# QUANTUM VS CLASSICAL COMPLEMENTARY PNT

## Are quantum sensors the next big thing for PNT, or are they overhyped?

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# INTRODUCTION

IF YOU LOSE ACCESS TO GPS IN AN UNFAMILIAR PLACE, HOW WOULD YOU DETERMINE WHERE YOU ARE AND HOW TO GET TO YOUR DESTINATION?

Due to its accuracy and availability, dependency on GPS has become commonplace in all domains, from civil aviation and critical infrastructure to cellular communications, global finance, and the military. However, to maintain robust positioning, navigation, and timing (PNT) capabilities, it is crucial to develop techniques and technologies that do not rely on GPS. The field of complementary PNT aims to provide PNT solutions that are complementary or alternative to GPS, which is often treated as the primary source of PNT. Complementary PNT may be needed when GPS signals are unavailable (e.g., underground or underwater), in the event of GPS interference, or in case of GPS satellite failures. Currently, no complementary PNT capabilities can offer the same level of global availability and accuracy as GPS, but many techniques serve as critical complements or alternatives to GPS in a wide range of applications.

Emerging quantum sensors are often cited as promising technologies to enable complementary PNT solutions with improved capabilities or reduced cost. Atomic clocks are already widely used for precision timing, and emerging quantum inertial sensors, magnetometers, and gravimeters may also play a role for positioning and navigation as they are developed further. However, quantum sensors are just a subset of the broader complementary PNT ecosystem, which comprises a wide range of sensing and timing technologies with varying technological maturity and attributes. To determine the optimal solution for a given complementary PNT need, it is necessary to consider both quantum and classical technologies and analyze their advantages and limitations. This report provides a summary of various complementary PNT techniques and identifies where quantum technologies are likely to fit within the broader complementary PNT ecosystem.

# OVERVIEW OF COMPLEMENTARY PNT TECHNIQUES AND TECHNOLOGIES

What is now called “complementary PNT” encompasses PNT techniques and technologies that long predate the inception of GPS. Inertial navigation was used for rocket guidance as early as WWII [1, 2]. Crystal oscillators were developed for timekeeping in the early 1900s and used for radio communications soon after [3, 4]. Even the sextant, which is still used for marine navigation, was first developed centuries ago.

Whether due to rapid technological advances or proven dependability, many complementary PNT techniques and technologies are ubiquitous. Inertial sensors are part of the core sensor suite for airborne, marine, and undersea platforms. Electronic and crystal oscillators are commonplace in consumer electronics, automotive and industrial systems, aerospace and military systems, and communications systems. Other technologies, such as cameras, radar, and lidar, are used for PNT in airborne platforms and autonomous vehicles. While none of these technologies provide the same level of global, all-weather availability and accuracy as GPS or other Global Navigation Satellite Systems (GNSSs), many of them provide important complements or alternatives to GNSS.

Emerging technologies, such as quantum sensors, offer the potential for enabling improved performance for certain complementary PNT techniques. However, it is currently unclear to what extent quantum technologies will impact the future complementary PNT ecosystem. Research efforts to improve the stability and accuracy of atomic clocks could enable longer holdover times in the absence of GNSS or other time transfer techniques [5]. Commercially available quantum magnetometers are already used in applications such as oil exploration and atmospheric research, and they are currently being investigated for magnetic anomaly-aided

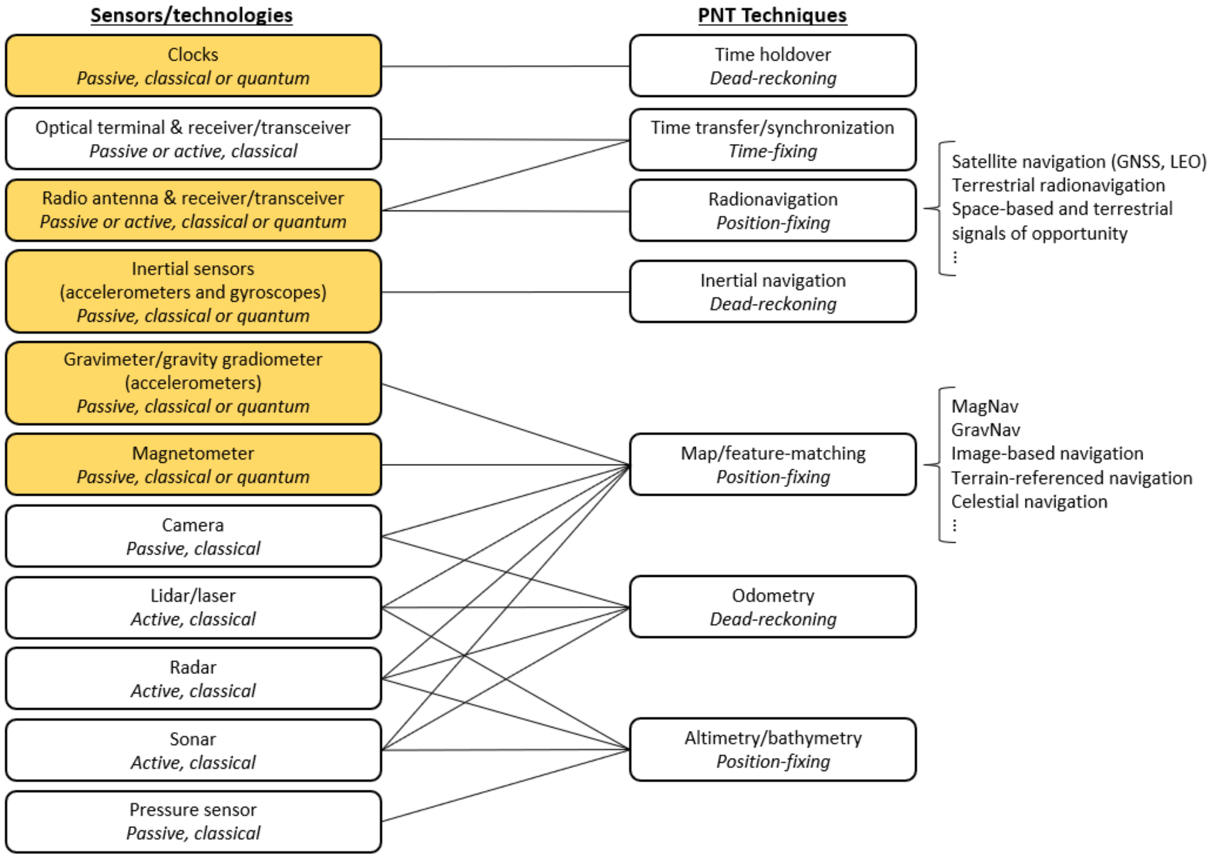

Figure 1. Sensors/technologies and the PNT techniques they enable. Sensors/technologies that have the potential for practical quantum options are noted in text and highlighted in yellow. LEO stands for low-earth orbit.

navigation (MagNav) [6]. Quantum inertial sensors and gravimeters are still largely under development but are being investigated for applications in inertial navigation and gravitational anomaly-aided navigation (GravNav) [7, 8, 9, 10, 11, 12].

Quantum and classical PNT technologies continue to improve in accuracy and maturity. In some cases, the performance of a PNT technique is strongly tied to the performance of the sensor or technology that enables it, such as the stability of a clock for precision timing. In other cases, the performance of a PNT technique is limited by other factors. For example, the positioning accuracy of map-matching techniques, in which environmental features are observed or measured and compared to a database of known features in order to localize oneself on a map, is strongly dependent on the availability, resolution, and quality of the map.

Figure 1 summarizes various sensors and technologies and connects them with the PNT techniques that they support. PNT sensors and technologies can be classified as either active or passive. Passive sensors only receive information from the environment, while active systems both transmit and receive information. Positioning techniques can be broadly classified as either dead-reckoning or position-fixing. In dead-reckoning, position is estimated relative to a previously known or estimated position. In position-fixing, position is estimated relative to one or more landmarks that have a common reference frame, such as GNSS satellites, a terrestrial feature like a building, or features in a map or database. The categories of dead-reckoning and position-fixing can also be extended to timing. In dead-reckoning, time is estimated relative to a previously known or estimated time, for example in a free-running clock for time holdover. In “time-fixing” or time-synching, time is estimated relative to an externally defined time (in a sense, the "time of a landmark"), such as Coordinated Universal Time (UTC).

A characteristic of dead-reckoning PNT techniques is that they accumulate error over time. For example, a free-running clock will invariably accumulate time error, no matter how stable it is. Similarly, an inertial navigation system will accumulate position and attitude errors. These accumulated position, attitude, and time errors can be corrected by position-fixing or time-synching. The time estimate of a free-running clock can be corrected by disciplining it to (i.e., steering it into synchronization with) a time reference such as UTC through a communications link, while accumulated errors in an inertial navigation system (INS) can be corrected by

## TIMING AND CLOCKS

Timing is an essential capability for navigation, communications, and remote sensing. It relies on a clock, which consists of an oscillator and a counter. An oscillator is anything that produces a stable, periodic signal, for example a pendulum, a pulsar, a piezoelectric crystal when a voltage is applied, or a laser/microwave source stabilized to an atomic reference. A counter is a device that counts or accumulates the periodic signal of an oscillator. Timing sources are used to maintain time in the absence of an absolute reference, and time-fixing is provided by comparing the local time estimate to an externally defined time (e.g., UTC).

## INERTIAL NAVIGATION

Inertial navigation is a dead-reckoning positioning technique used to measure changes in position or attitude. An inertial navigation system (INS) consists of an inertial measurement unit (IMU) containing accelerometers and gyroscopes and a processor which integrates the IMU measurements to estimate position, velocity, and attitude relative to an initial (or previous) position and attitude. Inertial navigation is ubiquitous in aircraft, unmanned aerial systems (UASs), missiles, marine vessels, and submarines.

## RADIONAVIGATION

Radionavigation is a position-fixing technique in which radio signals are used to measure the position, velocity, range, or bearing of a receiver/transceiver relative to one or more radio transmitters. Radionavigation can take a variety of forms; a few examples are listed below.

- GNSS receivers estimate their position and time by measuring their ranges to a constellation of GNSS satellites, whose positions are known and transmitted on the satellite signals.
- Receivers can determine their position using Doppler, and sometimes ranging, measurements from signals of opportunity (i.e., signals that are not meant for navigation and timing). Signals of opportunity include but are not limited to LTE signals [13] or broadband communications signals from LEO satellites [14, 15, 16].
- Terrestrial radionavigation systems such as pseudolites (which are terrestrial transceivers used to back up or complement GNSS) or Loran (which stands for long-range navigation) operate on principles similar to satellite-based navigation, except the transmitters are ground-based instead of space-based and provide regional coverage instead of global coverage.

## MAP/FEATURE-MATCHING

Map- or feature-matching is a position-fixing technique in which observations of the environment are matched to a map or database of known features. The features can be manmade (e.g., streets and buildings) or geophysical (e.g., terrain, gravitational anomalies, or magnetic fields). Map/feature-matching can take a variety of forms; examples are listed below.

- **MAGNETIC ANOMALY-AIDED NAVIGATION (MAGNAV)** is a technique in which measurements of the magnetic field are matched to maps or databases of the magnetic field and used to estimate position.
- **GRAVITATIONAL ANOMALY-AIDED NAVIGATION (GRAVNAV)** is a technique in which measurements of Earth's gravitational anomalies, or their gradients, are matched to maps or databases and used to estimate position.
- **IMAGE-BASED NAVIGATION** is a technique in which images of the surrounding environment (e.g., taken from an aerial platform using cameras or synthetic aperture radar) are matched to maps or databases and used to estimate position.
- **TERRAIN-REFERENCED NAVIGATION** is a technique in which measurements of height above terrain are combined with independent altitude/depth measurements and a map or database of terrain elevation to estimate position. Measurements of height above terrain can be made using altimeters, cameras, lidar/lasers, radar, or sonar [17].
- **CELESTIAL NAVIGATION** is a technique in which images of the stars or other celestial bodies, like the moon or artificial satellites, are compared to a database (often called a catalogue) to estimate position.

providing a position fix from an external source, such as GNSS.

Figure 1 is not an exhaustive list of all PNT options, but it summarizes the primary techniques that are useful for analyzing the potential impacts of quantum sensors compared to their classical counterparts. Note that some PNT approaches combine multiple different sensors or PNT techniques. For example, radionavigation and map/feature-matching techniques are often used to correct inertial drift; simultaneous localization and mapping (SLAM) [18, 19] incorporates principles of map/feature-matching and dead-reckoning techniques like inertial navigation or visual odometry using cameras, lidar, radar, and/or sonar (e.g., see [20, 21, 22]); and image-based, terrain-referenced, and celestial navigation using cameras, radar, and lidar/lasers require accurate pointing using tilt sensors and gyroscopes. Finally, note that radar and sonar can be used passively by receiving emissions from other sources, but for navigation they are more commonly used actively.

Tables 1 and 2 provide a deeper look at clocks, inertial sensors, and map/feature-matching techniques (including MagNav, GravNav, image-based/terrain-referenced navigation, and celestial navigation), some of which use either classical or quantum sensors. Radionavigation is also included in Table 2 since GNSS is ubiquitous and serves as the PNT "benchmark." The technologies listed in Tables 1 and 2 are not comprehensive but include some of the primary options for enabling each PNT technique.

# PROSPECTS FOR QUANTUM VS. CLASSICAL COMPLEMENTARY PNT CAPABILITIES

From the perspective of a user, the operation of classical and quantum sensors is identical: sensors transduce inputs such as acceleration, rotation, or external fields into outputs that can be used for PNT. Sensor quality describes the correspondence between input and output, regardless of the sensor internals. However, in areas where quantum sensors are predicted to offer better performance than classical sensors, the improvements derive from fundamental differences in operation between the two categories of sensor. This section outlines the projected value of quantum sensors and summarizes their potential impact on various complementary PNT techniques.

## Differentiating Value of Quantum Sensors

The primary advantage offered by many quantum sensors is the potential for *self-calibration*, meaning that quantum sensors could theoretically be operated without calibration by the user or reference to known inputs. In classical sensors, the relationship between input and output depends on engineered properties, which must be calibrated using known inputs and periodically recalibrated to account for aging and drift. For instance, the resonance frequency of a crystal oscillator depends on the crystal thickness, which can vary between oscillators due to manufacturing imperfections.

By contrast, quantum sensors relate inputs to outputs by probing the properties of quantum particles, which have quantized energy levels that can be individually addressed for initialization and measurement. Many quantum sensors are based on atoms, whose properties do not vary sensor-to-sensor and have a well-defined relationship to fundamental physical constants. To the extent that certain systematic shifts can be measured, such as the coupling of energy levels to other quantities such as ambient temperature, the sensor properties can often be determined to a high accuracy, which in turn enables improved accuracy and stability of the sensor without the need for regular recalibration. However, no quantum sensor can be perfectly self-calibrating, and the benefits of this characteristic will be limited in practice by imperfections in the classical components of quantum sensors (e.g., optical and electronic components such as lasers, photodetectors, and frequency counters) as well as any limitations on manufacturing tolerances in non-atom-based quantum sensors (e.g., superconducting quantum interference devices, or SQUIDs).

Table 1. Summary of classical and quantum technologies for timing and inertial navigation. See the list of acronyms in the glossary for reference.

| | **Types of Sensors/ Technologies** | **Maturity and Use Cases** | **Limitations on Technology/Technique** | **Future Trends/Projections** |
|---|---|---|---|---|
| Clocks | *Classical:* quartz crystal oscillators, electronic oscillators, MEMS oscillators | Ubiquitous in low size, weight, power, and cost (SWaP-C) consumer products and electronics, automotive and industrial systems, aerospace and military systems, and communications systems. | Worse long-term (>10 s) frequency stability *cf.* atomic clocks. Sensitive to crystal aging (frequency drift). | MEMS oscillators are disrupting the crystal oscillator market as their performance continues to improve [23] [24] [25], and they could be competitive with chip-scale atomic clocks (CSACs) with further development. |
| | *Quantum:* atomic clocks (microwave and optical) | Microwave atomic clocks are standard in satellite navigation (satnav), communications, and network timing protocols.<br>Microwave and optical clock R&D include basic research and early prototyping efforts to improve performance and reduce SWaP. | Worse short-term (<1-10 s) stability than crystal oscillators and thus are often paired with crystal oscillators to take advantage of the stabilities of each. | Lower-SWaP-C atomic clocks, like CSACs, have been proposed and analyzed for use in proliferated LEO constellations [14] [26]. With further SWaP reduction, optical clocks are expected to offer improvements in stability *cf.* microwave clocks. |
| Inertial Sensors | *Classical accelerometers:* mechanical (pendulous, vibratory, quartz resonator, PIGA), optical, MEMS<br>*Classical gyroscopes:* mechanical (spinning-mass/DTG, vibratory, resonator, HRG), optical (RLG, FOG), MEMS | Ubiquitous inertial sensors satisfying low-end commercial needs (such as smartphones and gaming devices) through high-end military, aerospace, and marine applications. | Complex mechanical sensors (e.g., PIGA) contain machined parts and are therefore expensive to make and can be unreliable. Most MEMS sensors are currently limited to low-end consumer applications. RLGs are notoriously complex and can be unreliable. | Classical inertial sensors are unlikely to dramatically increase in long-term stability, but reductions in SWaP-C and reliability improvements are likely.<br>Investment in MEMS sensors is resulting in improved stability and MEMS sensors reaching higher in the inertial performance spectrum [27] [28] [29] [30], although they are not expected to verge on the projected performance of quantum inertial sensors. |
| | *Quantum accelerometers:* atomic interferometer, atomic vapor<br>*Quantum gyroscopes:* atomic interferometer, NMR, solid-state defect | Laboratory research and prototypes under development. | Unclear if quantum gyros will ever surpass classical gyros in practical systems.<br>A future performance limiter for high-performance inertial navigation may be gravitational effects and the need for accurate gravity models/ measurements. | Linear accelerometry measurements are likely to maintain better long-term stability than classical mechanical accelerometers.<br>Overall performance may not provide significant enhancements beyond comparable-SWaP classical systems but could provide cost reduction per unit. Could be coupled with classical inertial sensors to take advantage of their short-term stability. |

In addition to the potential for near-self-calibration, quantum sensors may offer one or more of the following advantages over classical devices as they are developed further:

i. *Reduced size:* The fundamental limit on the size of the sensing element is set by the spatial extent of the quantum particles, so nanometer-scale volumes are possible in principle. However, the size of the full sensor system will be set by the supporting components (e.g., optical, electronic, cryogenic, and/or vacuum systems).

Table 2. Summary of classical and quantum sensors/technologies and the position-fixing techniques they enable. The discussion of image-based navigation will remain high level and not include topics such as visual odometry or SLAM, which operate on different principles than techniques like MagNav and GravNav and have become their own fields outright, especially in the context of autonomous vehicles (e.g., see [18, 19, 31, 32, 33, 34, 35]).

| | **Types of Sensors/ Technologies** | **Maturity and Use Cases** | **Limitations on Technology/Technique** | **Future Trends/Projections** |
|---|---|---|---|---|
| Radionavigation | *Classical:* radio antennas and receivers/ transceivers<br>*Note: potential quantum solutions exist (e.g., Rydberg atom electric field sensors), but this is not an active area of R&D.* | GNSS is the gold standard for globally available, all-weather PNT, providing a 3D position solution plus time. Other radionavigation techniques (signals of opportunity, terrestrial radionavigation) can serve specialized purposes in the absence of or as a complement to GNSS. | Susceptible to signal obstruction and interference. Expensive infrastructure (satellite constellations, terrestrial transmitters, control segments, etc.). Navigation using signals of opportunity requires knowing transmitter locations, which may not always be available. | GNSS will continue to be the PNT benchmark. With the increase in proliferated LEO constellations and terrestrial transmitters, the use of signals of opportunity for navigation and timing will increase. |
| MagNav | *Classical:* fluxgate, magneto-inductive/ resistive | Fluxgate magnetometers are likely to be useful in removing platform noise but currently do not have sufficient accuracy and stability to serve as the primary sensor for navigation [36]. | MagNav requires a magnetic map of the operating area, and performance is limited by factors including map resolution. MagNav has degraded navigation performance when magnetic field gradients are small ("dead zones"). Only provides a horizontal (2D) position fix. | Challenges with map availability/resolution and platform field effects must be addressed before this technique can be widely deployed. Long-term stability and accuracy offered by quantum magnetometers likely to be required for magnetic map creation and navigation. Complex installation and integration of sensors on host platforms. |
| | *Quantum:* atomic, solid-state, superconducting/ SQUID | Proof-of-principle tests demonstrated on aircraft with atomic magnetometers [36]. SQUIDs unlikely to be used due to higher SWaP. | | |
| GravNav | *Classical:* mechanical or optical accelerometer, MEMS, falling corner cube | Gravimeters and gravity gradiometers are commonly used in submarines to aid in inertial navigation and provide position fixes [17]. | GravNav requires a gravitational map of the operating area, and performance is limited by factors including map resolution. GravNav has degraded navigation performance when gravitational acceleration gradients are small ("dead zones"). Only provides a horizontal (2D) position fix. | Challenges with map availability/resolution must be addressed before this technique can be widely deployed. Long-term stability and accuracy offered by atomic gravimeters and gravity gradiometers likely to be advantageous for GravNav. Complex installation and integration of sensors on host platforms. |
| | *Quantum:* atomic, superconducting/SQUID | Preliminary tests of atomic interferometers have been performed in relevant environments [37]. | | |
| Image-based and Terrain-referenced Navigation | *Classical:* cameras, lidar/lasers, radar, sonar<br>*(No practical quantum sensor solutions)* | Cameras, lidar/lasers, and radar are commonly used in aircraft and missiles for navigation and terrain collision avoidance. Sonar is common in surface vessels and underwater platforms for navigation and obstacle avoidance. Can provide 2D or 3D position fixes. | Performance is limited by image/terrain feature variation (e.g., vision-based navigation does not work in darkness); map/database availability, quality, and resolution; and platform velocity and height/depth. | Heavy investment in feature-tracking and map-matching algorithms and AI/ML approaches for image-based navigation. Higher-precision terrain elevation mapping for higher-precision navigation applications. Complex installation and integration of sensors on host platforms. |
| Celestial Navigation | *Classical:* cameras<br>*(No practical quantum sensor solutions)* | Frequently used on low-dynamic aircraft, surface vessels, and underwater platforms (when surfaced) for position-fixing. | Requires clear view of sky. Degraded performance during daytime or under cloud cover. Only provides a horizontal (2D) position fix. | Celestial navigation using artificial satellites and other resident space objects (RSOs) can improve position-fixing accuracy compared to using stars only. Complex installation and integration of sensors on host platforms. |

ii. *Simultaneous sensing of multiple inputs:* Some quantum sensors could offer simultaneous measurement of multiple inputs or rejection of systematic shifts enabled by comparing the relative shifts of different energy states. These protocols are similar in concept to co-locating multiple classical sensors but instead arise naturally and may enable reduced complexity for sensing multiple quantities (e.g., vector magnetic fields and temperature [38, 39]).

iii. *Entanglement-enhanced performance:* In principle, quantum sensors can make use of entangled states, where an ensemble of quantum particles becomes collectively (rather than individually or independently) sensitive to sensor inputs, which can improve the signal-to-noise ratio of the measurement. Though proposals exist for entanglement-enhanced quantum PNT sensors (e.g., see [40]), entangled states are easily destroyed by interactions with the environment and are difficult to prepare, increasing sensor SWaP and complexity. Consequently, entangled states are unlikely to provide a significant practical performance enhancement for many PNT systems.

Table 3 provides a summary of the differentiating value of some of the most promising quantum sensors for PNT and their use cases. It also highlights some of the current or projected barriers to deployment. In many cases, the primary challenges are in sensor miniaturization and ruggedization, which will require advances in laser miniaturization, photonic integration, and magnetic shielding, among other technologies [41].

Table 3. Summary of the differentiating value and challenges facing deployment of quantum sensors for PNT.

| **Quantum Device** | **Maturity** | **Differentiating Value *cf.* Classical Devices** | **Most Promising Use Cases for PNT** | **Current/Potential Key Barriers to Deployment** |
|---|---|---|---|---|
| Microwave atomic clock | Commercially available | Improved long-term stability and frequency accuracy; less aging and temperature dependence. | Long-time holdover in the absence of satnav or communications link. | Some emerging microwave (e.g., cold atom, ion) clocks require further SWaP-C reduction and ruggedization. |
| Optical atomic clock | Advanced research | Improved long-term stability and frequency accuracy (compared to both classical devices and microwave clocks), less aging and temperature dependence. | Long-time holdover in the absence of satnav or communications link. | SWaP-C reduction and ruggedization, especially of lasers/frequency combs. |
| Atom interferometer-based inertial sensor | Advanced research/ Early prototypes | Improved long-term stability and accuracy; fewer precision-machined components and therefore potentially better reliability and lower SWaP-C. | Navigation-grade through strategic-grade inertial navigation. | Continued improvements in ring laser and fiber optic gyros may outpace development of atomic inertial sensors. |
| Atom interferometer-based gravimeter/ gravity gradiometer | Early commercial prototypes available | Improved long-term stability and accuracy; no need to recalibrate at new locations; fewer precision-machined components and therefore potentially better reliability and lower SWaP-C. | Gravitational anomaly map-making, GravNav, gravitational measurements to support high-performance inertial navigation. | Further improvements in overall system ruggedization. |
| NMR Gyroscope | Early prototypes | Low-SWaP; fewer precision-machined components; low noise and high scale factor stability; some can provide simultaneous magnetic field measurement. | Low-SWaP inertial sensing and high-rotation-rate PNT applications. | Continued improvements in classical IMUs may outpace development of NMR gyros. |
| Atomic Magnetometer | Commercially available | Improved long-term stability and accuracy. | Magnetic map-making and MagNav. | Developing techniques for overcoming platform effects may require advances in vector magnetometers. |

## Current and Potential PNT Performance

The selection of the optimal complementary PNT techniques and technologies for a given application will ultimately depend on a systems-level analysis including performance, SWaP-C, and other factors like platform integration and installation. To better understand the potential tradeoffs between quantum and classical technologies and associated PNT techniques, Figures 2 through 4 provide summaries of key metrics that can contribute to a systems-level analysis of various complementary PNT options.

Figure 2 provides a visual representation of the approximate long-term performance of various clocks and oscillators after one day of free-running operation as a function of the unit cost. In general, SWaP increases with performance and cost. Classical oscillators currently have more drift than atomic clocks, but as MEMS technologies are developed further, they may become competitive with low-SWaP atomic clocks like CSACs. In parallel, there are ongoing efforts to reduce the cost of CSACs [42], which could keep the low-SWaP atomic clock market competitive with improving MEMS systems. At the other end of the performance spectrum, emerging atomic clocks are expected to have improved long-term stability, with research focused on reducing SWaP.

Figure 3 summarizes different IMU performance grades as a function of cost. Again, SWaP tends to increase with performance and cost. The types of accelerometers and gyroscopes that constitute various IMU grades are also shown with dashed lines. While there is no single agreed-upon taxonomy of inertial sensor grades and their applications, they can generally be described as follows:

- Consumer grade, which is used for automobiles and consumer electronics like smartphones and gaming devices;
- Industrial grade, which is used for small UASs and robotics;
- Tactical grade, which is used for UASs and guided weapons;
- Navigation/aviation grade, which is used for commercial and military aircraft; and
- Marine/strategic grade, which is used for ships and submarines, long-range missiles, and spacecraft.

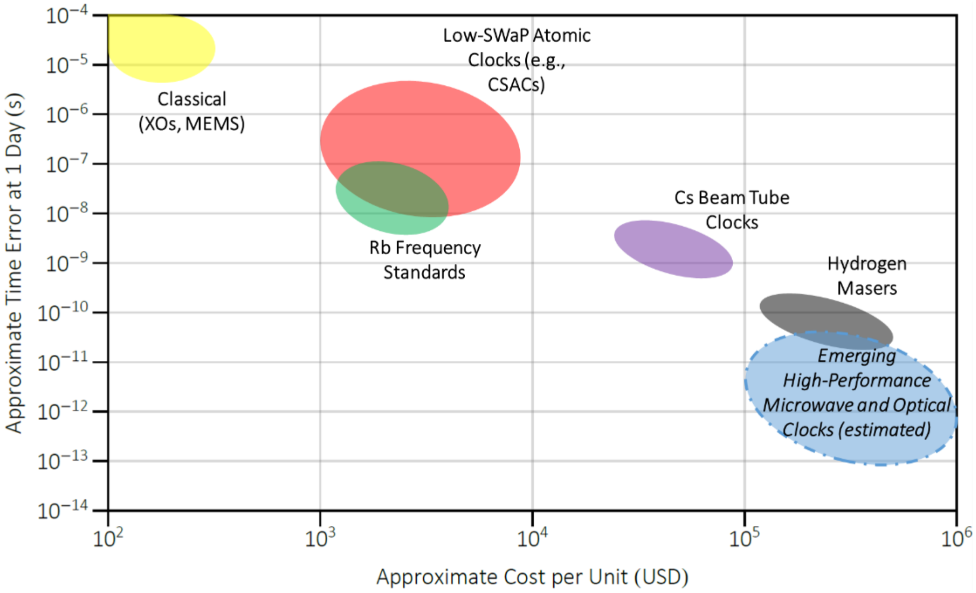

Figure 2. ***Atomic clocks and classical oscillators***. Approximate time error after one day of free-running operation of various clocks and frequency standards assuming no environmental perturbations as a function of the approximate cost per clock/oscillator. The time error metrics for this bubble chart were generated based on Ref. [5] and the cost metrics were estimated based on Refs. [43, 44, 45, 46]. There are also classical oscillators with lower cost (less than a dollar) and poorer performance than what is included on this chart. Actual costs vary over time depending on demand and manufacturing requirements, and thus certain clocks/oscillators may have cost metrics that fall outside these bubbles. In addition, purchasing single clocks vs. purchasing in bulk can cause price variations.

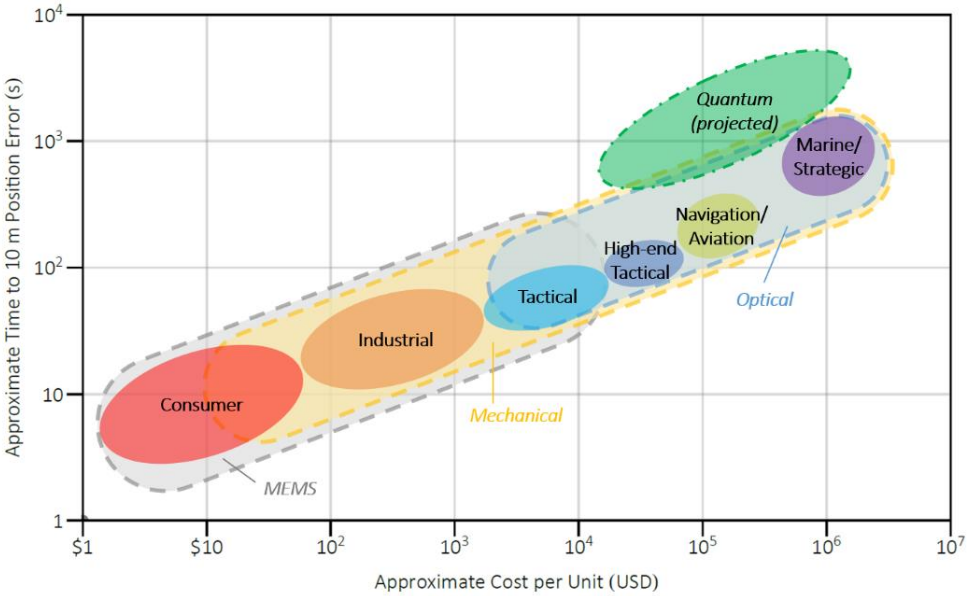

Figure 3. ***Inertial sensors.*** Approximate time until an inertial navigation system accumulates 10 m of position error as a function of the approximate cost per inertial measurement unit. This bubble chart was generated based on Refs. [8, 27, 28, 29, 30, 17, 47].

Some quantum inertial sensors are expected to achieve performance that is comparable to high-end navigation and strategic-grade systems at a lower SWaP-C, which is perhaps the strongest value proposition for these sensors.

Figure 4 summarizes the approximate positioning accuracies that can be achieved with various position-fixing techniques. GNSS currently offers the best accuracy, but active radiofrequency and vision-based techniques employing cameras, radar, and lidar are also highly accurate. Among the complementary PNT techniques shown in Figure 4, MagNav and GravNav are the only ones that are likely to employ quantum sensors (though future Rydberg atom electric field sensors may enable improved accuracies for certain use cases like terrestrial radionavigation). While these techniques may not be suitable for all platforms, MagNav and GravNav have three key advantages over some of the other position-fixing techniques.

Firstly, they use passive sensors, making them attractive for applications in which covertness is important. Secondly, they are difficult to jam or spoof, especially compared to radiofrequency techniques like radionavigation and radar.

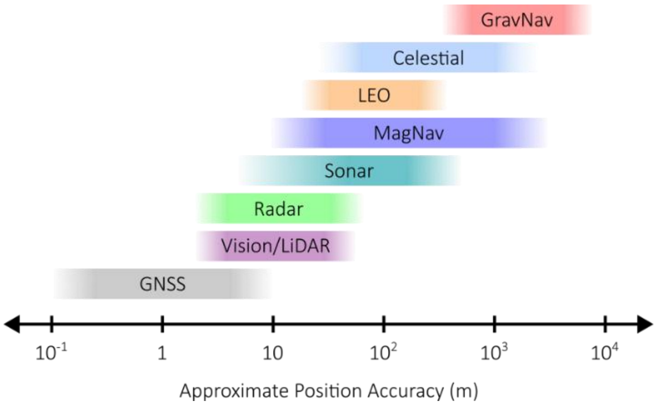

Figure 4. ***Position-fixing techniques.*** The approximate range of position accuracies for various position-fixing techniques (when they are available). This bar chart was generated based on the following references: GNSS [17, 48, 49, 50]; MagNav [51, 52, 53, 54, 55, 56, 57, 58, 6]; GravNav [17, 59, 60, 61]; Celestial [17, 51, 62, 63, 64]; Vision/lidar [17, 51, 65, 66, 67, 68]; Radar [17, 69, 70, 71, 72]; LEO signals of opportunity [17, 73, 74, 15, 16]; Sonar [17, 75, 76, 77]. Note: Many of these techniques are employed in tandem with inertial navigation to bound inertial drift. Some provide 3D positioning while others only provide 2D.

Thirdly, they can function over terrain that is flat or otherwise visually sparse and provide all-weather operation, whereas cameras used for image-based navigation or celestial navigation require an unobstructed view of the ground or stars, making them less suitable for use over open water or during daytime or cloud cover. (Similarly, lidar/lasers, radar, and sonar rely on detectable, discernable features in their respective "fields of view.") Magnetometers and gravimeters/gravity gradiometers are not subject to these same limitations, although the ultimate positioning accuracy will depend on the density of features available on the respective magnetic and gravitational maps and on the magnitude of the anomaly gradients. In addition, the ability to resolve features on a given map will depend on the velocity and altitude/depth of the platform. For example, fast-moving, low-flying platforms will observe higher magnetic or gravitational anomaly gradients than slow-moving, high-flying platforms, enabling improved accuracy.

Quantum atomic and solid-state magnetometers and atomic gravimeters/gravity gradiometers are likely to be useful in generating high-quality maps for MagNav and GravNav, respectively, owing to their high accuracies and good long-term stabilities. Once high-quality magnetic and gravitational anomaly maps are generated, the quality of the primary sensor employed for navigation may be less important, though the tolerable accuracy bounds may still mandate use of a quantum sensor.

When assessing different PNT techniques for a particular application, it is important to consider overall system cost, not just sensor cost. Factors like platform integration and installation, creating or accessing maps or feature databases, and service subscriptions (e.g., to a LEO satellite communications system) can all contribute to overall cost. In particular, platform integration and installation may pose barriers to deployment of a PNT technique, especially for existing platforms. These barriers may include mechanical installation (physically mounting the sensor on the platform) and/or data integration (integrating the sensor measurements into the navigation system).

It is also important to understand that some position-fixing techniques only provide 2D fixes, thus requiring fusion with other techniques if a 3D fix is desired. For example, MagNav may be deployed on aerial platforms with the aid of a barometric or radar altimeter, while GravNav may be deployed on underwater platforms with the aid of a depth/pressure sensor. In fact, many map/feature-matching techniques are commonly paired with other techniques that provide altitude or depth information.

## Perennial Classical Solutions

There are some applications in which quantum sensors for PNT are unlikely to ever replace their classical counterparts. Understanding these applications and the prospects for quantum sensors are critical to prioritizing future research and development and to selecting the complementary PNT solutions that best fit the need for various use cases.

Crystal oscillators will likely continue to remain the technology of choice in proliferated satnav receivers. For most familiar applications and platforms, such as civil aircraft, automobiles, cell phones, and other consumer electronics, the receiver clock is not the performance limiter. Other sources of error, such as satellite clock and ephemeris errors, atmospheric effects, multipath, and radiofrequency interference tend to dominate the effects of receiver clock instabilities, so using a better receiver clock will not yield improved navigation and timing accuracy. However, in certain high-precision satellite navigation applications such as surveying or remote sensing where these other sources of error can be mitigated or eliminated (e.g., by performing carrier phase ambiguity resolution with differential corrections), or when long time holdover periods are required, improving the receiver clock performance can provide enhanced positioning and timing accuracy [78].

Furthermore, MEMS oscillators are making rapid advances and now achieve stability and accuracy levels that are competitive with high-end crystal oscillators and may be competitive with low-end atomic clocks as they are developed further. The key advantages of MEMS oscillators over crystal oscillators include their extremely small size, complementary metal-oxide semiconductor

(CMOS) compatibility, strict manufacturing process control, ruggedness and reliability, and multi-frequency functionality on a single chip [23, 24, 25]. While recent advances in MEMS oscillators have made them comparable with crystal oscillators in terms of short-term stability, neither is likely to approach the long-term stability of high-end atomic clocks.

Classical inertial sensors (mechanical, optical, and MEMS) will likely continue to remain the technology of choice for inertial navigation applications requiring consumer through tactical grade performance levels. These lower-end classical inertial sensors will continue to fulfill PNT needs in automobiles, consumer electronics like cell phones and gaming devices, and robots for industrial applications, as well as tactical military platforms such as UASs and guided munitions. Depending on the future cost and practical performance of quantum inertial sensors, classical inertial sensors may remain the predominant solution for higher-end navigation applications as well. In addition, as is the case with atomic clocks utilizing crystal oscillators to take advantage of their short-term stabilities, quantum inertial sensors are also likely to require integration with classical inertial sensors to capture rapid platform accelerations/vibrations and improve short-term performance.

Lastly, Figure 4 shows that there are many viable position-fixing techniques that do not rely on quantum sensors, including image-based navigation, terrain-referenced navigation, and celestial navigation. These navigation techniques are likely to remain common solutions for airborne, marine, and submarine platforms due to the accuracy they can provide and the high maturity of technologies like cameras, radar, lidar/lasers, and sonar. While it is certainly not a given that these will remain “perennial solutions,” it is important to understand the advantages and disadvantages of techniques like MagNav and GravNav in light of well-established techniques that use classical sensors.

## OUTLOOK

While certain quantum technologies, such as atomic clocks, will continue to be deployed broadly, it is unclear to what extent other quantum sensors will impact the broader complementary PNT ecosystem. Quantum inertial sensors may provide improved long-term performance beyond current strategic-grade inertial navigation systems, but their practical benefit will likely depend on whether they can offer a reduced cost. Atomic and solid-state magnetometers are likely the most promising sensors for MagNav and generating magnetic maps owing to their long-term stability, but the field of MagNav itself requires further advances in signal processing, platform calibration, and support for generating high-quality magnetic maps. Atomic gravimeters and gravity gradiometers offer prospects for more reliable long-term operation than classical gravitational sensors, but their practical performance must be assessed further in relevant environments to determine whether they can support GravNav with sufficient position accuracies.

Position-fixing techniques such as MagNav and GravNav are attractive because they are passive and difficult to jam/spoof, but other all-classical passive techniques such as image-based/terrain-referenced navigation, celestial navigation, and radionavigation using signals of opportunity also continue to improve. In addition to positioning accuracy and sensor SWaP-C, it is also important to evaluate factors like overall system cost and platform integration and installation when considering different PNT techniques for particular applications. Past and present market and R&D trends and the heavy investment in complementary PNT techniques and technologies suggest the key takeaways outlined on the next page. As complementary PNT techniques and technologies continue to advance, it will be crucial to assess them holistically to determine the optimal sensor suite for a given application, regardless of whether those sensors are quantum or classical.

## KEY TAKEAWAYS

i. Atomic clocks, both microwave and optical, will continue to advance and provide improvements in long-term stability. They will continue to dominate applications such as satellite navigation, communications, and network timing protocols. Many atomic clocks will continue to be paired with crystal oscillators to ensure good short-term stability.

ii. Quantum inertial sensors are in early prototype stages, and their main value proposition is for high-performance inertial navigation at reduced SWaP-C relative to classical inertial sensors. While they may provide performance enhancement beyond strategic grade, they are unlikely to "change the game" in the way that atomic clocks have for timing.

iii. Atomic magnetometers, which are mature and commercially available, and solid-state magnetometers, which are in the prototype stages, are both promising for MagNav due to their enhanced long-term stability and accuracy. MagNav performance is currently limited by factors like platform effects/calibration and map availability/resolution, rather than by sensor quality. In the near-term, quantum magnetometers can be used for high-quality magnetic field map-making to facilitate future MagNav.

iv. Quantum gravimeters/gravity gradiometers are in advanced prototype stages and are promising for GravNav due to their long-term stability and accuracy. While current GravNav performance is generally limited by map availability/resolution and not by sensor quality, quantum gravimeters could be used for high-quality gravitational map-making to facilitate future GravNav. They could also be used in strategic-grade inertial navigation by providing gravitational acceleration information to the INS.

v. MagNav and GravNav are just two of several position-fixing techniques. Other map/feature-matching techniques that use classical sensors such as image-based navigation, terrain-referenced navigation, and celestial navigation are highly mature and are being actively improved to fill critical complementary PNT needs. Furthermore, there is increasing interest in radionavigation using LEO or terrestrial signals of opportunity. The main advantages of MagNav and GravNav over these other techniques is that they use passive sensors, can function in all weather and over sparse terrain, and do not rely on radio signals that can be degraded or obstructed. However, challenges like platform integration and installation must also be considered.

# GLOSSARY OF TECHNOLOGIES

| | **Keyword** | **Definition** |
|---|---|---|
| **Timing** | Oscillator | Anything that produces a stable, periodic signal, e.g., a pendulum, a pulsar, a feedback circuit containing a piezoelectric crystal, or a laser/microwave source stabilized to an atomic reference. |
| | Counter | A device that counts or accumulates the periodic signal of an oscillator. |
| | Clock | A device that consists of an oscillator and a counter. |
| | Crystal Oscillator | A device in which a periodic input voltage is applied across a crystal, which distorts and generates its own periodic voltage in response. When the periodic input voltage is tuned to the resonant frequency of the crystal, a stable periodic signal is produced. |
| | Atomic Clock | A device in which an oscillator's frequency is stabilized to the energy difference between two atomic states, producing a stable periodic signal. |
| **Accelerometry** | Accelerometer | A device that measures non-gravitational acceleration (i.e., specific force). |
| | Gravimeter | A device that measures the acceleration due to gravity. |
| | Atomic Interferometer | A device that measures accelerations using atoms experiencing inertial inputs. Using laser pulses that transfer momentum to the atoms, an atomic cloud is separated and then recombined after a free evolution period. After recombination, the relative populations in two internal states of the atoms depend on the positions of atom/laser interactions and the velocities of the atomic clouds, quantities that depend on gravity, external accelerations, and gravity gradients. |
| | Classical Accelerometer | A device in which a "proof mass" is suspended in a case by springs, a pendulum, a viscous fluid, or magnetic forces. When a force is applied to the case, the resulting displacement of the proof mass relative to the case or the restoring force required to hold the proof mass at center is measured. |
| | Gyroscope | A device that measures orientation, rotations, or angular velocity. |
| | Atomic Interferometer-based Gyroscope | A device that measures rotations using atoms. Using laser pulses that transfer momentum to the atoms, an atomic cloud is separated and then recombined to form a closed loop in space. The populations in each internal atomic state depend on the phase of the laser at the positions of atom/laser interaction. Rotations change the phase at the interaction locations, so the internal state of the atoms can be measured and converted to the rotation rate. |
| | NMR Gyroscope | A device that measures external rotations using an atomic vapor by detecting variation in the rate of Larmor precession, the rotation of an atom's magnetic dipole around an external magnetic field. The transmission of a laser beam through the atomic vapor depends on the orientation of the atomic dipoles relative to the laser polarization, so monitoring the laser transmission and comparing its modulation to the predicted rotation rate allows measurement of the external rotation rate. |
| | Classical Gyroscope | Classical gyroscopes come in three main types:<br>• Spinning-mass gyros rely on the principle of conservation of angular momentum to measure rotations. When a torque is applied to an axis perpendicular to the axis of rotation of the spinning mass, the mass rotates about the third axis, perpendicular to both the axis of rotation and the axis of the applied torque.<br>• Vibratory gyros rely on detecting the Coriolis acceleration of a vibrating element, such as a string, a beam, or a tuning fork. When a vibratory gyro is rotated along an axis perpendicular to the axis of vibration, it results in a measurable Coriolis acceleration in the third axis, perpendicular to both the axis of vibration and the axis of rotation.<br>• Optical gyros rely on the Sagnac effect to measure rotations. Two beams of coherent light are sent in opposite directions around a closed path, for example through a network of mirrors or an optical fiber. Rotation about the axis perpendicular to the plane of the closed path increases the propagation distance of one beam relative to the other. The resulting interference pattern between the two counterpropagating beams at the detector can be observed and used to measure rotation. |

| | Keyword | Definition |
|---|---|---|
| **Magnetometry** | Magnetometer | A device that measures an external magnetic field. Vector magnetometers measure one or more components of the magnetic field, while scalar, or total-field, magnetometers measure the magnitude of the magnetic field. |
| | Atomic Magnetometer | A device that measures magnetic field by probing the field-dependent energy difference between two internal states in atoms. The transmission of polarized laser light through an atomic vapor depends on the populations in each state, which can be modified by, for example, applying resonant radiofrequency radiation. By monitoring the phase or amplitude of variations in the light transmission, the applied frequency is stabilized to this energy difference. |
| | Solid-state defect (e.g., NV diamond) Magnetometer | A device that measures magnetic field by probing the field-dependent energy difference between multiple internal states in solid-state quantum particles. The emitted fluorescence from the particles under laser illumination depends on the populations in each state, which can be modified by applying resonant microwave radiation. By monitoring the fluorescence and varying the microwave frequency, the frequency difference between states can be measured and converted to magnetic field. |
| | SQUID Magnetometer | A device that senses magnetic flux (the product of magnetic field and sensor area) in a superconducting ring split into two regions by thin non-superconducting regions. The voltage drop measured across the non-superconducting regions depends on the magnetic flux and fundamental constants. |
| | Fluxgate Magnetometer | A device that senses vector magnetic fields by measuring magnetic saturation, a limit on the magnetization of certain materials as a function of external magnetic field. Reductions in the applied field needed to induce saturation correspond to the presence of additional external fields. |

## LIST OF ACRONYMS

| | |
|---|---|
| AI/ML | Artificial intelligence/machine learning |
| CSAC | Chip-scale atomic clock |
| DTG | Dynamically-tuned gyroscope |
| FOG | Fiber-optic gyroscope |
| HRG | Hemispherical resonator gyroscope |
| IMU | Inertial measurement unit |
| INS | Inertial navigation system |
| LEO | Low-earth orbit |
| MEMS | Micro-electromechanical system |
| PIGA | Pendulous integrating gyroscopic accelerometer |
| RLG | Ring laser gyroscope |
| RSO | Resident space object |
| SQUID | Superconducting quantum interference device |
| SWaP-C | Size, weight, power, and cost |
| UAS | Unmanned aerial system |
| UTC | Coordinated Universal Time |
| XO | Crystal oscillator |

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