

6. CONCLUSION

Although we presented a solution to the problem of blindly localizing and estimating signals in a line-of-sight environment, much work needs to be done to find better ways to optimize the objective functions–namely making the estimator sequential and able to exploit the Riemannian structure of the statistical manifold. Further work needs to be done also to identify optimal contrast functions that will induce metrics and connections that lead to faster converging algorithms. Lastly, the constraint of real-valued source signals needs to be lifted to allow for complex-valued source signals such as communication waveforms.

7. REFERENCES

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